Lecture 26: Percussion

This lecture will just try to outline the issues involved in percussion instruments and illustrate with a few examples. No attempt will be made to be comprehensive. A much better and more complete discussion can be found in the course book.

A percussion instrument is any instrument where you hit something to put some vibrational energy in, and then wait as the instrument converts the vibrational energy into sound. A simple example is a thin plate of metal. The metal can bend in a number of different ways (normal modes). When you hit it with a hammer, the plate starts bending in each of several of these modes at once. Depending on the place you hit and the properties of the hammer, different modes are excited different amounts. (Metal hammers excite a lot of very high frequency modes, which sounds bad, so people tend to go for mallets made with rubber, wood, leather, or other somewhat softer materials, which are in contact with the plate for longer and so don't excite high frequency modes as much.)

Percussion instruments invariably involve resonances in the instrument. Therefore the instrument has to be made in a way that it has some (bending or vibrational) resonances. There are several other design problems to overcome:

- How is sound production to be made efficient–or is it?
- How are overtones to be tuned to be in harmonic relation–or are they?
- How can the relative loudness of different overtones be controlled?
- How long does the resonance last, and how can that be controlled?

Consider a metal plate struck by a hammer, for instance. The sound production comes from the metal moving against the air. The larger the plate, the more air it pushes against. The thinner the plate, the further it moves with the same strength of hammer blow. Therefore, a large, thin plate makes more sound. This leads in the direction of the cymbal, which is a large thin plate of metal. Tuning the overtones in a metal plate is difficult. For a random shaped plate, they are not in tune, and it just makes a noise. A bell can be thought of as a plate which is in a very strange shape (no longer flat), and it turns out that the shape and thickness is cleverly designed to make several harmonics be approximately harmonic. The relative loudness of overtones can be controlled by the material of the hammer and the position of the hammer strike, at least to some extent. If I hold a metal plate with my hand, the resonance is quickly damped away into my hand. Suspending it by drilling a small hole through the plate and hanging it with a string eliminates this damping and lets the resonance last much longer.

Now we briefly survey some of the percussion instruments. One family of percussion instruments involves long thin bars, struck by a mallet or hammer. A uniform bar, much longer than it is wide and much wider than it is thick, and suspended so the ends are free, has resonant frequencies at about

$$f = \frac{f_{\text{fund}}}{9} \times \left(9\,,\, 25\,,\, 49\,,\, 81\,,\, \dots\right)$$

where the numbers are 3^2, 5^2, 7^2, and so forth. This is not a harmonic series.

In the glockenspiel, one makes a chromatic scale of bars with frequencies in a range around 800 to 4000 Hertz. The suspension and mallets are chosen so that only the fundamental is audible. There is a sharp attack which contains other harmonics, but they quickly die away, leaving a pure sine wave sustain.

The xylophone is similar, but with two modifications. First, to enhance the loudness, tubes are placed under the bars, with resonant frequencies matched to the resonant frequencies of the bars. The tubes support air resonances at the same frequency, strongly enhancing the conversion of sound from bar vibration into sound in the air. This allows the instrument to make sound efficiently at much lower frequencies, so the range can extend much lower than in the glockenspiel. Further, the bars (typically made of wood) are carved to be thinner in the middle than on the ends. This lowers the fundamental relative to the overtones. The amount of thinning is chosen so the first overtone has 3 times the frequency of the fundamental, which also happens to be the first overtone of the (open-closed) resonator tube. Because the instrument possesses a harmonic as well as the fundamental, it has a more interesting tone color.

Chimes are also based on long bars. They use that three of the frequencies, $81 = 9^2$, $121 = 11^2$, and $169 = 13^2$, are pretty close to $2:3:4$ relation. By putting a weight on the top of the bar, the upper overtones are pulled flat, making the relation more nearly $2:3:4$. The ear ignores the low tones and fills in the missing fundamental to hear a tone with harmonics.

Cymbals *revel* in the randomness of the overtones. They are thick near the center and thin at the edges to make sound production efficient and vibration retention longer lasting. Because the bending motion on the cymbal can be large, vibrations can transfer from overtone to overtone (something which does not usually happen; vibrations are usually linear). Therefore the cymbal can store vibration in low frequency forms and transfer it to high frequency (more easily heard and more efficiently converted to sound) some time after the cymbal is struck.

