2

The standard model: general features

The last section developed the general principles for writing down a relativistic quantum field theory. It showed what types of fields are possible, and explained that spin-one fields can only appear in an interacting, renormalizable theory if they are coupled via the gauge principle.

In this section, we write down specifically what the field content of the standard model is. The interactions will then follow as the most general set of renormalizable interactions, compatible with that field content. We then explore what the vacuum and the particle content are, and write down the complete interaction Hamiltonian in the particle basis.

We will not attempt to motivate theoretically, why the particle content of the standard model is what it is. We have no deep understanding of why the gauge group is $SU_c(3) \times SU_L(2) \times U_Y(1)$, for instance. We just take the field content as observed fact, and present it. The exception is the Higgs boson, which has not been observed. This is the weakest part of our understanding of the standard model. Note however that the field content of the standard model is not completely arbitrary; once the gauge group is known, the fermionic field content is somewhat constrained by the requirement of anomaly cancellation, which we discuss at the end of the chapter.

2.1 Particle content

The strong, weak and electromagnetic interactions are understood as arising due to the exchange of various spin-one bosons amongst the spin-half particles that make up matter. The gauged symmetry group of the standard model is $SU_c(3) \times SU_L(2) \times U_Y(1)$. The specific gauge bosons associated

with the generators of the algebra of the group are:

$$\begin{array}{ccccc} SU_c(3) & \times & SU_L(2) & \times & U_Y(1) \\ \downarrow & & \downarrow & & \downarrow \\ 8\ G^\alpha_\mu & & 3\ W^a_\mu & & B_\mu \\ \alpha = 1,\ldots,8 & & a = 1,2,3 & & \end{array} \tag{2.1}$$

The eight spin-one particles, $G^\alpha_\mu(x)$, associated with the factor $SU_c(3)$ are called *gluons* and the associated subscript 'c' is meant to denote 'color'. Gluons are thought to be massless. Any particle that transforms with respect to this factor of the gauge group, and so which couples to the gluons, is said to be colored or to carry color. This interaction is also called the "strong interaction," and any particle which couples to the gluons is said to be "strongly interacting." Three spin-one particles, $W^a_\mu(x)$, are associated with the factor $SU_L(2)$, and one, $B_\mu(x)$, with the factor $U_Y(1)$. The subscript 'L' is meant to indicate that only the left-handed fermions turn out to carry this quantum number. The subscript 'Y' is meant to distinguish the group associated with the quantum number (defined below) of *weak hypercharge*, denoted Y, from the group associated with ordinary electric charge, denoted Q. The electromagnetic group will be written as $U_{em}(1)$. The four spin-one bosons associated with the factors $SU_L(2) \times U_Y(1)$ are related to the physical bosons that mediate the weak interactions: $W^\pm$ and Z^0, and the familiar photon from QED, in a way we will explain in Section 2.3.

Apart from spin-one particles we are aware of a number of fundamental spin-half particles. Our knowledge to date about the character of the interactions of these fermions may be compactly summarized by giving their transformation properties with respect to the gauge group $SU_c(3) \times SU_L(2) \times U_Y(1)$. The fermions transform in a relatively complicated way with respect to this symmetry group. There are three copies (or families) of particles, each copy of which couples identically to all spin-one particles.

Leptons are, by definition, those spin-half particles which do not take part in the strong interactions. Six leptons are known to date. They are denoted by $e, \mu, \tau, \nu_e, \nu_\mu$ and ν_τ, and collectively by ℓ.

Hadrons, on the other hand, are defined as those particles that do take part in the strong interactions. The spectrum of known hadrons is rich and varied but, as we shall see, appears to be accounted for as the bound states of six quarks u, d, s, c, b and t, denoted collectively as q.

Because of the relatively large number of spin-half fields involved, a few words on notation may be appropriate. Spinors written in capital letters L, E, D, U, Q or script letters $\mathcal{E}, \mathcal{U}, \mathcal{D}$, and neutrinos ν_i are taken as Majorana spinors. The left and right handed components of these spinors are denoted by subscripts L, R. Spinors written in lower case Roman letters l_i, u_i, d_i, e, u, c, t, d, s, b or by μ, τ are Dirac spinors, which we will introduce in turn.

For example, the electron field is represented in quantum electrodynamics by the Dirac spinor, $e(x)$. Denote the left- and right-handed components of this spinor by e_L and e_R respectively:

$$e = \begin{pmatrix} e_L \\ e_R \end{pmatrix}. \tag{2.2}$$

In the standard model, however, the electron is represented by two Majorana fields, $\mathcal{E}(x)$ and $E(x)$, that are defined to contain the left- and right- handed parts of $e(x)$ respectively. That is,

$$\mathcal{E} = \begin{pmatrix} e_L \\ \epsilon e_L^* \end{pmatrix} \quad , \quad E = \begin{pmatrix} -\epsilon e_R^* \\ e_R \end{pmatrix}, \tag{2.3}$$

where the 2×2 matrix ϵ is defined in Eq. (1.79). The Dirac spinor, e, is therefore related to the Majorana fields, $\mathcal{E}$ and E, by projecting onto the left- or right-handed part:

$$e = P_L \mathcal{E} + P_R E. \tag{2.4}$$

The 'left-handed' electron field, $\mathcal{E}$, itself appears within an $SU_L(2)$-doublet with the field, ν, whose left-handed part contains the left-handed electron-neutrino. This doublet is denoted $L(x)$:

$$L = \begin{pmatrix} \nu \\ \mathcal{E} \end{pmatrix}. \tag{2.5}$$

The notation here is somewhat confusing; the matrix structure shown for L above does not show spinorial matrix structure, but shows matrix structure under the group $SU_L(2)$; each component, ν and $\mathcal{E}$, is a 4-component Majorana spinor. Generally, when possible spinorial structure is suppressed in what follows.

