

Lecture 18: Reflection and Impedance

Let us look again at the question of what happens when a sound wave in the air runs into the surface of a body of water (or a wall, or glass, or whatever). How much of the wave is reflected, and how much becomes a sound wave inside the water?

Recall first that the velocity of the air due to the wave, and the pressure of the air due to the wave, were related via

$$P - P_{\text{atmos}} = (\rho_{\text{air}} v_{\text{sound,air}})(v_{\text{air}} - v_{\text{“wind”}})$$

From now on I will write ΔP as the pressure difference from atmospheric and will ignore the possibility of wind speed, so v_{air} is the air motion because of the sound wave.

The relation between pressure change and velocity change in a sound wave is true of *all* media, it is not special to air. The key is that you have to use the speed of sound and density of the medium you are thinking about. Because it comes up so often, the product ρv_{sound} (a characteristic of a material) is called the **mechanical impedance** Z :

$$\text{Mechanical impedance} \quad Z \equiv \rho v_{\text{sound}} .$$

If you compare Z between air and most other materials, you find out *both* that ρ_{air} is small (typically 500–5000 times smaller than in another material), *and* that v_{sound} in the air is small (typically 3–10 times smaller than in another material). Therefore the mechanical impedance Z of a solid or liquid will be far larger than for air.

Now consider a sound wave in the air hitting the surface of a lake or other body of water. The sound wave approaching the water contributes a pressure which I will call ΔP_{\rightarrow} and an air velocity v_{\rightarrow} , which satisfy

$$\Delta P_{\rightarrow} = Z_{\text{air}} v_{\rightarrow} .$$

Note that I am measuring v in terms of the direction the sound wave is moving, that is, from the air into the water. There will be a sound wave in the water, moving away from the air. It will have a pressure and water velocity

$$\Delta P_{H_2O} = Z_{H_2O} v_{H_2O} .$$

To relate these, I have to think about what happens right at the surface. The air ends exactly where the water begins. Therefore, however fast the air is moving right at the surface, the water must be moving at the same speed. Also, the air pushes on the water exactly as hard as the water pushes on the air. Therefore,

$$v_{\text{air}} = v_{H_2O} \quad \text{and} \quad \Delta P_{\text{air}} = \Delta P_{H_2O} .$$

If you try to put these together with the results for the pressures and velocities, you will find that the only way to get *both* things to be true, is either to have the Z 's be equal, or to have

the ΔP 's both be zero. The reason is, of course, that I tricked you by leaving out something important. There will always be a reflected wave going off into the air. It satisfies,

$$\Delta P_{\leftarrow} = -Z_{\text{air}} v_{\leftarrow}$$

because the wave is going in the opposite direction. The total air motion and pressure are the sum of those from the two waves:

$$\Delta P_{\text{air}} = \Delta P_{\rightarrow} + \Delta P_{\leftarrow}, \quad v_{\text{air}} = v_{\leftarrow} + v_{\rightarrow}.$$

The next part is derivation and you can skip to the end if you don't want to see it.

[At this point I have 5 equations (3 relations between ΔP and v for each wave, and the two equalities at the water surface) in 6 unknowns (three pressures and three velocities). I can use the relations between pressures to express the v 's in terms of ΔP 's and write both equalities in terms of pressures:

$$\begin{aligned} \Delta P_{\rightarrow} + \Delta P_{\leftarrow} &= \Delta P_{H_2O} \\ \frac{\Delta P_{\rightarrow}}{Z_{\text{air}}} - \frac{\Delta P_{\leftarrow}}{Z_{\text{air}}} &= \frac{\Delta P_{H_2O}}{Z_{H_2O}} \end{aligned}$$

In the last expression, multiply through by Z_{air} . Then add the top expression:

$$2\Delta P_{\rightarrow} = \frac{Z_{\text{air}} + Z_{H_2O}}{Z_{H_2O}} \Delta P_{H_2O}, \quad (1)$$

$$\Delta P_{H_2O} = \frac{2Z_{H_2O}}{Z_{H_2O} + Z_{\text{air}}} \Delta P_{\rightarrow}, \quad (2)$$

$$\text{and also,} \quad \Delta P_{\leftarrow} = \frac{Z_{H_2O} - Z_{\text{air}}}{Z_{H_2O} + Z_{\text{air}}} \Delta P_{\rightarrow} \quad (3)$$

