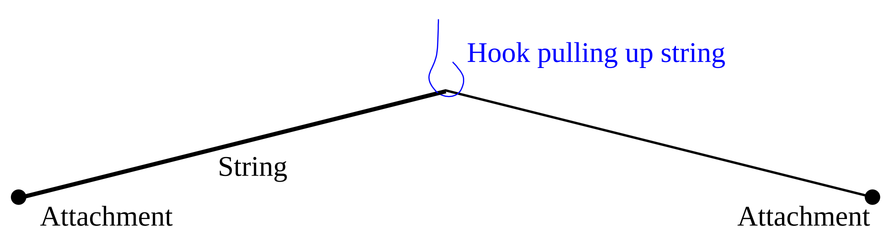
Lecture 25: String instruments

A string under tension will vibrate back and forth when it is plucked or bowed. A single string under tension has many resonant frequencies, which are in harmonic relation if the string is thin and uniform. This makes strings a particularly simple way to design an instrument which produces periodic tones which are rich in harmonic structure. This lecture will explain why a string under tension has resonances, why their frequencies are harmonically related, and how string instruments overcome the two other problems of musical instrument design, efficient sound production and the production of sustained tones, rather than tones which die off (though only the bowed instruments solve the latter problem.)

A string is under **tension** if something on each end of the string is pulling on it. A string on an instrument is usually not accelerating very fast, which means that the forces have to balance out on the string; so something on each end of the string must be pulling on it with about the same force. The tension is defined to be this force.

It is not easy to see that there is a large force pulling on each end of a string on a violin or guitar because the force is being maintained by a peg which is not moving. The force is maintained by friction on the peg, keeping it from turning. You only really realize how big the tension is when you try turning the peg. On a guitar this is done through a screw mechanism which gives a large mechanical advantage (leverage); the force on the string is much larger than the force you exert on the peg. To clarify, Figure 1 contains a very poor cartoon of the parts of a string instrument. The tension on the string is maintained by the peg pulling the string away from the tailpiece and the tailpiece pulling the string the other way. Of course, this means the string puts force on the peg and the tailpiece, pulling them towards the middle of the instrument. The body of the instrument is under compression from these two forces, and must be fairly mechanically strong.

An "ideal" string has no rigidity; its shape and its motion are completely determined by the tension and the mass per length of the string. The tension on the string tries to keep the string straight. When the string is not straight, it will "snap back" towards being straight. The mass per length of the string limits how fast this "snapping back" actually happens, and means that the string keeps moving past where it becomes straight. In pictures, think of it like this; imagine I attach a string on its two ends, under tension. Now, grab it somehow and pull it upward:



and release it. The two anchor points of the string are no longer pulling it left and right;

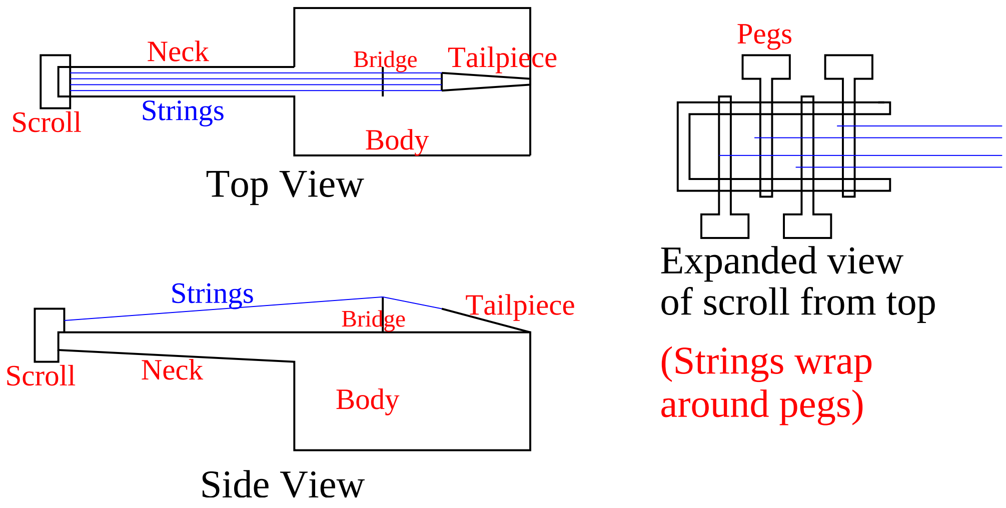
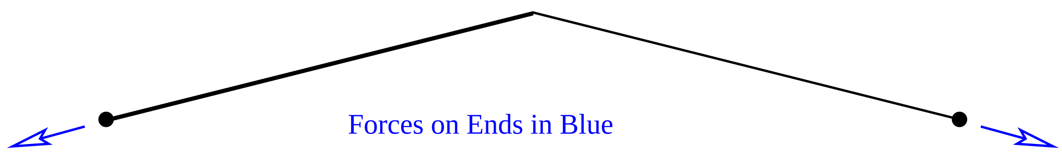
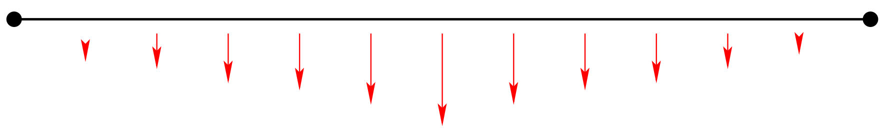