Members of successive generations are denoted by a generation index, m, that runs from 1 to 3. The generations are numbered in increasing order with respect to the mass of the corresponding charged lepton:

$$\begin{aligned} &\nu_m \text{ denotes } \nu_1 = \nu_e, \;\; \nu_2 = \nu_\mu, \;\; \nu_3 = \nu_\tau \\ &e_m \text{ denotes } e_1 = e, \;\; e_2 = \mu, \;\; e_3 = \tau \\ &u_m \text{ denotes } u_1 = u, \;\; u_2 = c, \;\; u_3 = t \\ \text{and} \quad &d_m \text{ denotes } d_1 = d, \;\; d_2 = s \text{ and } d_3 = b. \end{aligned} \tag{2.6}$$

The transformation properties of the fermions are summarized by giving

the representation of the gauge group in which they transform. A standard way to label the representations of $SU_L(2)$ and $SU_c(3)$ is with their dimension. So the two-dimensional spinor representation of $SU_L(2)$ is written $\mathbf{2}$ (familiar from the physics of spin as the spin-half representation) and the two three-dimensional representations of $SU_c(3)$ would be $\mathbf{3}$ or $\overline{\mathbf{3}}$. The trivial (invariant) representation is written as $\mathbf{1}$. The transformation properties of the known fermions with respect to $U_Y(1)$ may be specified by giving the corresponding eigenvalue of the generator, Y, called the *weak hypercharge*. Y is normalized so that the action of $U_Y(1)$ on a field with eigenvalue y is given by: $\psi \to \exp(i\omega(x)y)\psi$.

With these conventions the particle content of the standard model may be summarized as follows:

$$\begin{array}{llll} P_L L_m & = \begin{pmatrix} P_L \nu_m \\ P_L \mathcal{E}_m \end{pmatrix} & \text{transforms as} & \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right) \\ P_R E_m & & & \left(\mathbf{1}, \mathbf{1}, -1\right) \\ P_L Q_m & = \begin{pmatrix} P_L \mathcal{U}_m \\ P_L \mathcal{D}_m \end{pmatrix} & & \left(\mathbf{3}, \mathbf{2}, +\frac{1}{6}\right) \\ P_R U_m & & & \left(\mathbf{3}, \mathbf{1}, +\frac{2}{3}\right) \\ P_R D_m & & & \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}\right) \end{array} \tag{2.7}$$

Here the first number represents the $SU_c(3)$-representation, the second number is the $SU_L(2)$-representation and the final number is the eigenvalue of the weak hypercharge, Y. In the case of $SU_L(2)$ doublets, we have named their upper and lower $SU_L(2)$components, $L_m = (P_L \nu_m \;\; P_L \mathcal{E}_m)^T$ and $Q_m = (P_L \mathcal{U}_m \;\; P_L \mathcal{D}_m)^T$. We could in principle do this for the three separate colors of the Q, U, and D fields; but it turns out to be useful to do so for the $SU_L(2)$ content but not for the $SU_c(3)$ content.

Since the left- and right-handed pieces of a Majorana spinor are the complex conjugates of one another, they must transform in complex-conjugate representations. It follows then that:

$$\begin{array}{llll} P_R L_m & = \begin{pmatrix} P_R \nu_m \\ P_R \mathcal{E}_m \end{pmatrix} & \text{transforms as} & \left(\mathbf{1}, \mathbf{2}, +\frac{1}{2}\right) \\ P_L E_m & & & \left(\mathbf{1}, \mathbf{1}, +1\right) \end{array}$$

$$P_R\,Q_m = \begin{pmatrix} P_R\,\mathcal{U}_m \\ P_R\,\mathcal{D}_m \end{pmatrix} \qquad \left(\overline{\mathbf{3}},\mathbf{2},-\frac{1}{6}\right) \tag{2.8}$$
$$P_L\,U_m \qquad \left(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3}\right)$$
$$P_L\,D_m \qquad \left(\overline{\mathbf{3}},\mathbf{1},+\frac{1}{3}\right)$$

We note in passing that if the standard model were to be supplemented to include a right-handed neutrino field, N_m, this field would be a singlet,

$$P_R\,N_m \qquad \text{transforms as:} \quad (\mathbf{1},\mathbf{1},0)\,, \tag{2.9}$$

with respect to the gauge group $SU_c(3)\times SU_L(2)\times U_Y(1)$. We will discuss such a singlet some more in Chapter 10, see also problem 2.3.

Apart from fermions, the Lagrangian must also involve the fields representing the spin-one gauge bosons. These fields and their transformation rules are denoted as follows:

$$\begin{array}{lll} G^\alpha_\mu & \text{transforms as:} & (\mathbf{8},\mathbf{1},0) \\ W^a_\mu & & (\mathbf{1},\mathbf{3},0) \\ B_\mu & & (\mathbf{1},\mathbf{1},0)\,. \end{array} \tag{2.10}$$

