Physics 551 Homework 10

Due Friday 14 November 2014

1 Time-dependent Perturbations and Squeezing

Consider the Simple Harmonic Oscillator with a time dependent spring constant [again using dimensionless units]

$$H = \frac{-\partial_x^2}{2} + \frac{1+S(t)}{2} x^2 \tag{1}$$

where $S(t)$ is a small time-dependent contribution to the spring constant. The function $S(t)$ is zero for all $t < 0$ and all $t > t_1$, but is nonzero (though small) in between these times. The oscillator is in the ground state $|\psi(t=0)\rangle = |0\rangle$ at time $t = 0$.

First, give a simple symmetry argument to show that $\langle x \rangle = 0 = \langle p \rangle$ at all times.

Second, use time-dependent perturbation theory to determine the interaction-picture amplitude for the system to be in the state $|2\rangle$ at time t_1, to leading (linear) order in small $S(t)$. In terms of the Fourier transform $\tilde{S}(\omega)$, what value or values of ω contribute?

Third, assume that the final state $|\psi(t_1)\rangle_I$ is a squeezed state. What value of the squeezing parameter ζ has the same $|2\rangle$ amplitude (size and phase)? (Expand the squeezed state to linear order in ζ.)

Fourth, find the amplitude to be in $|4\rangle$, in the interaction picture, to second order in small $S(t)$. Expand the state with squeezing parameter ζ which you found at leading order to determine what $|4\rangle$ content it will have at order ζ^2, and show that the two results are compatible. (This is an actual test of whether it is really a squeezed state, rather than just a claim.)

2 Sudden and adiabatic approximations

Consider the Simple Harmonic Oscillator with a force term

$$H = \frac{-\partial_x^2}{2} + \frac{1}{2} x^2 - F(t) x \,, \tag{2}$$

where the force takes the particular time dependence

$$F(t) = \frac{F_0}{1 + \exp(-t/\tau)} \,. \tag{3}$$

Suppose that the oscillator starts in the ground state at $t \to -\infty$, where $F(t) = 0$. We know that the oscillator state will be a coherent state at all times.

- For what range of τ values do you expect the *sudden approximation* to be valid? According to the sudden approximation, what should be the coherence parameter $\alpha(t)$ at large $t \gg \tau$?

Hint: in terms of the original simple harmonic oscillator, the oscillator at $t \gg \tau$ has a shifted zero-point x_0. Therefore, in terms of the original $F = 0$ oscillator, the final ground state has a fixed nonzero value of α. Time evolution should have α rotate around this shifted value, not around the original $\alpha = 0$ ground-state value.

- For what range of τ values do you expect the *adiabatic approximation* to be valid? According to this approximation, what should be the coherence parameter α at large $t \gg \tau$?

- Use the result of a previous homework to write an explicit expression for the coherence parameter $\alpha(t)$ in terms of a single integral. Now perform this integral explicitly, using contour methods, for the case $t \gg \tau$ (late times). **Warning**: this may be hard. If you seem to be getting no-where, give up and work on other problems! But at least give it the old college try!

  Show that your explicit expression reduces to the sudden and adiabatic versions in the appropriate limits, and comment on the size of the first corrections in each case.

3 Spin flipping

When a spin-$\frac{1}{2}$ particle is in a magnetic field the two spin states are split by the Zeeman effect,

$$H = \mu S_i B_i \,, \tag{4}$$

with μ the magnetic moment and S_i, B_i the spin operator and the magnetic field respectively. For electrons

$$\mu = -2g\frac{\mu_B}{\hbar} \,, \quad g = 1.0011596521807 \,, \quad \mu_B = \frac{e\hbar}{2m_e c} = 9.27400968 \times 10^{-24} \text{ J/T} \tag{5}$$

(Joules per Tesla). We want to create a population inversion by applying a strong magnetic field in the z direction to a cold sample for a long time, so the spins will thermally relax to the lower state, and then suddenly flipping the field to the $-z$ direction, so the spins are in the higher-energy state.

Suppose there is a small stray x-component to the magnetic field, $B_x = 10^{-6}$ Tesla. Write the Hamiltonian in the z-basis in terms of the B_z and B_x values. When we flip the B_z field, how fast does B_z need to change with time as it crosses through zero to ensure that more than half the spins end up in the higher-energy state? Use the Landau-Zener formula. Your intermediate steps should be expressions but your final answer should be a number in Tesla per second.