Figure 1: Parts of a string instrument. How the peg works is shown in a blow-up of the scroll of the instrument.

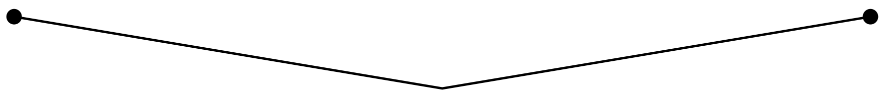
they are pulling left and a little down, and right and a little down:



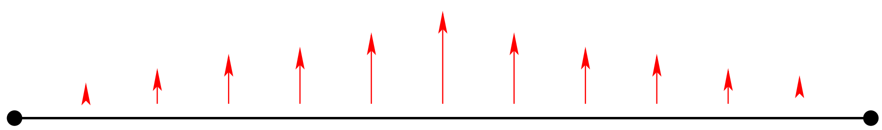
and if I add up the forces, there is a downwards force on the string. That will start the string moving. The bigger the tension, the faster the string will move. The heavier the string, the slower it will move. A moment later, the string will be straight, but moving:

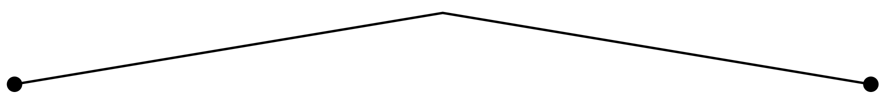


Now there is no *net* force on the string; it is pulled to the left from one side and the right from the other, and these forces cancel. However, since it is moving, it will continue to move until something pulls on it to stop:



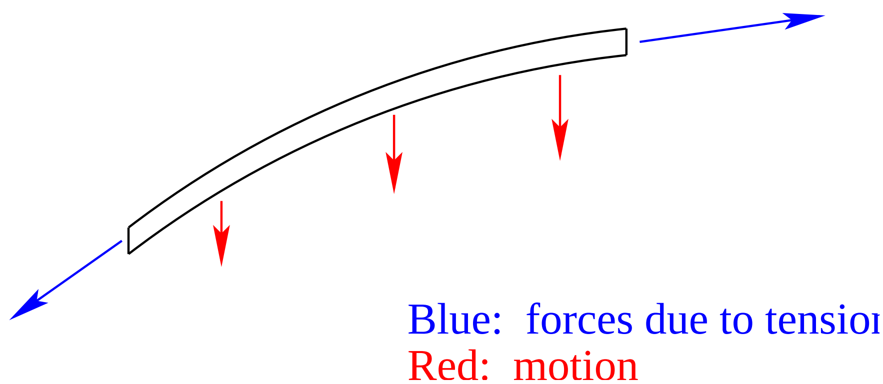
at which point the forces on the end are each pulling it somewhat upward. The motion of the string will reverse:





and we see the string is undergoing periodic, resonant oscillations.

To calculate the possible patterns and frequencies of oscillation that a string can make, we have to think about the string "little bit by little bit." Think of a short bit of string. Call the tension T (unfortunately the same letter as the period, but for this lecture T will be tension), the mass per unit length μ (measured in kg/m, meaning how many kg weight a meter of string would weigh; usually μ is much less than a kg per meter), and imagine that the little bit of string may be bent, and may be moving:



The forces will *not* balance to zero if the string bit is curved. The string bit can be moving, and if the motion is not uniform, the curvature will change.

Recall that, for the air, the thing which made the air move was a *difference* of pressure across a bit of air. Here, for the string, it is a *difference* of slope for the string between the two ends. For the air, the thing which made the pressure change was a *difference* of the air speed, between the front and back of a bit of air. For the string, what makes the slope change is a *difference* of the motion between the front and back. Therefore, there is an analogy between the case of a string and of air:

Air	String
Pressure P	Tension T
density ρ	mass per length μ
Compression	slope of string
Difference in compression	difference in slope
Air motion $v_{\rm air}$	String motion $v_{\rm string}$
Difference in $v_{\rm air}$	Difference in $v_{\rm string}$

The analogy means that bending and motion of the string propagates as a wave along the string, just as compression and motion in the air propagates as a sound wave. The speed of the "sound wave" on a string is,

$$v_{\rm sound,string} = \sqrt{\frac{T}{\mu}}$$

just as the speed of sound in air is $\sqrt{P/\rho}$ (up to that correction $\sqrt{c_p/c_v}$, which is absent for the string).