This representation content is merely a short form for the invariance of the Lagrangian under the following symmetries:

$$\begin{aligned}
\delta L_m &= \left[\left(-\frac{i}{2}\omega_1(x)+\frac{i}{2}\omega_2^a(x)\tau_a\right)P_L+\left(\frac{i}{2}\omega_1(x)-\frac{i}{2}\omega_2^a(x)\tau_a^*\right)P_R\right]L_m \\
\delta E_m &= \left[i\omega_1(x)P_L-i\omega_1(x)P_R\right]E_m \\
\delta Q_m &= \left[\left(\frac{i}{6}\omega_1(x)+\frac{i}{2}\omega_2^a(x)\tau_a+\frac{i}{2}\omega_3^\alpha(x)\lambda_\alpha\right)P_L+\right. \\
&\qquad \left.+\left(-\frac{i}{6}\omega_1(x)-\frac{i}{2}\omega_2^a(x)\tau_a^*-\frac{i}{2}\omega_3^\alpha(x)\lambda_\alpha^*\right)P_R\right]Q_m \\
\delta U_m &= \left[\left(-\frac{2i}{3}\omega_1(x)-\frac{i}{2}\omega_3^\alpha(x)\lambda_\alpha^*\right)P_L+\left(\frac{2i}{3}\omega_1(x)+\frac{i}{2}\omega_3^\alpha(x)\lambda_\alpha\right)P_R\right]U_m \\
\delta D_m &= \left[\left(\frac{i}{3}\omega_1(x)-\frac{i}{2}\omega_3^\alpha(x)\lambda_\alpha^*\right)P_L+\left(-\frac{i}{3}\omega_1(x)+\frac{i}{2}\omega_3^\alpha(x)\lambda_\alpha\right)P_R\right]D_m \\
\delta G^\alpha_\mu &= \partial_\mu\omega_3^\alpha(x)-f^\alpha{}_{\beta\gamma}\,\omega_3^\beta(x)\,G^\gamma_\mu \\
\delta W^a_\mu &= \partial_\mu\omega_2^a(x)-\epsilon^{abc}\,\omega_2^b(x)\,W^c_\mu \\
\delta B_\mu &= \partial_\mu\omega_1(x).
\end{aligned} \tag{2.11}$$

In these expressions the generators of $SU_L(2)$ have been explicitly written as $T_a=\frac{1}{2}\tau_a$ where $\tau_a, a=1,2,3$ denotes the two-by-two *Pauli* matrices that

act on the $SU_L(2)$-indices:

$$\tau_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\ \tau_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\ \tau_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}. \tag{2.12}$$

(The same matrices appeared in discussing the spin structure of fermions in Section 1.3. We use the notation τ_i when they act on $SU_L(2)$ indices and σ_i when they act on spinorial indices.) Similarly, the generators of $SU_c(3)$ are given explicitly by $T_\alpha=\frac{1}{2}\lambda_\alpha$ where $\lambda_\alpha, \alpha=1,...,8$ denote the three-by-three *Gell-Mann* matrices given in Eq. (1.186).

The electric charge Q of a field is defined in terms of the hypercharge Y and the $SU_L(2)$ charge's T_3 component, according to $Q=T_3+Y$. Note that the electromagnetic group is *not* directly the $U_Y(1)$component of the standard model gauge group, and electric charge Q is *not* one of the basic charges particles carry under $SU_c(3)\times SU_L(2)\times U_Y(1)$; rather it is a derived quantity.

2.2 The Lagrangian

The most general renormalizable Lagrangian involving these fields is

$$\begin{aligned}
\mathcal{L}_{fg} &= -\frac{1}{4}G^\alpha_{\mu\nu}G^{\alpha\mu\nu}-\frac{1}{4}W^{a\mu\nu}W^a_{\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}-\frac{g_3^2\Theta_3}{64\pi^2}\epsilon_{\mu\nu\lambda\rho}G^{\alpha\mu\nu}G^{\alpha\lambda\rho} \\
&\quad -\frac{g_2^2\Theta_2}{64\pi^2}\epsilon_{\mu\nu\lambda\rho}W^{a\mu\nu}W^{a\lambda\rho}-\frac{g_1^2\Theta_1}{64\pi^2}\epsilon_{\mu\nu\lambda\rho}B^{\mu\nu}B^{\lambda\rho}-\frac{1}{2}\overline{L}_m\not{D}L_m \\
&\quad -\frac{1}{2}\overline{E}_m\not{D}E_m-\frac{1}{2}\overline{Q}_m\not{D}Q_m-\frac{1}{2}\overline{U}_m\not{D}U_m-\frac{1}{2}\overline{D}_m\not{D}D_m,
\end{aligned} \tag{2.13}$$

in which the gauge field-strengths are given by

$$G^\alpha_{\mu\nu} = \partial_\mu G^\alpha_\nu-\partial_\nu G^\alpha_\mu+g_3 f^\alpha{}_{\beta\gamma}G^\beta_\mu G^\gamma_\nu\,, \tag{2.14}$$
$$W^a_{\mu\nu} = \partial_\mu W^a_\nu-\partial_\nu W^a_\mu+g_2\epsilon_{abc}W^b_\mu W^c_\nu\,, \tag{2.15}$$
$$B_{\mu\nu} = \partial_\mu B_\nu-\partial_\nu B_\mu\,. \tag{2.16}$$

