Lecture 27: Piano, a Percussion String Instrument

One of the problems–often the most severe problem–in the design of a percussion instrument was, that it is difficult to get the resonant overtones to fall in a harmonic series. Well, a string fixed on both ends automatically has an overtone series which is harmonic (frequencies in ratio 1:2:3:4:5:. . .). Why not have a percussion instrument where the things being struck are long, thin strings under tension?

This is an old idea; hammer dulcimers existed in antiquity. However, the piano takes another idea, also very old, of having a keyboard, and combines them. In a hammer dulcimer, the player holds the hammers in the hands and directly strikes the strings with them. In a keyboard instrument, there is a row of keys which each actuate some sound producing device, such as a hammer or pick. This has a very substantial advantage for the performer; it makes it easy to hit the right sound producer and to hit many at a time. A keyboard is such a good idea that it is used in a very wide variety of instruments. For instance, in a harpsichord, each key operates something to pluck a string; in a celesta, each key operates a hammer which strikes a metal rod (as in a glockenspiel); in an organ, each key operates a valve which lets compressed air flow into a pipe. In the piano, each key operates a hammer which strikes a string (or more than one string tuned to the same frequency).

There are two big design complications associated with this idea:

- The action of the hammer must be quite complicated. The harder you push the key, the harder the hammer must be thrown at the string. The hammer must be thrown at the string, so that it is not left resting against the string (which would damp the vibration away again). It is generally not desired that the string should ring until the vibration dies away; the player should be able to control how long the string vibrates. There must therefore be dampers, one for each key, and the way the key is pressed and released must allow the player to control when the damper will and will not press against the string and absorb the vibration.
- The instrument is best if it has an enormous frequency range. Pianos conventionally have a range of more than 7 octaves, from A_0 to C_8. This (you can check) is about a factor of 154 in frequency. If all the strings are of the same thickness and are under the same tension, then if the shortest string is a few centimeters long, the largest must be a few *meters* long.

The first problem is solved by having a tremendously complicated "action" actuated by pressing each key. As the key is depressed, the force is transferred into throwing a hammer at the strings. Simultaneously, as the key is depressed, the damper is drawn back from the strings. The harder the key is pushed, the harder the hammer is thrown (hence "piano-forte," or "soft-loud")–this is the key advantage over the older harpsichord. The damper remains

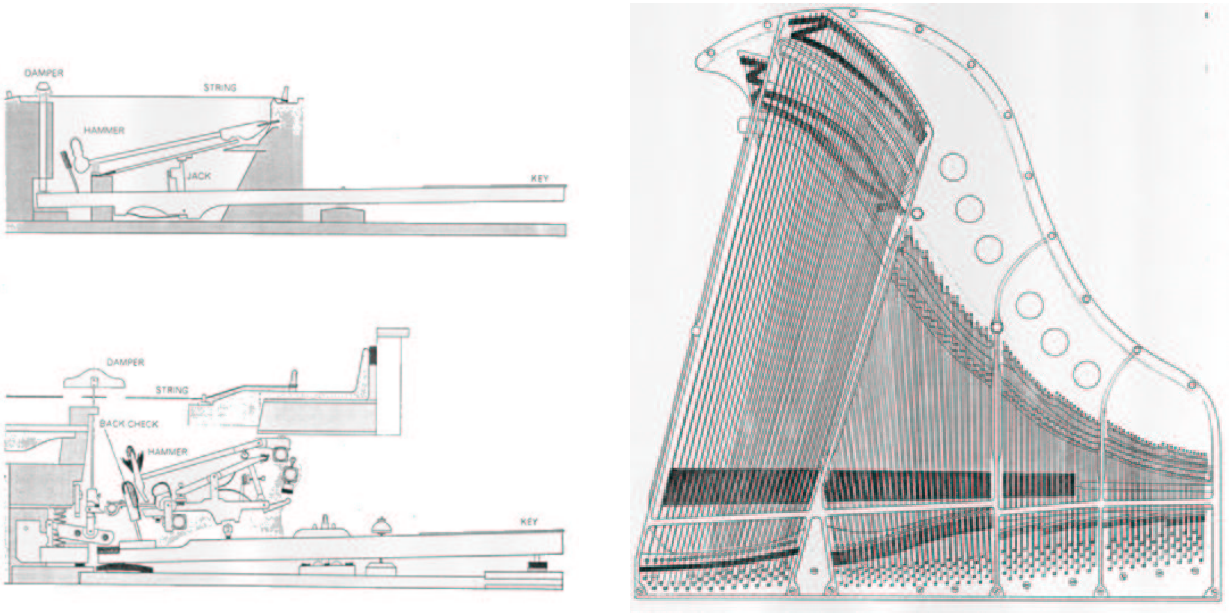


Figure 1: Left: illustration of a piano action, intended just to give the idea of how complicated it is. Right: illustration of how the strings are arranged to fit more compactly inside the body of the piano.

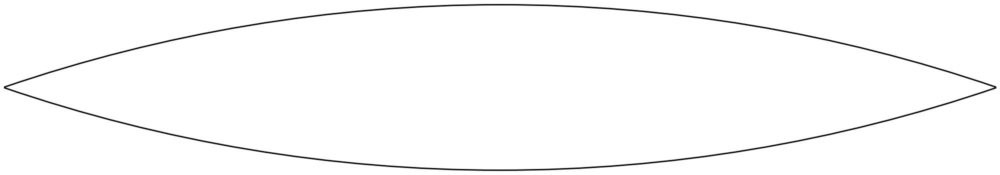
up as long as the key remains depressed. The key is counterweighted so that it will rise as soon as the finger is lifted off, and the damper will move up and stop the vibration of the string. Therefore, how long you hold down the key controls the "sustain." To make life more interesting and the design of the action more difficult, there are pedals, which modify what the action does. The exact function of each pedal varies between different piano designs, but usually includes a "sustain" pedal which removes all the dampers from the strings. The action must be designed so that pushing the pedal moves something which changes the way the action behaves. Piano actions are terribly complicated, as illustrated in figure 1.

