# Physics 673 Homework 6

Due 3 April 2007

## 1 Top quark production

In this problem we will compute the total cross-section to produce $t\bar{t}$ pairs in a proton-antiproton collider (such as the Tevatron) and in a proton-proton collider (such as the LHC). Since this involves working with parton distribution functions, the problem will have a numerical aspect.

### 1.1 Tree level cross-sections

At lowest order and at tree level, top quark pairs are produced by two processes: $gg \to t\bar{t}$ and $q\bar{q} \to t\bar{t}$. Here $g$ is a gluon and $q, \bar{q}$ are any other quark flavor and its antiparticle.

Compute the squared matrix element, averaged over initial spins and colors and summed over final spins and colors, for a $q\bar{q}$ pair to annihilate into a $t\bar{t}$ pair. Express your answer as a function of the Mandelstam variables $\hat{s}, \hat{t}, \hat{u}$ and the top quark mass squared $m_t^2$. Feel free to neglect the mass of the light quark; but you *cannot* neglect the top quark mass. [Here $\hat{s}$, $\hat{t}$, $\hat{u}$ are the Mandelstam variables for the partons; we reserve $s, t, u$ for the Mandelstam variables describing the full momentum of the system, ie, those referring to the proton rather than just a quark drawn from the proton.]

Do not compute the squared matrix element for the other process (which would involve extending a problem from last homework to the finite mass case); just verify that the following result,

$$
\begin{aligned}
|\bar{\mathcal{M}}|^2_{gg\to t\bar{t}} \; = \; g_s^4 \Bigg[ & \frac{3(m^2-\hat{t})(m^2-\hat{u})}{4\hat{s}^2} - \frac{m^2(\hat{s}-4m^2)}{24(m^2-\hat{t})(m^2-\hat{u})} \\
& + \frac{(m^2-\hat{t})(m^2-\hat{u}) - 2m^2(m^2+\hat{t})}{6(m^2-\hat{t})^2} - \frac{3(m^2-\hat{t})(m^2-\hat{u}) + 3m^2(\hat{u}-\hat{t})}{8\hat{s}(m^2-\hat{t})} \\
& + \frac{(m^2-\hat{u})(m^2-\hat{t}) - 2m^2(m^2+\hat{u})}{6(m^2-\hat{u})^2} - \frac{3(m^2-\hat{u})(m^2-\hat{t}) + 3m^2(\hat{t}-\hat{u})}{8\hat{s}(m^2-\hat{u})} \Bigg] ,
\end{aligned} \qquad (1)
$$

reduces to the result of the last homework in the $m \to 0$ limit. Here $m$ is short for $m_t$ the top quark mass.

Integrate each expression over final state phase space to determine the total cross-section for each process as a function of $\hat{s}$, remembering that for massless initial states

$$\frac{1}{2p_0 2p'_0 |v_{\mathbf{p}} - v_{\mathbf{p}'}|} = \frac{1}{2\hat{s}} \tag{2}$$

and that

$$\int \frac{d^3k d^3k'}{(2\pi)^6 2k^0 2k'^0}(2\pi)^4\delta^4(p+p'-k-k') = \frac{1}{8\pi\hat{s}}\int d\hat{t}\,. \tag{3}$$

You will have to determine the integration range for $\hat{t}$ and the relation between $\hat{u}$, $\hat{t}$, and $\hat{s}$–remember the top quark mass. Express your answer as a function of $\alpha_{\mathrm{s}}$, $\hat{s}$, and $m_t^2$.

## 1.2 Total production rate

Consider the collision of a proton with an antiproton. Recall that the parton distribution functions $q(x), \bar{q}(x), g(x)$ are defined as the probability density of finding a quark, antiquark, or gluon in a proton containing momentum fraction $x$.

If a proton contains a quark of momentum fraction $x$ and an antiproton contains an antiquark of momentum fraction $x'$; and if the squared center of mass energy of the proton-antiproton system is $s$, what is the Mandelstam $\hat{s}$ for quark-antiquark system?

Write an expression in terms of a double integral (integrals over $x$, $x'$) for the total cross-section for

1. a $q$ in the proton and a $\bar{q}$ in the antiproton to annihilate into a $t\bar{t}$ pair;
2. a $\bar{q}$ in the proton and a $q$ in the antiproton to annihilate into a $t\bar{t}$ pair;
3. a $g$ in the proton and a $g$ in the antiproton to annihilate into a $t\bar{t}$ pair.