This relates the pressure of the incoming sound wave to the pressure of the sound wave in the water. You see that the sound wave in the water has about twice the pressure of the one in the air, since Z_{H_2O} is much larger than Z_{air} ; so the relation is almost $\Delta P_{H_2O} = 2\Delta P_{\rightarrow}$. That makes it sound like the wave in the water is not so small after all. But you have to remember the relation between intensity and pressure, and that what really counts is the intensity. The intensity of the sound wave in the water is,

$$I_{H_2O} = \frac{\Delta P_{H_2O}^2}{Z_{H_2O}} = \frac{4Z_{H_2O}}{(Z_{\text{air}} + Z_{H_2O})^2} \Delta P_{\text{air}}^2 = \frac{4Z_{\text{air}}Z_{H_2O}}{(Z_{\text{air}} + Z_{H_2O})^2} I_{\text{air}}.$$

Similarly, you can take equation (3) and square both sides to find that

$$I_{\text{reflected}} = \frac{(Z_{H_2O} - Z_{\text{air}})^2}{(Z_{H_2O} + Z_{\text{air}})^2} I_{\text{air}}.$$

That ends the derivation and you can start reading again.]

Now rewrite that for an incident (incoming) sound wave going from a medium of impedance Z_1 to a medium of impedance Z_2 . There is a transmitted wave (the one which makes it into the new medium) and a reflected wave, of intensities,

$$\boxed{\frac{I_{\text{transmitted}}}{I_{\text{incident}}} = \frac{4Z_1Z_2}{(Z_1+Z_2)^2} \quad \text{and} \quad \frac{I_{\text{reflected}}}{I_{\text{incident}}} = \frac{(Z_1-Z_2)^2}{(Z_1+Z_2)^2}}$$

When I_1 and I_2 are very different (as for air and water), these expressions mean that almost all the energy is reflected and not transmitted: $I_{\text{transmitted}}/I_{\text{incident}} \simeq 4Z_1/Z_2$ if Z_1 is smaller. Note that the result here is for right-angle incidence. It is different at a different angle, but I will not present that just so as not to confuse matters further.

For air and water, since

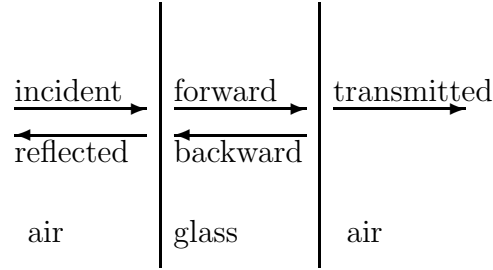
$$\begin{aligned} Z_{\text{air}} &= \rho_{\text{air}} v_{\text{sound,air}} = 1.2 \text{ kg/m}^3 \times 344 \text{ m/s} = 400 \frac{\text{kg}}{\text{m}^2 \text{ s}} \\ Z_{\text{water}} &= \rho_{H_2O} v_{\text{sound,H}_2\text{O}} = 1000 \text{ kg/m}^3 \times 1400 \text{ m/s} = 1400000 \frac{\text{kg}}{\text{m}^2 \text{ s}} \\ \frac{Z_{\text{air}}}{Z_{\text{water}}} &\simeq \frac{1}{3500} \end{aligned}$$

which is indeed tiny. This means, using the formulas above, that when sound reflects off water, only 4/3500 of the intensity makes it through. That is a reduction of almost 30 dB in the intensity of the sound. Note that sound, moving through the water, reflects just as efficiently to stay in the water and let only a tiny amount into the air.

One other thing about the reflection problem just considered. The pressure of the incident wave and of the transmitted wave were of the same sign. What about the reflected wave? This turns out to depend on whether $Z_1 < Z_2$ (as for air to water), or $Z_1 > Z_2$ (as for water to air). For $Z_1 < Z_2$, the reflected wave has the same sign of ΔP and opposite sign of v . For the other case, $Z_1 > Z_2$, the reflected wave has the same sign of v and opposite sign of ΔP . You can see this in equation (3) above. This proves important in many problems, and so it is worth thinking about a little more.

The way to think about it is, that when Z is small, the medium is easy to move around (like the air—you don't have to push on it hard to make it move). If the other medium is hard to move around, then the velocities of the incoming and outgoing waves will need to cancel, which happens (since they are waves going in opposite directions) when the pressures are the same. When it is the high Z medium (water) which the sound wave is coming from, then the medium is hard to move. It only takes a tiny pressure to get the other medium to move, so the incoming and outgoing waves must have opposite pressure from each other.