The gauge covariant derivatives are:

$$\begin{aligned}
D_\mu L_m &= \partial_\mu L_m+\left[\frac{i}{2}g_1B_\mu-\frac{i}{2}g_2W^a_\mu\tau_a\right]P_L\,L_m \\
&\qquad +\left[-\frac{i}{2}g_1B_\mu+\frac{i}{2}g_2W^a_\mu\tau_a^*\right]P_R\,L_m\,,
\end{aligned} \tag{2.17}$$
$$D_\mu E_m = \partial_\mu E_m+ig_1B_\mu(P_R\,E_m)-ig_1B_\mu(P_L\,E_m)\,, \tag{2.18}$$
$$D_\mu Q_m = \partial_\mu Q_m+\left[-\frac{i}{2}g_3G^\alpha_\mu\lambda_\alpha-\frac{i}{2}g_2W^a_\mu\tau_a-\frac{i}{6}g_1B_\mu\right]P_L\,Q_m$$

$$+\left[\frac{i}{2}g_3 G^\alpha_\mu \lambda^*_\alpha + \frac{i}{2}g_2 W^a_\mu \tau^*_a + \frac{i}{6}g_1 B_\mu\right] P_R\, Q_m\,, \quad (2.19)$$

$$D_\mu U_m = \partial_\mu U_m + \left[-\frac{i}{2}g_3 G^\alpha_\mu \lambda_\alpha - \frac{2i}{3}g_1 B_\mu\right] P_R\, U_m + \left[\frac{i}{2}g_3 G^\alpha_\mu \lambda^*_\alpha + \frac{2i}{3}g_1 B_\mu\right] P_L\, U_m\,, \quad (2.20)$$

$$D_\mu D_m = \partial_\mu D_m + \left[-\frac{i}{2}g_3 G^\alpha_\mu \lambda_\alpha + \frac{i}{3}g_1 B_\mu\right] P_R\, D_m + \left[+\frac{i}{2}g_3 G^\alpha_\mu \lambda^*_\alpha - \frac{i}{3}g_1 B_\mu\right] P_L\, D_m\,. \quad (2.21)$$

It is worth emphasizing at this point why certain terms do *not* appear in $\mathcal{L}_{fg}$, particularly mass terms for the fermionic fields. The reason is that only terms which are singlets under $SU_c(3) \times SU_L(2) \times U_Y(1)$ can appear in the Lagrangian–otherwise it would not respect gauge invariance, that is, it would change under a gauge transformation. The rules for telling if a combination of fields is a singlet under $SU_c(3)$ or $SU_L(2)$ are summarized in Appendix B; basically the rule is that all color and $SU_L(2)$ indices must "tie off" against each other. The rule for $U_Y(1)$ is even easier; the charges of the fields must add to zero.

Consider for instance the would-be mass term for the E field,

$$\mathcal{L}_{\text{would-be}} = -\frac{m_{mn}}{2}\overline{E}_m E_n\,.$$

Write $\overline{E}_m E_n = \overline{E}_m P_L\, E_n + \overline{E}_m P_R\, E_n$, and just consider the P_R term. $P_R\, E$ has hypercharge -1. The hypercharge of $\overline{E} P_R$ is also -1. To see this, note that

$$\overline{E} P_R = E^\dagger \beta P_R = E^\dagger P_L\, \beta \quad (2.22)$$

is actually the conjugate field of $P_L\, E$, and has the opposite charge as $P_L\, E$. Therefore, the combination $\overline{E} P_R\, E$ is hypercharge -2 and is not a gauge singlet. The combination $\overline{E} P_L\, E$ is hypercharge 2 and is also not allowed. One can quickly check that no combination of two spinor fields is hypercharge neutral, so no such mass is permitted. The kinetic terms *are* invariant because $P_L\, \gamma^\mu = \gamma^\mu P_R$; so the left handed component of a field couples to the Hermitian conjugate of the left handed component and the gauge dependence does cancel.

The spectrum of this theory may be analyzed by perturbing in the gauge couplings, g_i, $i = 1, 2, 3$. (We return to the accuracy of this approximation in more detail later.) The unperturbed part of the Lagrangian becomes in this case those terms that are quadratic in the fields. The spectrum of this unperturbed theory is therefore that of a system of free spin-half and

spin-one particles, as was described in the previous chapter. Following the discussion leading up to Eq. (1.110)–Eq. (1.125), their masses may be read off from the Lagrangian and are zero!

Since the perturbative semiclassical analysis should apply to at least the electroweak part of the theory, the Lagrangian, Eq. (2.13), cannot be the whole story. In fact, as is clear from the discussion of Subsection 1.6.2, the vanishing of the masses is a consequence of the $SU_c(3) \times SU_L(2) \times U_Y(1)$ invariance of the theory and can be evaded only if this symmetry is spontaneously broken by the ground state. We must couple the known particles to some hitherto undiscovered sector whose ground state is not $SU_L(2) \times U_Y(1)$ invariant.

The simplest way to do so is to add a weakly coupled spin-zero particle to the theory with a potential that is minimized at a non-zero field value. The transformation properties of this scalar field are largely determined if we require that no new spin-half fields are to be included. Since the scalar field is supposed to produce a mass for the fermions after it develops a *v.e.v.* it must have Yukawa couplings with the fermions. But all of the fermions are either singlets or doublets under $SU_L(2)$ so the new scalar field must itself be either a doublet or a triplet if it is to combine with two fermions into a gauge-invariant Yukawa interaction. It turns out that a scalar triplet cannot by itself couple in a way that can generate masses for all the known massive fermions, but a scalar doublet can. The simplest choice is therefore to add a single (complex) scalar doublet, called the Higgs field:

$$\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}, \quad (2.23)$$

transforming as $(1, \mathbf{2}, 1/2)$. Its complex conjugate,

$$\tilde{\phi} \equiv \begin{pmatrix}\phi^{0*} \\ -\phi^{+*}\end{pmatrix} = \epsilon\phi^*, \quad (2.24)$$

then transforms as $(1, \mathbf{2}, -1/2)$. It must be emphasized that apart from the conditions that the appropriate masses be generated, there is precious little known about this symmetry-breaking sector. Although the Higgs-doublet model we are using is the simplest, its foundations are much less firm than are those of the rest of the theory. This is explored further in the problem section of this chapter.