Repeat for the case of a $pp$ (proton-proton) collision. Hint: the $\bar{q}$ parton distribution in an antiproton equals the $q$ distribution in the proton, and vice versa. The gluon distributions are the same.

Your expressions should depend only on $s$, $m_t$, and the parton distribution functions $u(x), \bar{u}(x), d(x), \bar{d}(x), \ldots, g(x)$.

## 1.3 Actual evaluation

Use the data provided in the table, interpolation, and numerical integration to make a plot of the partial rate of $t\bar{t}$ production by each of the three processes mentioned above, as a function of the center of mass energy $\sqrt{s}$, in the range from 500 GeV to 20 TeV. If possible, put the plots for $pp$ and for $p\bar{p}$ on the same plot. Make it a log-log plot!

Now answer the following questions:

- If the Tevatron (initial CM energy 1.8 TeV) was intended as a top quark discovery machine, why was it worth their while to make it a $p\bar{p}$ machine rather than a (much simpler!) $pp$ machine?

- How much did the energy boost from 1.8 TeV to 1.96 TeV increase the $t\bar{t}$ production rate at the Tevatron?

- The LHC (design CM energy 14 TeV) will also study top quark physics. Why was it OK to go with a $pp$ machine?

| $x$ | $xg$ | $x(u-\bar{u})$ | $x\bar{u}$ | $x(d-\bar{d})$ | $x\bar{d}$ | $xs = x\bar{s}$ | $xc = x\bar{c}$ |
|---|---|---|---|---|---|---|---|
| 0.0010 | 34.0939 | 0.0792 | 1.7278 | 0.0473 | 1.7338 | 1.5954 | 1.4282 |
| 0.0020 | 23.0507 | 0.1087 | 1.2667 | 0.0673 | 1.2743 | 1.1521 | 1.0051 |
| 0.0030 | 18.0777 | 0.1313 | 1.0477 | 0.0823 | 1.0567 | 0.9424 | 0.8067 |
| 0.0040 | 15.1118 | 0.1503 | 0.9119 | 0.0945 | 0.9222 | 0.8128 | 0.6851 |
| 0.0050 | 13.0836 | 0.1670 | 0.8162 | 0.1050 | 0.8277 | 0.7219 | 0.6002 |
| 0.0100 | 8.1073 | 0.2313 | 0.5659 | 0.1430 | 0.5825 | 0.4869 | 0.3841 |
| 0.0150 | 5.9592 | 0.2784 | 0.4469 | 0.1681 | 0.4675 | 0.3776 | 0.2863 |
| 0.0200 | 4.7153 | 0.3158 | 0.3726 | 0.1865 | 0.3965 | 0.3108 | 0.2280 |
| 0.0250 | 3.8882 | 0.3467 | 0.3200 | 0.2005 | 0.3466 | 0.2645 | 0.1884 |
| 0.0300 | 3.2938 | 0.3726 | 0.2801 | 0.2113 | 0.3089 | 0.2299 | 0.1596 |
| 0.0400 | 2.4889 | 0.4136 | 0.2224 | 0.2263 | 0.2542 | 0.1811 | 0.1201 |
| 0.0500 | 1.9660 | 0.4439 | 0.1819 | 0.2351 | 0.2155 | 0.1476 | 0.0942 |
| 0.0600 | 1.5980 | 0.4658 | 0.1517 | 0.2396 | 0.1860 | 0.1231 | 0.0759 |
| 0.0700 | 1.3250 | 0.4815 | 0.1282 | 0.2411 | 0.1623 | 0.1042 | 0.0623 |
| 0.0800 | 1.1154 | 0.4922 | 0.1094 | 0.2402 | 0.1427 | 0.0891 | 0.0519 |
| 0.0900 | 0.9494 | 0.4986 | 0.0942 | 0.2376 | 0.1261 | 0.0769 | 0.0437 |
| 0.1000 | 0.8154 | 0.5015 | 0.0816 | 0.2337 | 0.1119 | 0.0668 | 0.0371 |
| 0.1100 | 0.7053 | 0.5014 | 0.0711 | 0.2287 | 0.0994 | 0.0583 | 0.0317 |
| 0.1200 | 0.6139 | 0.4990 | 0.0622 | 0.2229 | 0.0885 | 0.0511 | 0.0273 |