Now think about a pane of glass. Instead of three waves—incoming, transmitted, and reflected—there are five:



This problem is more complicated. The problem is that the time when the incoming wave has its peak pressure need not be the time when the forward and backward waves have their peak pressures. I will not drag you through a derivation of what happens in this case, but I will quote two limits for the answer.

First, think about a very thin panel of glass. The forward wave has the same sign of pressure as the incident wave. The backward wave, though, has the opposite sign of pressure, because of the remarks about reflected waves we made above. That means that the forward wave can have a bigger pressure, and still satisfy the matching condition where the air and glass meet. That means that much more sound will get through. The only reason the sound does not *all* go through, is that the short time the wave is moving in the glass gets the pressure peak of the forward and backward moving waves to occur at slightly different times. This depends on the wavelength of the sound and thickness of the glass. The answer, after much work, turns out to be,

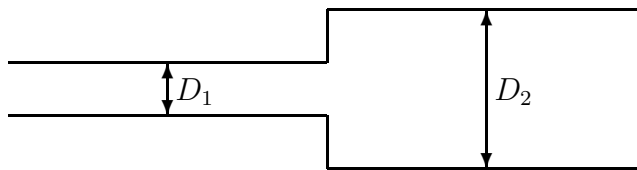
$$\frac{I_{\text{transmitted}}}{I_{\text{incident}}} \simeq \left[\frac{Z_{\text{air}}}{Z_{\text{glass}}} \right]^2 \left[\frac{v_{\text{sound,glass}}}{\pi f d_{\text{glass}}} \right]^2, \quad (4)$$

where d_{glass} is the thickness of the glass. This formula shows that glass lets through more low frequency sound than high frequency sound, something you have probably all experienced or can easily check at home. The sounds from outside when you close the window are muffled; not just quieter, but different in timbre, specifically, losing 6 dB of loudness per octave frequency compared to the original sound.

Now, what about a very thick piece of stuff? In this case, essentially you can forget about the backward wave in considering the transmission from air into the glass. Then you just get the product of the amount of transmission from air to glass, times the transmission from glass back to air.

If we put typical numbers for the walls of a musical instrument into equation (4), we find something like 30 to 40 dB of attenuation. That is, the walls of the instrument hold in essentially all of the sound bouncing around inside of the instrument. Therefore, we can think of the sound inside an instrument as being perfectly contained by the walls of the instrument. This leads us to think about sound traveling in a tube.

Consider sound traveling in a tube, where the walls of the tube hold the sound in perfectly. What happens if the tube widens abruptly, say, from diameter D_1 to diameter D_2 , both much smaller than the wavelength of the sound?



Consider the sound wave traveling down the narrow part of the tube, reaching this widening. Instead of talking about the air pressure and the air velocity, talk about the air pressure and the air flow rate. Surely, the rate the air is flowing has to be the same just before the widening as just after, since air cannot go anywhere else. The pressure must also be the same. But the air flow rate is just,

$$\text{flow} = A v_{\text{air}} = \frac{\pi D^2}{4} v_{\text{air}}$$

We have to have $A_1 v_1 = A_2 v_2$ and $\Delta P_1 = \Delta P_2$. If we simply define a quantity called **Acoustic Impedance**,

$$Z_A \equiv \frac{\rho v_{\text{sound}}}{A}$$

then the relation between pressure and air flow is,

$$\Delta P = Z_A \text{ flow} .$$

In computing how much air reflects from the juncture of the two pipes and how much is transmitted, we can re-use all our previous work, just using Z_A instead of Z . The transmitted and reflected powers are,

$$\boxed{\frac{\text{Power}_{\text{transmitted}}}{\text{Power}_{\text{incident}}} = \frac{4Z_{A1}Z_{A2}}{(Z_{A1} + Z_{A2})^2} \quad \text{and} \quad \frac{\text{Power}_{\text{reflected}}}{\text{Power}_{\text{incident}}} = \frac{(Z_{A1} - Z_{A2})^2}{(Z_{A1} + Z_{A2})^2}}$$

What about the sign of the pressure for the reflected wave? A wide pipe has a low Z_A (it is easy for the air to move), a narrow pipe has a high Z_A (the air moves very little). Therefore, we can repeat the arguments we made about high and low Z material. When a narrow pipe opens out, the large pipe will take an airflow easily. The reflected wave therefore has the same sign of airflow and opposite sign of pressure. When a wide pipe narrows, the pressure of the reflected wave is the same as the incident wave.