The new terms that may then appear in $\mathcal{L}$ are

$$\mathcal{L}_{\text{Higgs}} = -(D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi^\dagger\phi) - (f_{mn}\bar{L}_m P_R\, E_n\phi + h_{mn}\bar{Q}_m P_R\, D_n\phi + g_{mn}\bar{Q}_m P_R\, U_n\tilde{\phi} + \text{ h.c. })\,, \quad (2.25)$$

in which

$$V(\phi^\dagger\phi) = \lambda\left[\phi^\dagger\phi - \mu^2/2\lambda\right]^2 = \lambda(\phi^\dagger\phi)^2 - \mu^2\phi^\dagger\phi + \mu^4/4\lambda\,. \quad (2.26)$$

Unitarity requires that the constants λ and μ^2 be real and stability demands that λ be positive. In order to ensure that the ground state not be $SU_L(2)\times U_Y(1)$invariant we further require that μ^2 be positive.

The gauge transformation rules for ϕ are explicitly

$$\delta\phi = \frac{i}{2}\omega_2^a\tau_a\phi + \frac{i}{2}\omega_1\phi, \quad (2.27)$$

so its covariant derivative must be

$$D_\mu\phi = \partial_\mu\phi - \frac{i}{2}g_2W_\mu^a\tau_a\phi - \frac{i}{2}g_1B_\mu\phi\,. \quad (2.28)$$

The complete Standard Model Lagrangian then becomes

$$\mathcal{L}_{SM} = \mathcal{L}_{fg} + \mathcal{L}_{\rm Higgs}\,. \quad (2.29)$$

The following general features of $\mathcal{L}_{SM}$ bear special mention:

(i) $\mathcal{L}_{fg}$, $\mathcal{L}_{\rm Higgs}$ and $\mathcal{L}_{SM}$ are the most general Lagrangian consistent with the given particle content and invariance under $SU_c(3)\times SU_L(2)\times U_Y(1)$. If the predictions made from such an $\mathcal{L}$ are wrong, then either the particle-content or renormalizability or the gauge group is wrong.

(ii) Because of $SU_c(3)\times SU_L(2)\times U_Y(1)$ invariance, all masses vanish in the absence of $\mathcal{L}_{\rm Higgs}$.

(iii) There are six parameters in $\mathcal{L}_{fg}$ of which only four enter into physical predictions (since Θ_1 and Θ_2 turn out to be removable by suitable fermion phase redefinitions, which we will not discuss). $\mathcal{L}_{\rm Higgs}$, on the other hand, contains no less than 15 parameters (as we shall see these may be taken to be the 10 masses, the Higgs self-coupling and the 4 Kobayashi Maskawa angles). In this sense $\mathcal{L}_{\rm Higgs}$ parameterizes most of our ignorance and is the part of the theory that is the least understood. All of the couplings also turn out to be small (modulo some restrictions to which we return for g_3), allowing the use of perturbation theory to calculate the predictions of $\mathcal{L}$.

2.3 The perturbative spectrum

The first step in analyzing the consequences of the standard model is to find its spectrum. We do so semiclassically, following the procedure of Subsection 1.6.2. For these purposes it is convenient here, as it was there, to use

the gauge freedom to transform to *unitary gauge*. In the present context unitary gauge is defined by the following condition:

$$\phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H(x)) \end{pmatrix}, \quad (2.30)$$

where $H(x)$ is a real field and v is a real constant that minimizes the scalar potential. It may be shown that it is always possible to reach Eq. (2.30) from an arbitrary initial field configuration *via* a gauge transformation. The motivation for this gauge choice is that it ensures that no vector-scalar cross terms survive in the quadratic terms once we expand about the ground state. It is worth noting in passing that the gauge, Eq. (2.30), does not fix those gauge invariances that leave the Higgs *v.e.v.* invariant. In the present context, as is shown later in this section, this means that the electromagnetic gauge invariance still remains to be fixed.

v is determined by minimizing the potential in Eq. (2.26) and satisfies

$$v^2 = \mu^2/\lambda\,. \quad (2.31)$$

In order to read off the particle masses we must identify the unperturbed Lagrangian, $\mathcal{L}_0$. This is equal to that part of $\mathcal{L}_{SM}$ that is quadratic in the fluctuations. The expansion of $\mathcal{L}_{fg}$ is trivial and just contributes the spin-half and spin-one kinetic terms to $\mathcal{L}_0$. Everything else comes from the expansion of $\mathcal{L}_{\rm Higgs}$. Using the following result,

$$D_\mu\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial_\mu H\end{pmatrix} - \frac{i}{2\sqrt{2}}\begin{pmatrix} g_2W_\mu^3 + g_1B_\mu & g_2W_\mu^1 - ig_2W_\mu^2 \\ g_2W_\mu^1 + ig_2W_\mu^2 & -g_2W_\mu^3 + g_1B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v+H\end{pmatrix}, \quad (2.32)$$

the expansion of the scalar-field kinetic term becomes:

$$-(D_\mu\phi)^\dagger(D^\mu\phi) = -\frac{1}{2}\partial_\mu H\partial^\mu H - \frac{1}{8}(v+H)^2g_2^2(W_\mu^1 - iW_\mu^2)(W^{1\mu} + iW^{2\mu}) - \frac{1}{8}(v+H)^2(-g_2W^{3\mu} + g_1B^\mu)(-g_2W_{3\mu} + g_1B_\mu)\,. \quad (2.33)$$

The scalar potential term contributes

$$V = \frac{\lambda}{4}\left[(v+H)^2 - \mu^2/\lambda\right]^2 = \frac{\lambda}{4}(2vH + H^2)^2 = \lambda v^2H^2 + \lambda vH^3 + \frac{\lambda}{4}H^4\,. \quad (2.34)$$