| $x$ | $xg$ | $x(u-\bar{u})$ | $x\bar{u}$ | $x(d-\bar{d})$ | $x\bar{d}$ | $xs = x\bar{s}$ | $xc = x\bar{c}$ |
|---|---|---|---|---|---|---|---|
| 0.1300 | 0.5367 | 0.4943 | 0.0547 | 0.2165 | 0.0788 | 0.0449 | 0.0236 |
| 0.1400 | 0.4713 | 0.4879 | 0.0484 | 0.2097 | 0.0701 | 0.0396 | 0.0205 |
| 0.1500 | 0.4152 | 0.4800 | 0.0429 | 0.2025 | 0.0624 | 0.0349 | 0.0178 |
| 0.1600 | 0.3669 | 0.4708 | 0.0381 | 0.1951 | 0.0555 | 0.0309 | 0.0156 |
| 0.1700 | 0.3250 | 0.4606 | 0.0340 | 0.1875 | 0.0493 | 0.0274 | 0.0137 |
| 0.1800 | 0.2884 | 0.4494 | 0.0304 | 0.1798 | 0.0437 | 0.0243 | 0.0120 |
| 0.1900 | 0.2565 | 0.4376 | 0.0273 | 0.1721 | 0.0388 | 0.0215 | 0.0105 |
| 0.2000 | 0.2285 | 0.4251 | 0.0245 | 0.1645 | 0.0343 | 0.0191 | 0.0093 |
| 0.2200 | 0.1819 | 0.3988 | 0.0199 | 0.1494 | 0.0267 | 0.0150 | 0.0072 |
| 0.2400 | 0.1453 | 0.3714 | 0.0162 | 0.1349 | 0.0207 | 0.0119 | 0.0057 |
| 0.2600 | 0.1163 | 0.3437 | 0.0133 | 0.1211 | 0.0158 | 0.0093 | 0.0044 |
| 0.2800 | 0.0932 | 0.3159 | 0.0109 | 0.1080 | 0.0121 | 0.0073 | 0.0035 |
| 0.3000 | 0.0748 | 0.2886 | 0.0089 | 0.0958 | 0.0091 | 0.0057 | 0.0027 |
| 0.3200 | 0.0599 | 0.2622 | 0.0072 | 0.0845 | 0.0068 | 0.0045 | 0.0022 |
| 0.3400 | 0.0480 | 0.2368 | 0.0059 | 0.0741 | 0.0050 | 0.0035 | 0.0017 |
| 0.3600 | 0.0383 | 0.2125 | 0.0047 | 0.0647 | 0.0037 | 0.0027 | 0.0013 |
| 0.3800 | 0.0305 | 0.1897 | 0.0038 | 0.0561 | 0.0027 | 0.0021 | 0.0010 |
| 0.4000 | 0.0243 | 0.1683 | 0.0030 | 0.0483 | 0.0019 | 0.0016 | 0.0008 |
| 0.4250 | 0.0181 | 0.1436 | 0.0022 | 0.0397 | 0.0013 | 0.0011 | 0.0006 |
| 0.4500 | 0.0134 | 0.1214 | 0.0016 | 0.0324 | 0.0008 | 0.0008 | 0.0004 |
| 0.4750 | 0.0098 | 0.1014 | 0.0011 | 0.0260 | 0.0005 | 0.0006 | 0.0003 |
| 0.5000 | 0.0072 | 0.0838 | 0.0008 | 0.0207 | 0.0003 | 0.0004 | 0.0002 |
| 0.5250 | 0.0051 | 0.0685 | 0.0005 | 0.0163 | 0.0002 | 0.0003 | 0.0002 |
| 0.5500 | 0.0037 | 0.0551 | 0.0004 | 0.0126 | 0.0001 | 0.0002 | 0.0001 |
| 0.6000 | 0.0018 | 0.0342 | 0.0001 | 0.0072 | 0.0000 | 0.0001 | 0.0000 |
| 0.7000 | 0.0003 | 0.0104 | 0.0000 | 0.0018 | 0.0000 | 0.0000 | 0.0000 |
| 0.8000 | 0.0000 | 0.0019 | 0.0000 | 0.0003 | 0.0000 | 0.0000 | 0.0000 |
| 0.9000 | 0.0000 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Table 1: Parton distribution functions at the renormalization scale $Q^2 = (2m_t)^2$, **TIMES** $x$, tabulated as a function of Bjorken $x$. The bottom quark PDF is negligible. The table is also available as a data file from the course page.