Beats

When two notes are played at once, the intensities add:

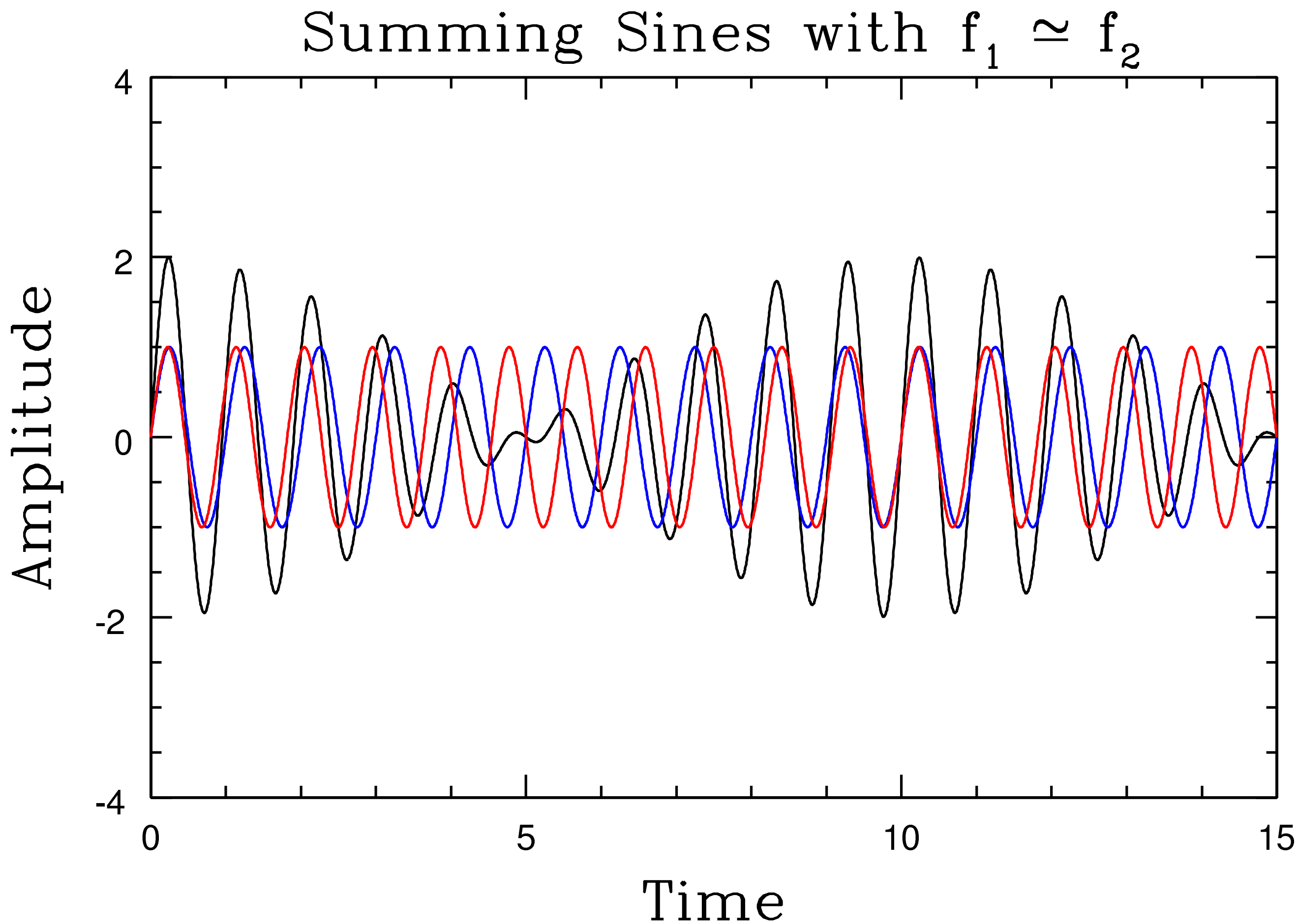
$$I_{\text{tot}} = I_1 + I_2$$

But when the frequencies are almost the same, it happens in a very strange way!