The Yukawa couplings may be expanded in an identical way:

$$\begin{aligned}\bar{L}_m P_R E_n \phi &= \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{\nu}_m \\ \bar{\mathcal{E}}_m \end{pmatrix}^T P_R E_n \begin{pmatrix} 0 \\ v+H \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} (v+H)\, \bar{\mathcal{E}}_m P_R E_n \,, \end{aligned} \tag{2.35}$$

and similarly for Q, d, and D, and

$$\begin{aligned}\bar{Q}_m P_R U_n \tilde{\phi} &= \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{\mathcal{U}}_m \\ \bar{\mathcal{D}}_m \end{pmatrix} P_R U_n \begin{pmatrix} v+H \\ 0 \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} (v+H)\, \bar{\mathcal{U}}_m P_R U_n \,. \end{aligned} \tag{2.36}$$

Combining all of these results gives the expansion of $\mathcal{L}_{\rm Higgs}$ to be:

$$\begin{aligned}\mathcal{L}_{\rm Higgs} = & -\frac{1}{2}\partial_\mu H \partial^\mu H - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4 \\ & - \frac{1}{8} g_2^2 (v+H)^2 |W^1_\mu - i W^2_\mu|^2 \\ & - \frac{1}{8} (v+H)^2 (-g_2 W^3_\mu + g_1 B_\mu)^2 \\ & - \frac{1}{\sqrt{2}} (v+H) \left[f_{mn} \bar{\mathcal{E}}_m P_R E_n + \text{ h.c.} \right] \\ & - \frac{1}{\sqrt{2}} (v+H) \left[g_{mn} \bar{\mathcal{U}}_m P_R U_n + \text{ h.c.} \right] \\ & - \frac{1}{\sqrt{2}} (v+H) \left[h_{mn} \bar{\mathcal{D}}_m P_R D_n + \text{ h.c.} \right] . \end{aligned} \tag{2.37}$$

2.3.1 Boson masses

$\mathcal{L}_{\rm Higgs}$ contains all of the mass terms, although some of these are not diagonal. They are, in more detail:

2.3.1.1 Spin-zero particles

Comparing the H^2 term of $\mathcal{L}_{\rm Higgs}$ with the standard form, $-\frac{1}{2} m_H^2 H^2$, gives

$$m_H^2 = 2\lambda v^2 = 2\mu^2 \,. \tag{2.38}$$

2.3.1.2 Spin-one particles

The relevant terms in this case are:

$$-\frac{1}{8} g_2^2 v^2 |W^1_\mu - i W^2_\mu|^2 - \frac{1}{8} v^2 (- g_2 W^3_\mu + g_1 B_\mu)^2 \,. \tag{2.39}$$

The fields W^1_μ and W^2_μ only appear in the combination $W_1^\mu W_{1\mu} + W_2^\mu W_{2\mu}$ and do not mix with any other fields. Their masses can therefore be read by inspection. Comparing this term to

$$-\frac{1}{2} M_1^2 W^1_\mu W^{1\mu} - \frac{1}{2} M_2^2 W^2_\mu W^{2\mu} \,, \tag{2.40}$$

gives the masses:

$$M_1^2 = M_2^2 = \frac{1}{4} g_2^2 v^2 \,. \tag{2.41}$$

It is not an accident that these masses are equal. They are equal because the particles W_1 and W_2 are related by a symmetry that is not spontaneously broken, even when $v \neq 0$. To see this, consider performing a constant gauge transformation, $\partial_\mu \omega^a = 0$. The ground-state scalar field configuration then transforms as

$$\begin{aligned}\delta \begin{pmatrix} 0 \\ v \end{pmatrix} &= \frac{i}{2} \omega_2^a \tau_a \begin{pmatrix} 0 \\ v \end{pmatrix} + \frac{i}{2} \omega_1 \begin{pmatrix} 0 \\ v \end{pmatrix} \\ &= \frac{i}{2} \begin{pmatrix} [\omega_2^1 - i \omega_2^2] v \\ [\omega_1 - \omega_2^3] v \end{pmatrix} , \end{aligned} \tag{2.42}$$

which vanishes provided that $\omega_2^1 = \omega_2^2 = 0$ and $\omega_1 = \omega_2^3 \equiv \omega$. This particular combination of $SU_L(2) \times U_Y(1)$-transformations is therefore a symmetry of the ground state.

Under this symmetry the W fields transform according to Eq. (2.11):

$$\delta W^a_\mu = -\epsilon^{abc} \omega_2^b W^c_\mu \,, \qquad \text{or} \,, \qquad \delta \begin{pmatrix} W^1_\mu \\ W^2_\mu \end{pmatrix} = \omega \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} W^1_\mu \\ W^2_\mu \end{pmatrix} . \tag{2.43}$$

This shows that W^1_μ and W^2_μ transform into one another under this symmetry. The condition $\omega_2^3 = \omega_1$ implies that the generator of this unbroken symmetry is $T_3 + Y$. Now we saw earlier that the electric charge, Q, of a field is related to the $SU_L(2) \times U_Y(1)$-generators by $Q = T_3 + Y$. It is precisely the electromagnetic gauge invariance, $U_{em}(1)$, which is unbroken by the vacuum. W^1_μ and W^2_μ must therefore correspond to the two degrees of freedom associated with the distinct particle and antiparticle states required for an electrically charged particle. It is convenient in these cases to deal with fields that diagonalize the generator of electric charge. This corresponds, in the present case, to writing W_1 and W_2 as the real and imaginary parts of a complex, charged field:

$$W^\pm_\mu \equiv \frac{1}{\sqrt{2}} (W^1_\mu \mp i W^2_\mu) \,, \tag{2.44}$$

which satisfies $\delta W_\mu^\pm = \pm i\omega W_\mu^\pm$ under electromagnetic gauge transformations, Eq. (2.43).