Bells are plates which have been bent so much that it is no longer useful to think of them as plates. By carefully designing the shape and the way the thickness varies, the bell is given a series of overtones of frequencies

$$f = f_{\text{main}} \times \left(\frac{1}{2}\,,\, 1\,,\, 1.2\,,\, \frac{3}{2}\,,\, 2\,,\, \frac{5}{2}\,,\, 3\,,\, 4\,, \dots\right) .$$

Depending on the strike point, different harmonics are louder. Several harmonics roughly form a tone and harmonics.

The simplest drums consists of a membrane (traditionally made from an animal hide) stretched over a circular frame. The mathematical problem of finding the vibrational modes of an ideal membrane stretched over a perfectly stiff frame have been solved long ago, and unfortunately, the resonant frequencies are in a nearly random succession, getting closer and closer together in frequency as the frequency goes up.

[To find the resonant frequencies for this ideal situation, one should solve the wave equation

$$\nabla^2 z = \frac{1}{c^2_{\text{sound,membrane}}} \frac{d^2 z}{dt^2}$$

in cylindrical coordinates. Here z is the height of the membrane. To find periodic (resonant) solutions, one should use $z = z(r)e^{-i\omega t}$. The boundary conditions are that z is regular at $r = 0$ and vanishes at $r = R$ the radius of the drumhead. The problem can be solved by separation of variables, writing $z = z(r)e^{in\theta}$, with n an integer; the equation for $z(r)$ becomes,

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + \frac{n^2}{r^2} + \frac{\omega^2}{c^2} \right) z(r) = 0$$

which is called the **Bessel equation**. The solutions are some nasty functions, $J_n(r)$, called Bessel functions of the n'th degree. A great deal is known about these functions but there is no simple statement about where they have zeros (which one must know to determine the condition on ω^2/c^2 for $J_n(R) = 0$ to hold).]

For the ideal drumhead, relative to the lowest frequency, the vibration frequencies are,

$$f = f_{\text{lowest}} \times (\mathbf{1}, 1.59, 2.14, \mathbf{2.30}, 2.65, 2.92, 3.16, 3.50, \mathbf{3.60}, 3.65, 4.06, 4.15, \ldots) \ .$$

If you see a pattern, you know something no-one else knows. The modes in bold are the ones where the drumhead is moving up and down at the middle; for the other modes, the drumhead moves up and down on either side of the middle but the exact middle is at rest. If you strike a drum with these resonant frequencies, it will sound like noise which is not periodic and not of a definite pitch (which is true of a lot of drums). The different resonances appear as different patterns on the drumhead; striking the drumhead at a particular point excites the patterns which vibrate a lot at that point more than the patterns which vibrate at other points. This is why the drum sounds very different when struck in different places.

Many drums have the underside of the drumhead closed off rather than open to the air. That means that, when the drumhead moves up and down, it compresses and decompresses the air inside. This acts as an extra "spring," stiffening the drumhead–but only for those modes where the drumhead as a whole moves up and down, which are the ones in bold in the previous list. These frequencies are raised, especially the first one. They also damp down the fastest, especially the first one. By clever design, for a kettle drum the resonant frequencies become (naming the next to lowest frequency the "prime" frequency),

$$f = f_{\text{prime}} \times (0.85, 1, 1.51, 1.68, 1.99, 2.09, 2.44, 2.89, \ldots)$$

The sequence $1, 1.51, 1.99, 2.44, 2.89$ is almost $1, 1.5, 2, 2.5, 3$ which is the harmonic series of a missing fundamental. Your ear reconstructs this as "almost" a definite pitch.

There are more sophisticated drums which make more nearly periodic sounds. Perhaps the most sophisticated are the tin drums used in the Carribian, which are discussed extensively in the text.

In conclusion, the challenges of designing a percussion instrument are to make it loud, to make it sustain, to make it produce a periodic sound, and to make it possible to control the relative loudness of different overtones. Not all instruments solve all of these problems, nor is it always the intention to. Percussion is really a grab-bag of a large number of very distinct instruments.