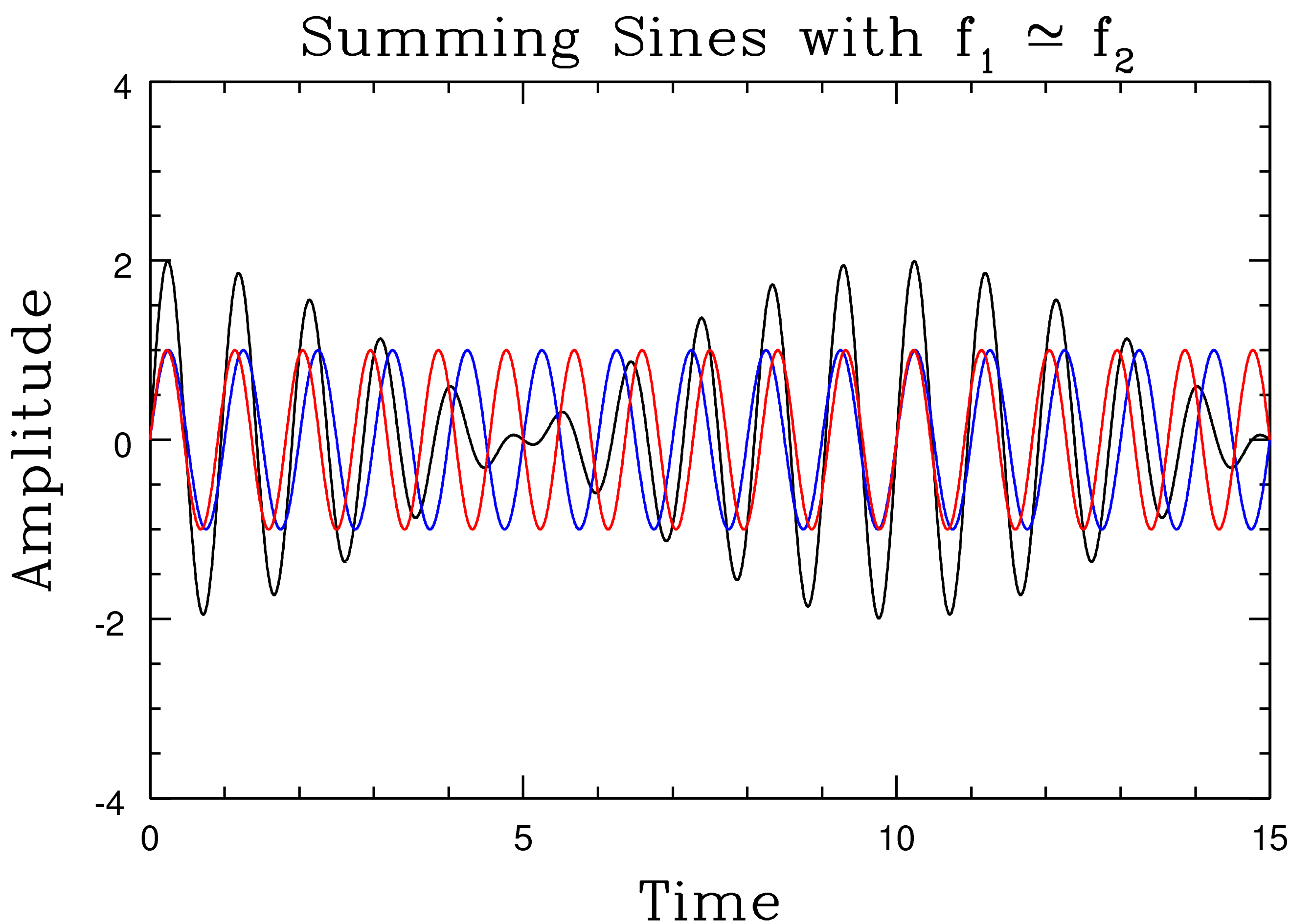
Loudness oscillates–sounds alternately louder and softer.