The mass term appropriate to such a charged field is $-M_W^2 W_\mu^+ W^{-\mu}$. Comparing with the Lagrangian, Eq. (1.121), therefore gives the $W^\pm$ mass to be

$$M_W = M_1 = M_2 = \frac{1}{2} g_2 v. \tag{2.45}$$

The remaining vector fields that appear in the mass term are W_μ^3 and B_μ. They also only appear in one particular combination, $g_1 B_\mu - g_2 W_\mu^3$. We may normalize this combination (in order not to alter the standard form for the kinetic terms) to define the mass eigenstate:

$$\begin{aligned} Z_\mu &\equiv \frac{-g_1 B_\mu + g_2 W_\mu^3}{\sqrt{g_1^2 + g_2^2}} \\ &\equiv W_\mu^3 \cos\theta_W - B_\mu \sin\theta_W \,. \end{aligned} \tag{2.46}$$

This last equation defines the weak-mixing angle or Weinberg angle, θ_W, given by

$$\begin{aligned} \cos\theta_W &= \frac{g_2}{\sqrt{g_1^2 + g_2^2}} \\ \sin\theta_W &= \frac{g_1}{\sqrt{g_1^2 + g_2^2}} \,. \end{aligned} \tag{2.47}$$

In terms of this field the mass term, Eq. (1.122), is

$$-\frac{1}{8} v^2 (g_1^2 + g_2^2) Z_\mu Z^\mu \,, \tag{2.48}$$

from which the mass may be read off:

$$M_Z^2 = \frac{1}{4}(g_1^2 + g_2^2) v^2. \tag{2.49}$$

The final mass eigenstate is the combination of W_μ^3 and B_μ that is orthogonal to Z_μ :

$$A_\mu = W_\mu^3 \sin\theta_W + B_\mu \cos\theta_W = \frac{g_1 W_\mu^3 + g_2 B_\mu}{\sqrt{g_1^2 + g_2^2}} \,. \tag{2.50}$$

This is massless, as are the gluons, G_μ^α , that gauge $SU_c(3)$. The masslessness of A_μ corresponds to the fact that the linear combination $Q = T_3 + Y$ is unbroken even when $v \neq 0$. A_μ is the corresponding massless gauge boson required for this unbroken symmetry. Since Q is the electric charge, we expect A_μ to have the couplings of the usual photon.

To summarize the relations between field bases, writing $c_W \equiv \cos\theta_W$ and $s_W \equiv \sin\theta_W$,

$$\begin{aligned} W_\mu^3 &= c_W Z_\mu + s_W A_\mu & \qquad Z_\mu &= c_W W_\mu^3 - s_W B_\mu \\ B_\mu &= -s_W Z_\mu + c_W A_\mu & A_\mu &= s_W W_\mu^3 + c_W B_\mu \,, \\[1ex] \sqrt{2}\, W_\mu^+ &= W_\mu^1 - i W_\mu^2 & \sqrt{2}\, W_\mu^1 &= W_\mu^+ + W_\mu^- \\ \sqrt{2}\, W_\mu^+ &= W_\mu^1 + i W_\mu^2 & \sqrt{2}\, W_\mu^2 &= i W_\mu^+ - i W_\mu^- \,, \\[1ex] \sqrt{g_2^2 + {g_1}^2}\, W_\mu^3 &= g_2 Z_\mu + g_1 A_\mu & \sqrt{g_2^2 + {g_1}^2}\, Z_\mu &= g_2 W_\mu^3 - g_1 B_\mu \\ \sqrt{g_2^2 + {g_1}^2}\, B_\mu &= -g_1 Z_\mu + g_2 A_\mu & \sqrt{g_2^2 + {g_1}^2}\, A_\mu &= g_1 W_\mu^3 + g_2 B_\mu \,. \end{aligned} \tag{2.51}$$

2.3.2 *The custodial* $SU(2)$

Notice that there is a relation amongst the three quantities M_W, M_Z and θ_W:

$$\frac{M_W}{M_Z} = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} = \cos\theta_W \,. \tag{2.52}$$

It is natural to ask how much this relation depends on the details of how the symmetry $SU_L(2) \times U_Y(1)$ is broken, since any information that can restrict the arbitrariness in the symmetry breaking sector is welcome. Consider therefore the most general form for the spin-one mass matrix that is consistent with the symmetry-breaking pattern $SU_L(2) \times U_Y(1) \rightarrow U_{em}(1)$:

$$\begin{pmatrix} M_W^2 & & & \\ & M_W^2 & & \\ & & M_3^2 & m^2 \\ & & m^2 & M_0^2 \end{pmatrix} . \tag{2.53}$$

This form has a simple explanation. As we saw above, unbroken electromagnetic gauge invariance dictates that the upper left two-by-two block of the matrix be proportional to the unit matrix: $M_W^2\ I_{2\times 2}$. It similarly implies that the upper-right and the lower-left blocks must vanish. The lower-right two-by-two block is a-priori an arbitrary symmetric matrix, subject to the one constraint that one of its eigenvalues must vanish. The vanishing of one of the eigenvalues corresponds to the masslessness of the photon. It is a general consequence of the fact that the electromagnetic gauge invariance is unbroken.

