Physics 225

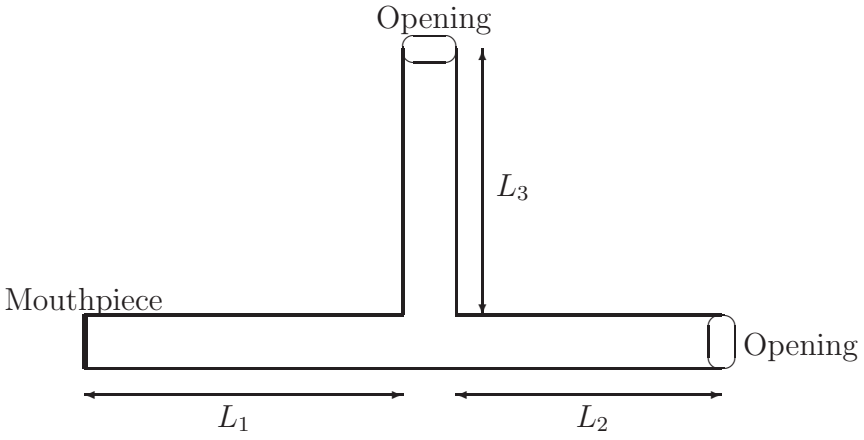
Homework Assignment 10

Due 10 April 2008

Grade on this homework will replace
your lowest homework grade on HW 1–9.

1 Weird instrument

A strange music design experimenter decides to build an instrument where a cylindrical pipe branches into two pipes:



The pipes are all cylinders, 1 cm in radius, and the musician will put a clarinet mouthpiece at the closed end labeled "mouthpiece" above. I will call the three pieces of pipe the "mouthpiece pipe" (length L_1) and the two "branch pipes" (lengths L_2 and L_3).

Approximate that the open ends of the two pipes are "ideal" openings, that is, the input impedances are $Z_L = 0$ at each of these openings. Also ignore losses in the pipe, that is, take the intrinsic impedance of the pipe to be $Z_0 = \rho c_s/S$ with S the cross-sectional area of the pipe. Neglect the length of the junction.

1. Write an expression for the impedance of a branch pipe at the junction, as a function of frequency and the length of the pipe.

2. Suppose the two open branches have lengths $L_2 = 0.2\,\text{m}$ and $L_3 = 0.2\,\text{m}$. We are interested in the behavior of the instrument at 220 Hz, which is a musical A. At this frequency, what is the impedance of each branch pipe at the junction?

3. Explain why the impedance which the mouthpiece tube "feels" at the junction (the load impedance felt by the mouthpiece tube) is given by adding the reactances of the two branches: that is, explain why

$$\frac{1}{Z_{L,\text{mouthpipe}}} = \frac{1}{Z_{\text{branch 1}}} + \frac{1}{Z_{\text{branch 2}}}\,.$$

   Hint: Impedance Z is pressure over flow. $1/Z$ is flow over pressure.

4. Compute the input impedance at the mouthpiece. (To do this, use the "load" impedance you computed in part 3. as Z_L.) How long should the mouthpiece pipe be to have an impedance peak right at the mouthpiece for the frequency 220 Hz?

5. **EXTRA CREDIT** (10 points): For those who want to do something a little advanced for extra credit: for the case where $L_1 = 30\,\text{cm}$, $L_2 = 20\,\text{cm}$ and $L_3 = 15\,\text{cm}$, make a plot of the input (mouthpiece) impedance as a function of frequency from 0 to 2000 Hertz, INCLUDING the finite wall resistances and the real (radiation) part of the open pipe impedances $Z_L = \rho c_s \pi/\lambda^2$. Are the overtones in harmonic relation, ie, do they fall in a series like f_0 times (1,3,5,...) or $(1, 2, 3, \ldots)$?