This phenomenon is called "beating" or "beats".

Why do beats happen? Consider adding sine waves:



Red wave gets ahead of blue wave. Where they equal–they add. When they are opposite–they subtract.



Consider two sines of frequency $f + \frac{\Delta f}{2}$ and $f - \frac{\Delta f}{2}$–that is, average f and difference Δf. Add them:

$$\sin\left[2\pi f t + \pi \Delta f t\right] + \sin\left[2\pi f t - \pi \Delta f t\right]$$

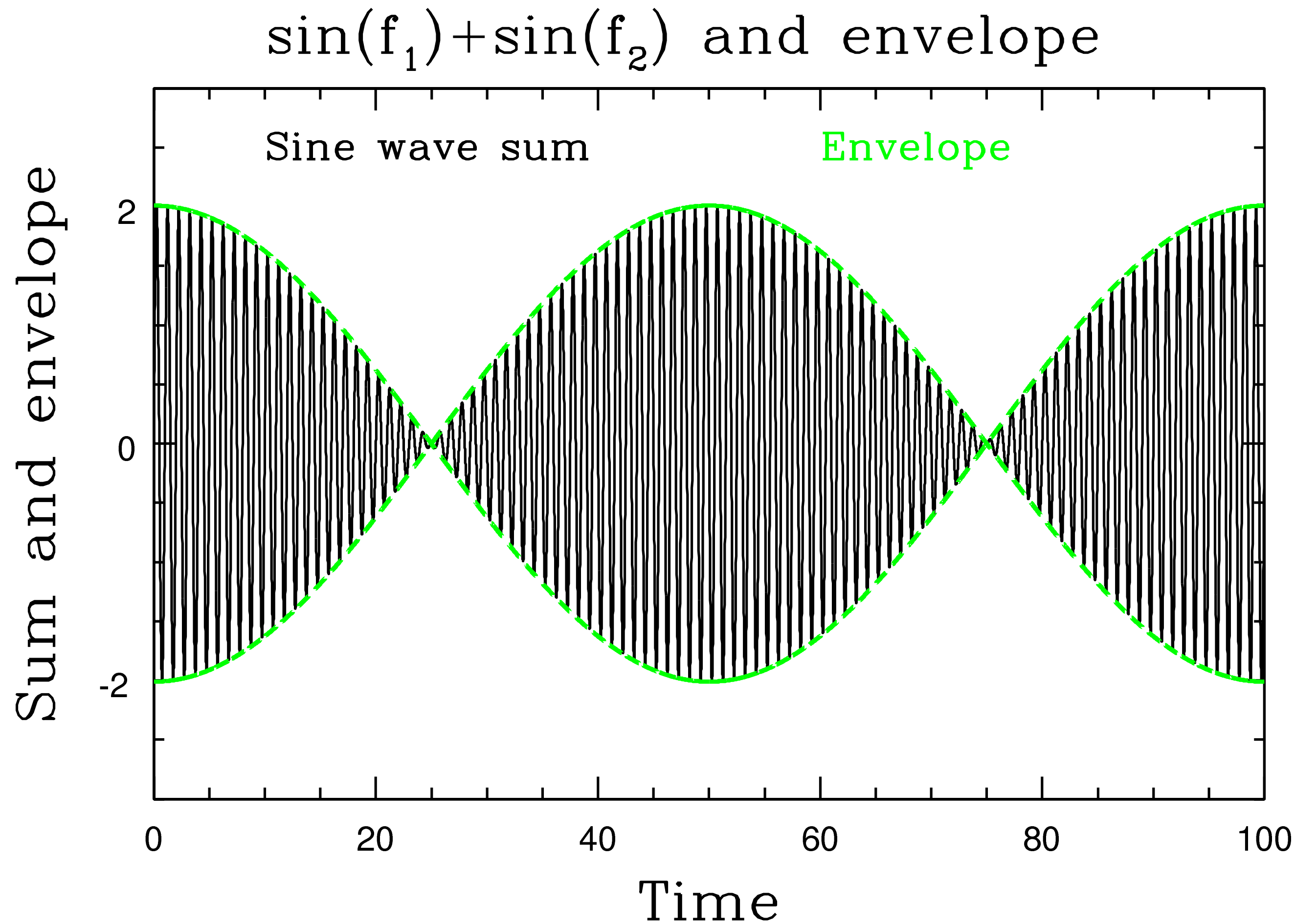
Use that

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$

to find,

$$\begin{aligned}
& \sin\left[2\pi f t + \pi\, \Delta f\ t\right] + \sin\left[2\pi f t - \pi\, \Delta f\ t\right] \\