What are the normal modes of vibration for a string? Invariably, the ends of a string are fixed, which means that the string cannot move there. "Cannot move" is like "air cannot move" which is like what happens at the closed end of a pipe. Therefore, the normal modes of vibration of a string are those of a closed-closed pipe, and the frequencies are

$$f = \frac{v_{\rm sound,string}}{2L} \times 1, 2, 3, 4, 5, \ldots$$

which means that strings automatically have a harmonic overtone series. The vibration patterns have nodes: the pattern with $f = 2f_{\rm fundamental}$ has a node at the midpoint (1/2) of the string, the pattern with $f = 3f_{\rm fundamental}$ has nodes at the 1/3 and 2/3 points, the pattern with $f = 4f_{\rm fundamental}$ has nodes at 1/4, 2/4, and 3/4 of the way along the string, and so forth. If you gently touch a finger to a point on the string, it is analogous to opening a register hole at the same point on a tube. Any pattern which has motion at that point will be damped, any pattern which does not will continue to resonate. Therefore, putting a finger gently at the middle of the string will "kill" the modes with frequencies of 1,3,5,7,... of the fundamental and leave those with frequencies of 2,4,6,... of the fundamental–that is, it will double the frequency the string plays. Putting the finger at the 1/3 point triples the frequency. Putting it 1/5 of the way along the string makes the frequency go up 5 fold (2 octaves and a major 3'rd). (So does putting at finger at the 2/5, 3/5, or 4/5 point.) This way of playing harmonics of the strings is widely used on the harp and used somewhat on the other string instruments (bowed instruments, guitar, etc). However, it does not give enough tuning options to be sufficient by itself.

There are 3 remaining design issues in making a string instrument. They are,

- How is the instrument designed to be able to produce many different frequencies (preferably a chromatic scale with a range of at least 3 octaves)?
- How is sound to be produced efficiently?
- How is a sustained sound, rather than a sound which dies off, to be produced?

First, let us address the question of producing many frequencies. One method is to have many strings, one for each pitch needed. This is the solution used by the harp. The added advantage is that the instrument can play several notes at once. To some extent this method is used by other string instruments too; most have several strings (violin family instruments have 4, most guitars have 6, some guitar family instruments have more).

Another method is to have a way to change the length of a string. This is done on the violin and guitar family instruments by having the string run, for most of its length, slightly above a **fingerboard**. By using a finger to clamp the string to the fingerboard, the effective length of the string is changed from ending at the nut (a bar the string runs over just before going into the peg box in the scroll) to ending where the finger presses the string. On guitars and some other instruments, little metal ridges called **frets** are placed along the fingerboard. If the string is pushed down between two frets, it automatically ends at the fret, rather than the finger, which ensures accurate intonation. (This is not done in the violin family of instruments to leave open more freedom in intonation, such as slides and vibrato, which is possible but harder on a guitar.)

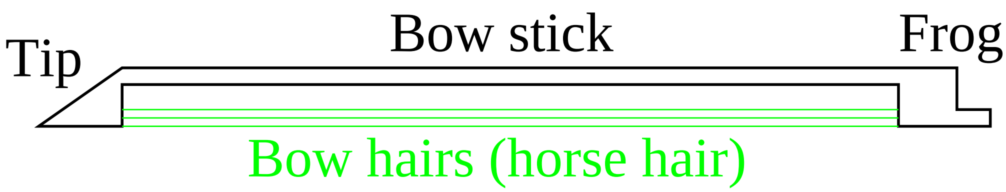
What about efficient sound production? A string is so thin that its motion through the air barely makes any air move, and is very inefficient in making sound. (Turn off the amp on an electric guitar to see this.) What makes sound efficiently is a large surface area moving back and forth by a substantial amount. To move by a substantial amount, it typically needs to be thin. This is accomplished by having the string go over a **bridge**, a piece of wood standing up vertically from the body of the instrument. The string bends by an angle as it crosses the bridge, as shown in figure 1, which means that the string exerts a downward force on the bridge. The bridge acts as the other end of the vibrating section of the string (opposite end to the nut, finger, or fret). The bridge absorbs some of the vibration energy of the string and transfers it into the body of the instrument. On some instruments (piano, harp) there is a large flat board called a **sound board**, which acts as the large vibrating surface which carries the sound into the air. On violin and guitar family instruments, this role is played by the front plate of the instrument, and to a lesser extent by the back and sides as well. The presence of an air cavity in these instruments is also helpful.

