Physics 551 Homework 13

Due Friday 12 December 2014

1 Commutator of boosts and Thomas precession

A Lorentz transformation

$$\Lambda^\mu{}_\nu = \exp \omega^\mu{}_\nu \tag{1}$$

in which the space-space ω_{ij} are all zero, but there are nonzero ω_{0i}, is called a *boost*. Consider the boost transformation generated by $\omega^0{}_x = \epsilon_x = \omega^x{}_0$ and all other components being zero. Find $\Lambda^\mu{}_\nu$ for this boost to *second* order in ϵ_x, and call it Λ_1. Similarly, find Λ_2 which exponentiates the choice where $\omega^0{}_y = \epsilon_y = \omega^y{}_0$ and all other components are zero, to second order.

Compute $\Lambda_1\Lambda_2$ and $\Lambda_2\Lambda_1$ to second order in ϵ's and show that, at this order, the difference is a rotation. What is the magnitude and axis of the rotation?

Consider an object in classical circular motion in the xy plane, centered on the origin and with speed v and radius r. Describe the particle's acceleration as a (continuous) series of boosts. Do the boosts all commute? After a complete circular orbit, by what angle and about what axis has the object been rotated due to the non-commutation of the boosts? Work to the lowest nontrivial order in v/c. This phenomenon is called *Thomas precession*.

2 Conserved Dirac current

Show that if Ψ obeys the Dirac equation then the current

$$j^\mu \equiv \bar{\Psi}\gamma^\mu\Psi \tag{2}$$

is conserved, $\partial_\mu j^\mu = 0$. (Use "my" metric and gamma-matrix conventions or those of the book, at your convenience.)

3 Large hard sphere

Consider scattering from a hard sphere: $V(r) = 0$ for $r > r_0$ and $V(r) \gg E$ for $r < r_0$. Assume $kr_0 \gg 1$ (the sphere is large compared to the wave length of the particle).

1. Use classical arguments to guess the total scattering cross-section.

2. For what range of ℓ does the sphere's boundary r_0 occur in the classically forbidden region, and for what ℓ does it occur in the classically allowed region?

3. Explain why, if r_0 occurs in the classically forbidden region, the phase shift will be small (and therefore not contribute much to the total cross-section).

4. Show that, if r_0 occurs in the classically allowed region, the phase shift is large. Since different ℓ will involve different large phases, assume that, for averaging the value of $\sin^2(\delta_\ell)$ over several ℓ values, we can treat the phase as *random*. In this approximation, what is $\langle \sin^2 \delta_\ell \rangle$?

5. Estimate σ_{tot} by summing the contributions from all the ℓ values where δ_ℓ is large. Does your result agree with your guess? If not, why do you think it is different?

4 Atoms, E&M, Density Matrices and Wigner-Eckart

Here is a "review" problem with a bit of several topics all together.

An H atom is initially in the 4f0 state (that is, $n = 4, \ell = 3, m = 0$ so $L^2 |\psi\rangle = 12\hbar^2 |\psi\rangle$ and $L_z |\psi\rangle = 0$). It emits a single photon.

What is the most likely final n, ℓ, and what are the possible m values, after the decay? Assume that the decay occurs to the n, ℓ values you find.

The final L_z-value of the hydrogen atom is *entangled* with the final J_z-value of the photon. Write the reduced density matrix after tracing out the photon state. What is the associated entanglement entropy? *Hint*: use angular momentum conservation and the Wigner-Eckart theorem to express the initial (total) angular momentum as a mixture of possible final states with the same total m-value, with coefficients given by the appropriate Clebsch-Gordan coefficients. If necessary write out the density matrix explicitly, but you may be able to trace over the photon state without doing so.