= \ & \sin(2\pi f t)\cos(\pi\, \Delta f\ t) + \cos(2\pi f t)\sin(\pi\, \Delta f\ t) \\
& \quad + \sin(2\pi f t)\cos(\pi\, \Delta f\ t) - \cos(2\pi f t)\sin(\pi\, \Delta f\ t) \\
= \ & 2\sin(2\pi f t)\cos(\pi\, \Delta f\ t)
\end{aligned}$$

$\sin(2\pi f t) \times \cos(\pi \, \Delta f \, t)$ is a sine wave, $\sin(2\pi f t)$, at the average frequency, times an **envelope**:



Beat frequency

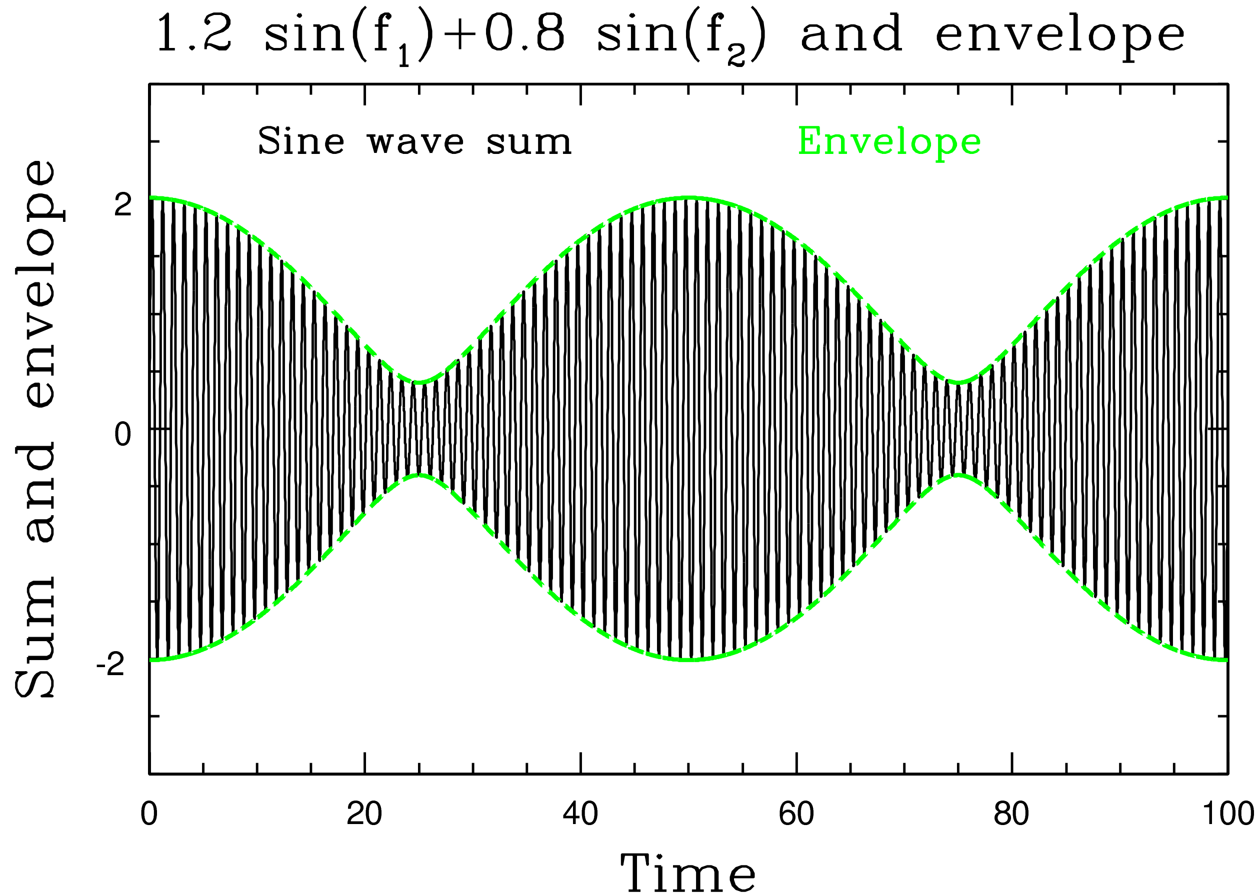
The audible "beat phenomenon" is this envelope. There is one "beat" (one quiet spot) every time $\cos(\pi\,\Delta f\ t)$ goes from being zero to being zero.

