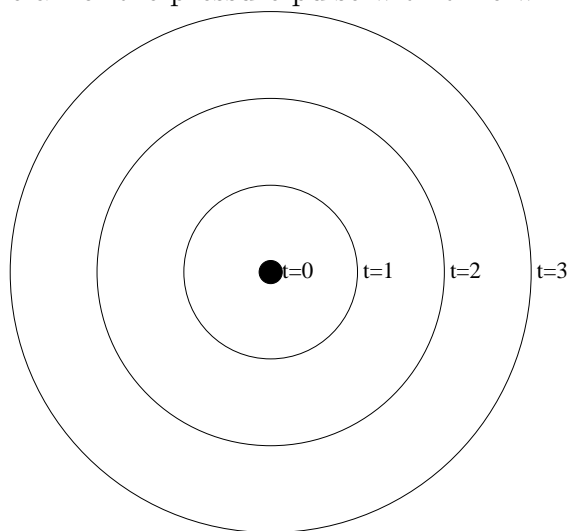


## Lecture 17: The Doppler Effect

The doppler effect is the phenomenon we have all noticed, that a sound produced by a moving source, or which you hear while you are moving, has its perceived frequency shifted. When a car is coming towards you, the car stereo sounds higher pitched and the tempo sounds faster. As it drives away, the sound is lower pitched and the tempo is slower. Let us see why, and by how much.

The first thing to understand and remember is that sound *is a phenomenon in the air*. It is, it is each piece of air pushing on the piece next to it. Once a sound is produced by a source, it will propagate out at the speed of sound in the air—no matter what the source of the sound does. That is, if a source of sound makes a sudden bang (pressure pulse) at time 0, then the location in the air of the pressure pulse with time will look like:



This is what happens *no matter where* the thing which made the bang goes to. That is, the pressure from a sudden banging is a sphere centered *where the sound was made*, *not* at the current location of the sound source. For a moving source, letting out a series of “bangs,” the location of pressure peaks will therefore look like what is shown in figure 1. In the figure, the blue dots are the location of the sound maker when each bang was produced, and the circles are the locations of the pressure peaks radiating out from those bangs. Notice that, because the sound maker is moving to the right, the pressure peaks are closer together on the right and further apart on the left.

Now remember that a sound is just a series of pressure peaks, which are tightly separated in time, by the period of the wave. Therefore, instead of thinking of the sound waves in these two pictures as the locations of pressure peaks due to “bangs,” you can think of them as pressure peaks in a periodic sound wave. In front of the moving object in figure 1, the peaks are closer together. That means that the wave length is shorter, which means it is a higher frequency wave. Behind, the peaks are farther apart, meaning that it is a longer

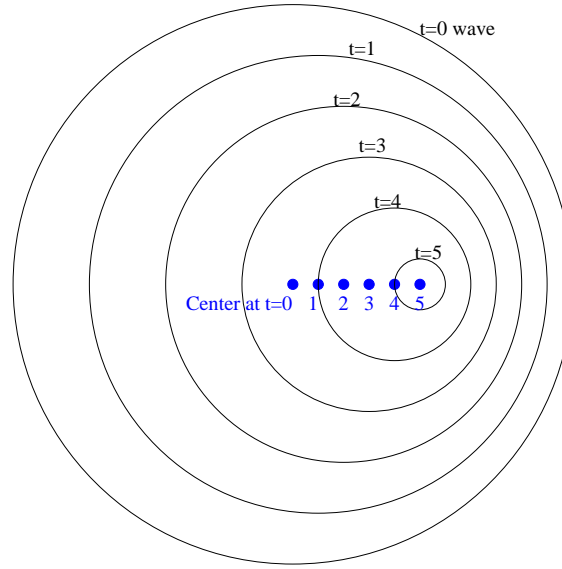


Figure 1: Pressure peaks radiated by a moving sound source. The blue dots represent the time-series of locations of the sound source when it made a series of “bangs,” the black circles represent the locations of the pressure peaks at a time slightly after the last “bang” was made.

wavelength sound, at a lower frequency. This is the gist of the Doppler effect.

Let us now actually calculate the size of the effect.

Suppose a sound source is moving right at you, at velocity  $v$ . At time 0, it emits a pressure peak. At time  $\Delta t$ , it emits a second pressure peak. If its distance from you at time 0 was  $x$ , its distance from you at time  $\Delta t$  was  $x - v\Delta t$  (it is nearer, since it is moving towards you).

