Physics 610 Homework 9

Due Wednesday 28 November 2012

1 Spinor-Scalar Scattering in Yukawa Theory

Consider Yukawa theory, with one Dirac fermion ψ and one real scalar field ϕ, with Lagrangian

$$\mathcal{L} = -\bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{M^2}{2}\phi^2 - \frac{\lambda}{24}\phi^4 - \phi\bar{\psi}(y - i\gamma^5 y')\psi\,. \tag{1}$$

Assume that y and y' are small but comparable to each other.

Consider the process in which a fermion of momentum p scatters off a scalar of momentum k, to the final state containing a fermion of momentum p' and scalar of momentum k'.

1.1 Matrix element

At lowest order in the Yukawa couplings (which will be second order), the scattering arises from two diagrams. Draw the diagrams, and use them to write an expression for the matrix element for the scattering process to this order.

1.2 Nonrelativistic case

Consider the case where p, k are both close to rest, $p^\mu = (E, 0, 0, p_z)$ with $p_z \ll E \simeq m$ and $k^\mu = (E', 0, 0, -p_z)$ with $E' \simeq M \gg p_z$. For this case, at lowest (zero) order in p_z, use the explicit expressions for the spinors $u(p, \sigma)$ and $\bar{u}(p, \sigma)$ and for the gamma matrices to simplify your expression for the matrix element for each of the four cases:

- initial spin $\uparrow$ and final spin $\uparrow$,
- initial spin $\uparrow$ and final spin $\downarrow$,
- initial spin $\downarrow$ and final spin $\uparrow$,
- initial spin $\downarrow$ and final spin $\downarrow$.

Your result should depend only on y, y', and the two masses.

1.3 Spin averaging

The book presents a derivation for the case where $y' = 0$ but $y \neq 0$. So consider instead the case $y = 0$ but $y' \neq 0$. For this case, consider generic p, k (make no assumption about the relative size compared to the two masses). Compute the squared matrix element, averaged over the initial spin and summed over the final spin of the fermion. Call it $|\overline{\mathcal{M}}|^2$.

Express your result in terms of the Mandelstamm variables and the particle masses.

1.4 Limits

Find the limit of $|\overline{\mathcal{M}}|^2$ from the last subsection, in each of the following limits:

- the nonrelativistic limit
- the ultra-relativistic limit in which one neglects both masses: $m^2, M^2 \ll s, |t|, |u|$.

Verify that the nonrelativistic limit coincides with the spin averaged and summed square of what you found directly two subsections ago.

1.5 Total cross-section

Consider the ultra-relativistic limit. Work in the center of mass frame. Express the squared matrix element as a function of s and of the cosine of the angle between $\vec{p}$ and $\vec{p}'$, $\cos\theta_{pp'}$. (It might help to first find expressions for t, u in terms of this angle.)

Express the cross section as an integral over the angle $\theta_{pp'}$, of a function of $\cos\theta_{pp'}$. Don't do the integral (it may result in a log divergence!)