Physics 673 Homework 7

Due 17 April, 2007

1 SU(3) with a Fundamental Higgs

Consider SU(3) gauge theory. First consider a theory with one complex fundamental representation scalar field

$$\phi = \begin{bmatrix} \phi_r \\ \phi_g \\ \phi_b \end{bmatrix}$$

with ϕ_r, ϕ_g, ϕ_b complex scalar fields. The only gauge invariant, renormalizable potential possible for this field is

$$V(\phi) = m^2 \phi^\dagger \phi + \frac{\lambda}{2} (\phi^\dagger \phi)^2 \,. \tag{1}$$

Assume that $m^2 < 0$. What is the (classical) VEV squared $\phi_0^\dagger \phi_0$ taken on by the field? Argue that (by using up our gauge freedom) one can choose ϕ to take the form

$$\phi = \begin{bmatrix} 0 \\ 0 \\ v \end{bmatrix} \tag{2}$$

with v real and equal to the VEV found above.

Recall that an infinitesimal gauge transformation on ϕ is a rotation of form

$$\phi \rightarrow \left[1 + ig\alpha_a \frac{\lambda_a}{2} \right] \phi \,, \tag{3}$$

with $a = 1, \ldots, 8$, α_a the infinitesimal gauge change parameters, and λ_a the Gell-Mann matrices. Identify which Gell-Mann matrices *do* rotate ϕ from being in the form of Eq. (2), and which do not. Explain (or show) why the gauge fields corresponding to the transformations which *rotate* v become massive, but those corresponding to transformations which do not change v remain massless. Prove that the transformations which do not change the vacuum value of ϕ will form a *subgroup* of the group SU(3). What is its dimension? What group is it?

Provide a counting argument that there *had* to be a subgroup left invariant, because ϕ does not provide enough degrees of freedom to provide longitudinal components to all of the gauge bosons A^a_μ of SU(3). Given that one degree of freedom of ϕ (the radial mode) is not available to be "eaten," what is the largest number of elements in SU(3) which can be made massive, and therefore what is the smallest dimension that the remaining subgroup could have been?

2 SU(3) with an adjoint Higgs

Repeat the last problem for a real adjoint scalar field Φ_a. The easiest way to work with Φ_a is to write it as a 3×3 matrix,

$$\Phi \equiv \Phi_a \frac{\lambda_a}{2} \qquad \text{transforms as} \qquad \Phi_a \frac{\lambda_a}{2} \to \left[1 + ig\alpha_b \frac{\lambda_b}{2}\right] \Phi_a \frac{\lambda_a}{2} \left[1 - ig\alpha_c \frac{\lambda_c}{2}\right] . \tag{4}$$

Verify that this transformation rule is equivalent to the adjoint transformation rule

$$\Phi_a \to \Phi_a + igF_{ab}{}^{c}\Phi_b\alpha_c = \Phi_a + gf_{abc}\Phi_b\alpha_c \, . \tag{5}$$

The most general renormalizable potential is

$$V(\Phi) = m^2 \operatorname{Tr} \Phi^2 - \kappa \operatorname{Tr} \Phi^3 + \frac{\lambda}{2}(\operatorname{Tr} \Phi^2)^2 \, . \tag{6}$$

Show that, at equal values of $\operatorname{Tr} \Phi^2$, and regardless of the sign of κ, the κ term gives a lower energy for $\Phi \propto \lambda_8$ than for $\Phi \propto \lambda_3$, so the VEV will take the former form (up to an SU(3) rotation).[1] Determine the VEV v in terms of m^2, κ, and λ, making no assumptions about the sign of m^2 or κ. Determine which gauge transformations leave this VEV unchanged, and which rotate it. What is the unbroken gauge group "left over" when Φ takes on its vacuum expectation value? How many of the components of Φ were not "eaten"?

3 Gauge independence of physical quantities

Consider the abelian Higgs model: a U(1) gauge field A_μ is coupled to a charge-1 complex scalar ϕ,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) + \mu^2\phi^*\phi - \frac{\lambda}{2}(\phi^*\phi)^2 \, , \qquad\qquad D_\mu = \partial_\mu + ieA_\mu \, . \tag{7}$$

Note that $\mu^2 > 0$ means symmetry is broken. Gauge fix in R_ξ gauge, taking the VEV of the ϕ field to be in the real component of the field.

Both the wave-function and mass squared parameter μ^2 of the ϕ field receive logarithmic renormalization. Set up the calculation of the scalar field self-energy, but *only* calculate the coefficient of that part of the log divergence which is proportional to ξ.

Show that there *is* a logarithmic correction proportional to ξ *both* to the mass parameter and to the wave-function of the field ϕ. Show however that in the calculation of the *physical* mass of the Higgs scalar, these cancel each other and the 1-loop mass is ξ independent. (Recall that the physical mass is the value of p^0 where the full propagator has a pole.)

[1] If you want, verify that Φ can always be made diagonal by suitable gauge choice.