

Lecture 12: Loudness, Decibels, Logarithms

Let us continue with how loudness is perceived and how it is characterized.

Last time we saw that a louder sound will stimulate more nerve cells to fire, but not in direct proportion to the sound's intensity, because of the way the nerves attach to the hair cells.

Besides this effect, one must also consider what the outer hair cells do. This is hard, because their role is not fully understood. However, it appears that, rather than *transmit* nerve signals, they actually *receive* nerve signals and react to them by moving their stereocilia (hairs). That is, they actually push and move around the tectoral membrane, which their hair cells are attached to. Their role seems to be,

- to amplify the motion of the membranes in very soft sounds, extending the sound threshold down to quieter sounds,
- to change the tension of the membranes, in a way which narrows the region of the cochlea responding to a given frequency (that is, narrowing the critical band).

[That is, they appear to be an element in an active feedback loop, which increases the resonant quality factor of the membranes, so the resonance caused by a sound wave is narrower and larger at its center.] People have studied this role by doing experiments on living and dead cochleas, and on ones where one or the other type of hair cell has been damaged.

DeciBels

Because the range of intensities which the ear can respond to is so large, it is most practical to describe it using logarithms. This is different than our using logarithms to describe frequencies, where it really directly described the way we perceive the frequencies. For loudness, it is just a convenient way of compressing the scale into an easy to handle range.

It has become standard to use a unit called the **decibel** to describe sound loudness. The definition and its use are a little confusing, so I will spend some time on this.

One defines a **Bel** to be,

$$\# \text{ of bels} = \log_{10} \frac{\text{something}}{\text{some standard}}.$$

Then a **decibel** (**dB**) is defined to be,

$$\# \text{ of deciBels} = 10 \times \# \text{ of Bels}$$

This definition is used in many applications, which is why I have written it in such a vague way.

In this course we will only use dB to talk about intensities of sounds (and power of sound producers, but I won't emphasize that application). For this context, "something" will be sound intensity, and we need some standard intensity to use as a basis for comparison. People have chosen as a standard against which to compare sound intensities, the fixed intensity of 10^{-12} W/m^2 , because it is the softest sound your ear can hear when there are no other sources of sound present. (Actually you cannot quite hear that small a sound, but it is too late to change the standard.) Therefore, in sound, we define

$$\# \text{ of dB for a sound} = 10 \times \log_{10} \frac{\text{Intensity}}{10^{-12} \text{ W/m}^2}.$$

Some examples:

- The softest sound you can hear is $I = 10^{-12} \text{ W/m}^2$. This intensity, expressed in dB, is,

$$\text{loudness in dB} = 10 \times \log_{10} \frac{10^{-12} \text{ W/m}^2}{10^{-12} \text{ W/m}^2} = 10 \times \log_{10}(1) = 0 \text{ dB}.$$

- A very quiet sound you might encounter in music, say *pp* (pianissimo), might have an intensity of $I = 10^{-9} \text{ W/m}^2$. The dB rating would then be,

$$10 \times \log_{10} \frac{10^{-9} \text{ W/m}^2}{10^{-12} \text{ W/m}^2} = 10 \times \log_{10}(1000) = 10 \times 3 = 30 \text{ dB}.$$

- A normal speaking voice for a person-to-person conversation in a quiet environment typically uses an intensity of about $I = 10^{-6} \text{ W/m}^2$. In dB, we would describe this as,

$$10 \times \log_{10} \frac{10^{-6} \text{ W/m}^2}{10^{-12} \text{ W/m}^2} = 10 \times \log_{10}(1000000) = 10 \times 6 = 60 \text{ dB}.$$

Now let's learn how to go backwards, from dB to intensity. If you don't like derivations, skip to the answer at the end. [The derivation is, that since

$$\# \text{ dB's} = 10 \times \log_{10} \frac{I}{10^{-12} \text{ W/m}^2}$$

dividing by 10 and exponentiating both sides gives,

$$10^{\# \text{ dB's}/10} = \frac{I}{10^{-12} \text{ W/m}^2}$$

or,] the answer is,

$$I = 10^{-12} \text{ W/m}^2 \times 10^{\# \text{ dB's}/10}$$

For instance, the acoustic reflex starts to kick in at a loudness (roughly *ff* fortissimo) of about 85 dB. What is the intensity?

$$I = 10^{-12} \text{ W/m}^2 \times 10^{85/10} = 10^{-12} \times 10^{8.5} = 10^{-12+8.5} = 10^{-3.5} = .00031 \text{ W/m}^2.$$

You should be able (try it!) to take the dB results from the previous examples and work backwards to get the intensities back out.

quick and dirty way: there are 10 dB for each factor of 10 above 10^{-12} W/m^2 . For instance, consider 10^{-8} W/m^2 . Starting with 10^{-12} W/m^2 , you have to multiply by 10 4 times to get up to 10^{-8} ; each of those is 10 dB, so you get 40 dB.

It is even easier to find the *difference* between the number of dB of two sounds, of intensity I_1 and I_2 :

$$\# \text{ of dB difference} = 10 \times \log_{10} \frac{I_1}{I_2}.$$

Other things to know to do dB quickly:

$$10 \times \log_{10}(2) = 3 \quad (\text{good approximation})$$

$$10 \times \log_{10}(5) = 7 \quad (\text{good approximation})$$

$$10 \times \log_{10}(3) = 5 \quad (\text{OK approximation})$$

Using these, we see that $2 \times 10^{-7} \text{ W/m}^2 = 53 \text{ dB}$, for instance. The first one means that Doubling intensity adds 3 dB.