The times that the two pressure peaks will reach you, are the time they were emitted plus the propagation time. For the first peak, that means,

$$t_{\text{arrival 1}} = 0 + \frac{x}{v_{\text{sound}}}$$

while for the second peak, it is

$$t_{\text{arrival 2}} = \Delta t + \frac{x - v\Delta t}{v_{\text{sound}}}.$$

The actual period and frequency of the sound—the period and frequency of the sound which is being produced—are,

$$T_{\text{produced}} = t_{\text{pulse 2}} - t_{\text{pulse 1}} = \Delta t \quad f_{\text{produced}} = \frac{1}{T_{\text{produced}}} = \frac{1}{\Delta t}.$$

The period and frequency *you perceive*, are the time difference of the arrivals of the pulses, and its inverse:

$$\begin{aligned}
 T_{\text{observed}} &= t_{\text{arrival } 2} - t_{\text{arrival } 1} \\
 &= \Delta t + \frac{x - v\Delta t}{v_{\text{sound}}} - \frac{x}{v_{\text{sound}}} \\
 &= \frac{v_{\text{sound}}\Delta t}{v_{\text{sound}}} + \frac{x - v\Delta t}{v_{\text{sound}}} - \frac{x}{v_{\text{sound}}} \\
 &= \frac{v_{\text{sound}}\Delta t + x - v\Delta t - x}{v_{\text{sound}}} \\
 &= \frac{v_{\text{sound}}\Delta t - v\Delta t}{v_{\text{sound}}} = \frac{v_{\text{sound}} - v}{v_{\text{sound}}} \Delta t
 \end{aligned}$$

and

$$f_{\text{observed}} = \frac{1}{T_{\text{observed}}} = \frac{v_{\text{sound}}}{v_{\text{sound}} - v} \frac{1}{\Delta t} = \frac{v_{\text{sound}}}{v_{\text{sound}} - v} f_{\text{produced}}$$

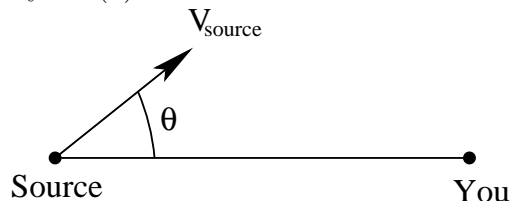
or,

Approaching Source:  $\frac{f_{\text{observed}}}{f_{\text{produced}}} = \frac{v_{\text{sound}}}{v_{\text{sound}} - v_{\text{source}}}$ .

Note that the time, distance, and  $\Delta t$  all do not matter to the final result.

The quick way to understand (and derive) this result is as follows. The time between when you receive two pressure peaks, is the time between when they were made, plus the time difference in their propagation times. That time difference is the distance away from you the source moved, between the pulses, divided by the speed of sound. The distance is the time times the speed of the source.

What if the source is moving away from you? Then the time the second pulse needs, to reach you, will be longer instead of shorter. We just got the *sign* of  $v_{\text{source}}$  backwards in the above. What if the source is moving at some general angle  $\theta$  with respect to the line between you and the source? Then how much closer the source gets to you, gets multiplied by  $\cos(\theta)$ :



Moving source:  $\frac{f_{\text{observed}}}{f_{\text{produced}}} = \frac{v_{\text{sound}}}{v_{\text{sound}} - v_{\text{source}} \cos \theta}$

What if you are the one who is moving? The answer is *not* just that you should use your speed for  $v_{\text{source}}$ . We actually have to re-think the calculation we did above.

First we should think about what is happening, to figure out what to expect in the answer. If you are moving towards a source, then not only are the pressure peaks moving

out towards you; you are moving in at them, so you will cross them more often. Therefore you should hear a higher frequency. Moving away, the pressure pulses are approaching you, but you are running from them, so they reach you less often, and you will hear a lower frequency.

Now let's do the calculation, considering first the case where you are moving straight at the sound producing source. This time, the sound source will not be moving, but you will. Suppose again that the source makes a pressure pulse at time 0 and another at time  $\Delta t$ . If you are at a distance  $x$  when the first pressure pulse occurs, then it will reach you at a time, when its distance from the source and your distance from the source are the same. Call that time  $t_1$ , and let us try to find it. The sound wave's distance from the source is  $v_{\text{sound}}t_1$ . Your distance from the sound is  $x - vt_1$ . These equal at

$$v_{\text{sound}}t_1 = x - vt_1 \quad \rightarrow \quad (v_{\text{sound}} + v)t_1 = x \quad \rightarrow \quad t_1 = \frac{x}{v_{\text{sound}} + v}$$

