Physics 673 Homework 1 Help

1 Problem 1

The hard part of this problem is to find the particle production. The key is to realize that turning on the "electric field" or the source J is shifting the position of the minimum of each harmonic oscillator. When you do that, the oscillator goes from being in its ground state to being in a *coherent state*. The beautiful thing about a coherent state is that it is just the ground state plus classical motion of the central value; the value of $\langle x\rangle$ and $\langle p\rangle$ is given by the classical expression and the energy exceeds the classical one by the classical expression in terms of these averaged coordinates. When you switch the linear J term back off, you have a coherent state of the original operator, due to the shift and $\langle p\rangle$ the oscillator took on while J was turned on. If you find an expression for this, you can find the excess energy put into the operator, and find the particle number from $n\omega = \Delta E$.

2 Problem 3

To help you find the classical solution $\varphi_{\rm cl}$: note that a classical solution is one which satisfies the classical equations of motion, which are the Euler-Lagrange equations. We are looking for a solution which furthermore is stationary in time. Therefore the Euler-Lagrange equation reduces to

$$\nabla_x^2 \varphi = \frac{dV(\varphi)}{d\varphi} = \lambda\phi\left(\phi^2 - \frac{m^2}{\lambda}\right) . \tag{1}$$

Look for a solution with $\varphi(x=0)=0$. Here are some hints:

The above equation has an analogue in classical mechanics, if you think of φ as the position of a particle, x as time, and $-V$ as the potential as a function of position φ. If I rename $x \to t$ and $\varphi \to x$, the equation is

$$\frac{d^2x}{dt^2} = dV(x)/dx \tag{2}$$

which is the equation for a particle in a potential $-V$. (Note the sign.) This potential has two maxima at $\pm m/\sqrt{\lambda}$ and one classical solution is for the particle to roll from one minimum at early time to the other at late time. You know that the total energy is conserved, which means that $\frac{(dx/dt)^2}{2} = -V(x) + V(x_0)$ with x_0 the starting (maximum) value. For the theory we actually care about, the analogous expression is

$$\frac{1}{2}(\nabla_x \varphi)^2 = \frac{\lambda}{4}\left(\varphi^2 - \frac{m^2}{\lambda}\right)^2 . \tag{3}$$

You may be able to take the square root and then explicitly integrate to find the solution.

An expansion in small fluctuations means you are going to make the substitution

$$\varphi(x,t) = \varphi_{\rm cl}(x) + \eta(x,t) , \tag{4}$$

insert it in the Lagrangian, and expand to second order in η. You will only find higher-than-second order terms from the potential term; drop them. You will also find linear terms in η but they will vanish up to a total derivative when you put in the explicit form of $\varphi_{\rm cl}$. As advertized you should find that the equation of motion for η is that of a field in a potential with the $3\tanh(y) - 1$ form (up to overall normalization and the relation between x and y, which you have to figure out for yourself).

See how far you can get with part 3 of the problem, but if you have trouble do not worry too much. It is actually not so easy to properly define and regulate the sums you need to be taking here. If you want help on this problem bother Keshav.