$$\cos(\pi/2) = 0 \quad \cos(3\pi/2) = 0 \quad \cos(5\pi/2) = 0 \quad \ldots$$

The cosine function is zero every π, so

The beat frequency is Δf.

What if tones are of different loudness?



quiet spot is not perfectly quiet–but beats are there.

(Why 10% loudness sensitivity gives 1% masking level.)

How small a beat frequency can you hear?

There must be enough time for one or two of the "quiet" spots to occur. Therefore, the beat period $1/\Delta f$ has to be shorter than the duration you play the notes.

Play 1 second: $\Delta f = 1$ Hz barely noticable.

Play 5 seconds: $\Delta f = 0.2$ Hz barely noticable.

Sustained notes $\Rightarrow$ must be in tune

Short notes $\Rightarrow$ can be further out of tune

How big a beat frequency can you hear?

It gets hard to distinguish the loud-soft pattern beyond 10 or 15 Hertz, though you still notice "something."

Fourier: a complex tone is usually a sum of sines at integer multiples of the fundamental (harmonics).

Ear: decomposes sound into harmonics, which each affect a different spot on the cochlea.

Result: beats can occur between a note and an overtone of another note between D at 147 Hz and D at 293 Hz if they have overtones

or between harmonics of different notes! as between A at 220 Hz and E at 330 Hz, which each have 660 Hertz as a harmonic.

Beat frequencies with harmonics

Q: What is the beat frequency when a 220 Hertz and a 331 Hertz tone are played together?

220		440		660		880	
	331			662			993

A: 2 Hertz (Not 1 Hertz)

Frequency Difference too big for beats

When a frequency difference is small enough for large overlap of the excited regions on the cochlea but too big for beat phenomena to be discernable, what happens?

There is a “harshness” or “rough” sensation, called **dissonance**.

[Note and minor 2’nd]

Also occurs between a note and another note’s harmonic, or between harmonics of two notes.

[Major 7’th: upper note and lower note’s 2’nd harmonic]

[Tritone: lower note’s 3’rd harmonic and upper note’s 2’nd harmonic]

Consonance

For some reason, the ear “likes” the exact overlap of two tones’ harmonics. **Consonance**.

But it “dislikes” or finds tension in the improper overlap of notes and harmonics. **Dissonance**

Music and musical scales are developed to have many consonant intervals.

Consonant intervals: to get harmonics to overlap, make frequencies small multiples of the same number, or in simple integer ratios:

Octave: $\frac{2}{1}$

1	2	3	4	5	6
	2		4		6

Fifth: $\frac{3}{2}$

2		4		6		8		10		12
	3			6			9			12

Fourth: $\frac{4}{3}$

3		6		9	12	15		18		21	24
	4		8		12		16		20		24

Maj. Third: $\frac{5}{4}$ Min. Third: $\frac{6}{5}$ Maj. Sixth: $\frac{5}{3}$ $\ldots$