Beware: decibels are used as a simple way to describe the wide range of loudnesses we can hear. They *do not* indicate how loud a sound will “sound,” in any simple way.

60 dB does NOT sound twice as loud as 30 dB.

The dB rating is not just “how loud it sounds.” Rather, each extra 10 dB means the sound is 10 times as intense. The rule of thumb from last time means, that it is perceived to be (“sounds”) roughly 2 times as loud. Therefore, 60 dB is perceived to be about $2 \times 2 \times 2 = 8$ times as loud as 30 dB. In intensities, it is 1000 times louder.

To complicate matters further, sound producing power is also often measured in dB, where,

$$\text{Sound Power in dB} = 10 \times \log_{10} \frac{\text{Power in Watts}}{10^{-12} \text{ W}}.$$

This does *not* correspond to the intensity in dB that you will hear.

It is a good idea to do an example. Suppose you have a 300 W driven stereo system, and it is turned up “pretty high,” so that for a loud sound in the music, it will draw 1/10 of its rated power, or 30 W. It is about 10% efficient (which is actually very good) at converting this into sound. Therefore it is making 3 Watts of sound power.

Power output in dB:

$$\begin{aligned} 10 \times \log_{10} \frac{3 \text{ W}}{10^{-12} \text{ W}} &= 10 \times \log_{10}(3 \times 10^{12}) \\ &= 10 \times (\log_{10}(3) + \log_{10}(10^{12})) = 10 \times (0.5 + 12) = 125 \text{ dB} . \end{aligned}$$

Suppose you do what you should never do, and you put your head 1 meter from the speaker. What intensity do you hear?

If the speaker is on the ground, the sound is radiating out from it in all directions, over the hemisphere surrounding it. The area of the hemisphere at distance 1 meter is, $2\pi R^2 = 2\pi \times (1 \text{ m})^2 = 6.28 \text{ m}^2$. Therefore the intensity is,

$$I = \frac{3 \text{ W}}{6.28 \text{ m}^2} = 0.48 \text{ W/m}^2$$

which in dB is,

$$10 \times \log_{10} \frac{0.48 \text{ W/m}^2}{10^{-12} \text{ W/m}^2} = 117 \text{ dB}$$

near the threshold of pain.

How does the dB rating fall off with distance (in an open field, no reflections or sound absorbers)? Every factor of 2 in distance, the sound spreads over 4 times as much area. Since a factor of 2 in intensity is a reduction of 3 dB, a factor of 2 in distance is a reduction of $3 + 3 = 6$ dB. (Since dB arise as the logarithm of the intensity, dividing the intensity by something involves subtracting from the number of dB.)

Therefore we find,

117 dB at 1 meter

111 dB at 2 meters

105 dB at 4 meters

99 dB at 8 meters

93 dB at 16 meters

87 dB at 32 meters

81 dB at 64 meters

75 dB at 128 meters

69 dB at 256 meters

63 dB at 512 meters

This is the bad way, though it eventually works. It would go faster if I used that a factor of 10 in distance is a factor of 100 in the area, which will bring down the intensity by $1/100$. Since $10 \times \log_{10}(1/100) = -20$, this lowers the number of dB by 20. Therefore,

117 dB at 1 meters is

97 dB at 10 meters

77 dB at 100 meters

57 dB at 1000 meters

Where is the intensity that of normal conversation, 60 dB? Where

$$I = 10^{-12} \text{ W/m}^2 \times 10^{60/10} = 10^{-6} \text{ W/m}^2$$

but the intensity is,

$$I = \frac{\text{power}}{\text{area}} = \frac{3 \text{ W}}{2\pi R^2}$$

Equating them,

$$\begin{aligned}\frac{3 \text{ W}}{6.28 R^2} &= I = 10^{-6} \text{ W/m}^2 \\ \frac{3}{6.28} \text{ W} &= R^2 \times 10^{-6} \text{ W/m}^2 \\ \frac{3 \text{ W}/6.28}{10^{-6} \text{ W/m}^2} &= R^2 \\ 480000 \text{ m}^2 &= R^2\end{aligned}$$

which gives $R = 700$ m. Realistically, sound usually encounters obstructions and absorbers over that distance, which is also far enough for the air to absorb the highest frequencies. Still, I have heard drummers at the statue of Cartier in Parc Mount Royal while walking on the trail on the top, at about this distance, and I have heard a bagpipe band in a valley while on a hill overlooking them at a comparable distance. So the answer is reasonable.

To re-iterate:

$$\# \text{ of dB's} = 10 \times \log_{10} \frac{I}{10^{-12} \text{ W/m}^2}, \quad I = 10^{-12} \text{ W/m}^2 \times 10^{\# \text{ of dB}/10}$$

- Multiplying or dividing the intensity by N adds or subtracts $10 \times \log_{10}(N)$ to the number of dB.
- Doubling or halving the intensity adds or subtracts 3 dB.
- Doubling distance from a source reduces the intensity by a *factor* of 4, subtracting 6 from the number of dB.
- The number of dB's is *not* simply proportional to the perceived loudness. Rather, each difference of 10 dB is an actual change in intensity of a *factor* of 10, and a difference in perceived intensity of *about* a *factor* of 2.