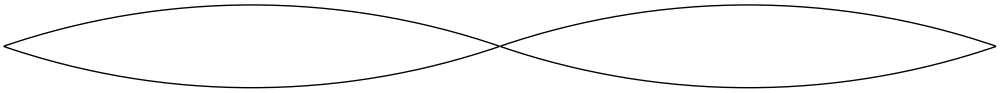
The second problem requires that, for compactness, some of the strings go at an angle above others, as also shown in the figure. Even so, it is impossible to make the strings as long as would be desirable. The speed of sound on the strings for the lowest notes must be made lower than on the higher notes, so they will not be as long as otherwise required. Either they must be made under low tension (which would lead to a wimpy sound), or they must be made thicker (the solution actually chosen). The lowest part of the piano register is therefore made of thick, wound strings which have a larger mass per length μ, and therefore a lower speed of sound. How many strings are wound, and how fat they are, depends on how big the piano is. The main advantage of a grand piano over an upright is that the lowest strings can be longer, and therefore do not need to be as heavily wound.

How is the vibration to be converted into sound? We already discussed that a string moving through air makes almost no sound. A large surface has to move back and forth against the air for efficient sound production. This is done in the piano, as in other string instruments, by having the piano strings pass over a bridge on a soundboard, a large flat piece of wood (typically spruce, a preferred material for other string instruments as well) with reinforcing braces. It is the vibration of the soundboard which produces the sound.

The problem of tuning a piano turns out to be quite complex. first, consider tuning each individual note. On the piano, most notes are played not by a single string, but by three. (This is not true of the low pitched, wound strings.) If there were only one string, or if the three were exactly in tune with each other, then their vibration energy would quickly be absorbed by the bridge into the soundboard. This would lead to a good attack but no sustain–a dead "plunk" sound. Instead, there are three strings which are deliberately slightly mis-tuned from each other. Right after they are struck, they pull up or down on the bridge in unison. Energy rapidly goes into sound production, giving a good attack to the note. But after half a beat frequency of the deliberate mis-tuning, one string is "up" when another is "down." That means that the 3 strings are no longer all pulling on the bridge at the same time. Their forces on the bridge mostly cancel out, and they no longer lose their energy so efficiently into the soundboard. This allows a better "sustain".

The other complication of piano tuning is that the strings do not actually have perfectly harmonic overtones. The very highest strings are very short, and the very lowest strings are very fat. In both cases, the strings do not actually act as perfectly "floppy." The rigidity of the strings is important. Compare the first and second vibration modes of a string:





For a string under tension, it is the slope of the string which tells how much energy is stored. Recall that, in the analogy between sound on a string and in the air, slope was analogous to pressure in the air. However, for a string with its own stiffness, it is how much bending of the string there is, that is, the curvature is what costs energy. For a given amount of slope, the curvature is higher for higher overtones. While, for an ideal string fixed at both ends, the overtone series is,

$$\text{ideal string:} \qquad f = f_0 \times \Big(1, 2, 3, 4, 5, \dots\Big)$$

for an ideal bar (the extreme case of the string's stiffness deciding what happens), fixed on both ends, the series is,

$$\text{ideal bar:} \qquad f = f_0 \times \Big(1, 4, 9, 16, 25, \ldots\Big)$$

For a string which has some stiffness, the series goes like,

$$\text{Real string:} \qquad f = \Big(f_T + f_b\,,\; 2f_T + 4f_b\,,\; 3f_T + 9f_b\,,\; 4f_T + 16f_b\,,\; \ldots\Big)$$

where f_T is the frequency of the fundamental because of the string tension and f_b is the frequency because of the stiffness of the string. Even in the top and bottom strings on the piano, the stiffness is only a small correction to the frequencies of the notes. Therefore, we can treat f_b as a correction in the above. Defining $f_0 = f_T + f_b$, and $A = f_b/f_T$ (the anharmonicity), the frequency series for a real string is,

$$\text{Real string:} \qquad f = f_0 \times \Big(1\,,\; 2{+}2A\,,\; 3{+}6A\,,\; 4{+}12A\,,\; \ldots\Big)\,.$$

We see that this correction will make the overtones' frequencies sharp compared to the fundamental. That is, for a string where you cannot neglect the stiffness, the overtones get sharper as you go up the overtone series. Remember that, even if this is a small effect, your ear is very sensitive to accuracy in frequencies, so it is still important.

What is most important to the listener is that there are no beats. When playing a note and its octave, this means that the octave must be in tune with the first overtone of the lower note–rather than being at exactly twice the frequency of the lower note. In the middle of the piano's range, the strings are long and thin, so A is almost zero. The overtones are harmonic, and tuning should be done in the normal way. However, the highest strings are short, so their stiffness is more important. The lowest strings are fat, so their stiffness is also more important. Therefore, at the very top and the very bottom of the range, the overtones are sharp and the intervals must be "stretched." That means that the top notes on the piano must be deliberately tuned sharp. For instance, G_7 must be more than twice the frequency of G_6, because the overtone of G_6 is sharp and G_7 needs to coincide with that overtone. C_1, on the other hand, must be tuned flat, so that its too-sharp overtone will line up with C_2. (Recall that, in tuning a piano, one starts in the middle and tunes towards the edges of the range.) This "stretch tuning" is particularly severe for the bottom range of upright pianos, where the strings are much shorter than would be ideal and must be very heavily wound. This is why the bottom octave or so of an upright piano sounds so bad, no matter how well tuned and how high quality the upright is.