Now we have to find the time when the second pressure pulse meets you. Call that time  $t_2$ . The distance from the source of the second pulse is  $v_{\text{sound}}(t_2 - \Delta t)$ , and your position is  $x - vt_2$ ; they equal at time,

$$v_{\text{sound}}(t_2 - \Delta t) = x - vt_2 \quad \rightarrow \quad (v_{\text{sound}} + v)t_2 = x + v_{\text{sound}}\Delta t,$$

or,

$$t_2 = \frac{x}{v_{\text{sound}} + v} + \frac{v_{\text{sound}}\Delta t}{v_{\text{sound}} + v}.$$

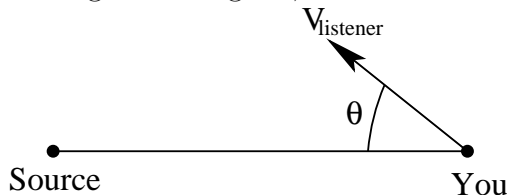
The period of the sound, as perceived by you, is the time difference between pressure peaks:

$$\begin{aligned} T_{\text{observed}} &= t_2 - t_1 = \frac{x}{v_{\text{sound}} + v} + \frac{v_{\text{sound}}\Delta t}{v_{\text{sound}} + v} - \frac{x}{v_{\text{sound}} + v} \\ T_{\text{observed}} &= \frac{v_{\text{sound}}}{v_{\text{sound}} + v} \Delta t \end{aligned}$$

from which it follows that,

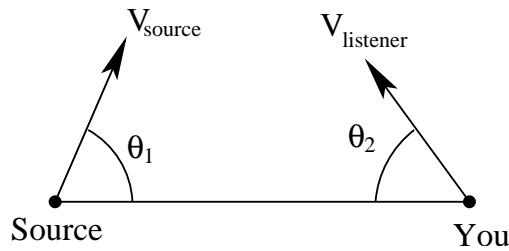
$$\frac{T_{\text{observed}}}{T_{\text{produced}}} = \frac{v_{\text{sound}}}{v_{\text{sound}} + v}, \quad \text{or} \quad \frac{f_{\text{observed}}}{f_{\text{produced}}} = \frac{v_{\text{sound}} + v_{\text{listener}}}{v_{\text{sound}}}.$$

The way that  $v_{\text{listener}}$  entered here, is that it is how fast you were moving towards the sound fronts. If you are moving straight away, you should flip its sign. If you are moving at some general angle  $\theta$ , then the result is multiplied by the cosine of that angle:



Moving listener: $\frac{f_{\text{observed}}}{f_{\text{produced}}} = \frac{v_{\text{sound}} + v_{\text{listener}} \cos \theta}{v_{\text{sound}}}$
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What if both are moving? You should not be too surprised by the answer:



Both Move: $\frac{f_{\text{observed}}}{f_{\text{produced}}} = \frac{v_{\text{sound}} + v_{\text{listener}} \cos \theta_2}{v_{\text{sound}} - v_{\text{source}} \cos \theta_1}$
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There are two cases in which these answers do not seem to make sense. The first is that if  $v_{\text{source}} \geq v_{\text{sound}}$  and the source is moving at you, it looks like the frequency can become *infinite*. This is actually **correct**! For a sound source moving at you, as it approaches the speed of sound, the frequency you hear becomes extremely sharp. What is happening is that the sound fronts, pictured in figure 1, are piling up and actually touching. All the pressure peaks are happening at once. If you try to go faster than the speed of sound, you run into the pressure pulses you created before you got up to that speed. For an airplane, that means that a huge overpressure in front of the plane is encountered just as it reaches the speed of sound. This is called the **sound barrier**, and it is why commercial airflights all (now that the Concorde is retired) fly slower than the speed of sound, and why it took so long to develop airplanes which could handle this and exceed the speed of sound. Surprisingly, it is much easier to fly an airplane at twice the speed of sound (Mach 2) than right at the speed of sound (Mach 1), for this reason.

The other case which looks strange is what happens if you are moving away from a sound source,  $\cos \theta_2 = -1$ , and  $v_{\text{listener}} \geq v_{\text{sound}}$ . Then the numerator becomes zero, and I find a *vanishing frequency*. Can that be right? This is **also correct**! This is the case where you are moving away from a sound source, at or above the speed of sound. Each pressure pulse is moving towards you, but you are moving back away from it at least as fast as it approaches you. That means that it never reaches you; *you can run away from a sound* and it never catches up (if you can go faster than  $v_{\text{sound}}$ ).