The sound board, instrument body, and instrument cavity typically have a number of resonances of their own, typically of fairly low Q. These tend to enhance any note or harmonic with a frequency lying close to a resonant frequency of the instrument. One of the design goals or features of string instruments is to make the resonances be widely scattered in frequency. In particular, a violin family instrument's apparent symmetry is much like your body's symmetry; once you look inside at the "guts" you find out it is nowhere near symmetric. The top plate's thickness is nonuniform and differs from left to right side; there is a reinforcing wood bar running most of the length of one side of the body, and a dowel (the soundpost) running between front and back of the instrument, offset on the side of the instrument without the reinforcement. Guitar reinforcement is also quite asymmetric. This ensures that the resonances reinforce many frequencies, rather than giving large reinforcements to a few frequencies.

The final question is how the instrument produces a steady sound. On all plucked, picked, strummed, or struck (hammer) string instruments, the answer is, that it does not. The string is set in vibration by some sudden process. The sound features an abrupt attack and then a decaying ringing. To make the sustain musically interesting, these instruments are usually constructed so the string's vibration will have quite a large Q (which is not too hard to do with a string instrument). In particular, the angle the string bends in going over the bridge is not too large, so the coupling between the string and the bridge is not too large.

For the bowed string instruments (violin family instruments and a number of other "traditional" instruments), the string *is* driven in a way which produces a steady tone. The way the bow acts on the string increases the vibration of the string in a way slightly analogous to how a reed increases the vibration in an air resonance.

Here is the idea. The bow is a wooden stick, supporting a mass of hairs (literally horse tail hairs) held under tension by the stick.



The hand holds the frog of the bowstick, which contains a mechanism for adjusting the tension on the bowhairs. Bowing the string means pushing the hairs against the string and dragging them across the string. This is generally done quite close to the bridge, not in the middle of the string.

The string vibrates back and forth underneath the bowhair. The vibration consists of the string moving with the bowhair, then moving against the bowhair, then moving with the bowhair, then against it, ... The friction of the bowhair against the string means that the hair is always pulling the string to move along with it. Pushing a car in the direction it is already moving adds energy, and pushing a car against the direction it is moving takes energy away. Similarly, when the bow is moving with the string, it is adding energy to the string's motion, but when the string moves back the other way, the bow actually takes energy out from the string.

The key to the operation of the bow is that it is a stick-slip action. While the bow moves with the string, the bowhair and string are actually sticking together–the string and the bowhair just above it are moving at the exact same speed. When the string moves back, it is slipping against the bow hair. The hairs are pulling on the string by friction, and friction turns out to be larger between two things which are sticking together than two things which are sliding against each other. This is why, when you are walking on ice, as long as you put your foot straight down, you are fine, until the moment the foot starts to slip–then there is no stopping it, because its friction against the ice suddenly reduces. Once you start to slip, you slip more easily. This is also the idea behind anti-lock brakes on cars. As soon as your tires start to skid, their friction against the road is reduced, and your car does not stop as well. Actually letting up on the brakes, until the road forces the wheel to turn again, increases your friction against the road and lets you brake faster.

The reason that bows are made with horse hair is that it turns out that hairs have an especially large difference between static (things sticking together) and dynamic (things slipping against each other) friction. This is probably because hairs are covered with tiny microscopic scales, which stick out and catch on the things touching them. (These scales are behind most of the properties of wool, such as its scratchiness and the way wool clothes shrink when they are washed.) To make the friction even larger when the bow sticks, the hairs are covered with tiny grains of rosen (dried pine tree sap).

Because the energy in the string is being continuously replenished by the bow action, the violin family instruments can afford to have a high bridge; the strings bend by a larger angle going over the bridge than in a guitar family instrument, which means that the vibration of the string is more quickly drained into the instrument body, and hence into sound. This (along with a more highly developed instrument body, violin makers would claim) allows the bowed instruments to play substantially louder than guitar family instruments (without amplification). The bridge is also arched, rather than straight, so the strings do not form a straight line across it, making it possible for the bow to play one string at a time. The large coupling to the bridge means that, when violin family instruments are plucked, the sound dies away faster than in a guitar instrument.

Besides the difficulty of good intonation on an instrument without frets, the main reason that beginners on violin family instruments sound so bad is that they have not learned to control the slip-stick action of the bowhair on the string. If the hairs are forced down with too much pressure or not enough bow speed, they tend to stick while the string is moving against the hairs, leading to a "crunching" sound. If the pressure is too small or the bow speed too fast, the hairs tend to slide across the string rather than sticking, leading to a wispy sound. The right range of pressure to get a proper stick-slip action varies depending on where on the spot on the string being played, the pressure, and the bow speed. These control parameters (bow placement and pressure) are also what determines loudness, and how fast the bow moves (which must be controlled depending on the length of the note).