The requirement that one eigenvalue be zero is equivalent to the vanishing of the determinant:

$$\det \begin{pmatrix} M_3^2 & m^2 \\ m^2 & M_0^2 \end{pmatrix} = M_3^2 M_0^2 - m^4 = 0\,, \tag{2.54}$$

implying the condition $m^2 = |M_0 M_3|$. The corresponding zero eigenvector may be written as

$$\begin{pmatrix} -\sin\theta_W \\ \cos\theta_W \end{pmatrix}. \tag{2.55}$$

Eq. (2.55) defines the mixing angle, θ_W, in the general case. We may now eliminate M_0^2 in favor of θ_W. The required relation is

$$\tan\theta_W = \frac{m^2}{M_3^2} = \left|\frac{M_0}{M_3}\right|. \tag{2.56}$$

The nonzero eigenvalue, M_Z, is then given in terms of M_3 and θ_W by

$$\begin{aligned} M_Z^2 &= \operatorname{tr}\begin{pmatrix} M_3^2 & m^2 \\ m^2 & M_0^2 \end{pmatrix} \\ &= M_0^2 + M_3^2 = M_3^2(1+\tan^2\theta_W) = M_3^2 \sec^2\theta_W. \end{aligned} \tag{2.57}$$

The mass relation implied by the symmetry breaking pattern $SU_L(2) \times U_Y(1) \to U_{em}(1)$ is therefore $M_3 = M_Z \cos\theta_W$. An alternative way of expressing the mass formula, Eq. (2.52), is therefore: $M_1 = M_2 = M_3 = M_W$.

The equality of M_3 and M_W within the standard model is a consequence of using a scalar $SU_L(2)$-doublet, ϕ, to break $SU_L(2) \times U_Y(1)$. The connection arises because of an *accidental symmetry* of the scalar self-couplings that determine the symmetry-breaking pattern that in turn determines the gauge boson mass matrix. The Higgs doublet, ϕ, may be thought of as four real scalar fields, corresponding to the real and imaginary parts of ϕ^0 and ϕ^+ in Eq. (2.23). An alternative way to write these four real fields would be as a column vector:

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}. \tag{2.58}$$

As we saw in Subsection 1.3.1, the kinetic terms for four real scalar fields can be written as $\partial_\mu \Phi^T \partial^\mu \Phi$ and so is always invariant under the multiplication of Φ by an arbitrary four-by-four orthogonal matrix, $\mathcal{O} \in O(4)$. Now, in general the interaction terms of the Lagrangian break this symmetry completely. However, for the standard model, the two requirements of

gauge invariance and renormalizability imply that the only possible scalar self-couplings are of form $V = V(\phi^\dagger\phi) = V(\Phi^T\Phi)$. Even though it was not required to be so, this potential is therefore also invariant under these general $O(4)$ transformations. Any such global symmetry that appears as a simple consequence of gauge invariance and renormalizability is known as an *accidental* symmetry.

Once Φ develops a *v.e.v.*,

$$\langle\Phi\rangle = \begin{pmatrix} v \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{2.59}$$

this $O(4)$-invariance gets broken to the three-by-three orthogonal, $O(3)$, transformations that shuffle the lower three components amongst themselves. Since this $O(3)$ is unbroken, it constrains the form that the mass matrix may take. The ϕ gauge couplings that ultimately produce the gauge boson mass matrix are also invariant under these $O(3)$ transformations if the W_μ^a's transform as a three-dimensional vector. Invariance of the mass matrix under this three-by-three transformation therefore implies that the upper-left three-by-three block of the spin-one mass matrix, Eq. (2.53), must be proportional to the unit matrix, implying $M_3 = M_1 = M_2 = M_W$ as required.

Since the group $O(3)$ is locally isomorphic to the group $SU(2)$, it is said that the symmetry-breaking sector has an accidental *custodial* $SU(2)$ invariance that is responsible for the mass formula, Eq. (2.52).

The utility of having such a symmetry understanding of this mass formula is that it points to the circumstances under which it might be altered and to how big the corrections might be. In fact, some of the interactions in the standard model, like the $\phi - B_\mu$ coupling and the Yukawa couplings, do *not* respect this custodial symmetry. We may expect, then, that radiative (quantum) corrections that involve these interactions can alter the mass relation. This is discussed in Section 7.5. Experimental verification of this relation is clearly of great importance since deviations point to detailed effects within the standard model, and potentially to indications of new physics.

2.3.3 Fermion masses

The terms quadratic in the fermion fields come from the Yukawa couplings after the shifting of the scalar field by v. The relevant terms are

$$\mathcal{L} = -\frac{v}{\sqrt{2}}\left[f_{mn}\bar{\mathcal{E}}_m P_R E_n + g_{mn}\bar{\mathcal{U}}_m P_R U_n + h_{mn}\bar{\mathcal{D}}_m P_R D_n + \text{h.c.}\right]. \tag{2.60}$$

(It now becomes clear why it was convenient to label separately the different $SU_L(2)$ components of the fields L and Q; the fact that the *v.e.v.* of the Higgs field breaks $SU_L(2)$ symmetry means that a Yukawa coupling introduces a mass which picks out one or the other component.)

The mass terms induced by the Yukawa couplings of fermions to the Higgs *v.e.v.* are in general not diagonal in the generation indices, m and n. They may be diagonalized following the procedure outlined in Subsection 1.3.2. To this end, redefine the spin-half fields as follows:

$$\begin{aligned} P_L\,\mathcal{E}_m &= U^{(e)}_{mn} P_L\,\mathcal{E}'_n\,, & P_R\,E_m &= V^{(e)}_{mn} P_R\,E'_n\,, \\ P_L\,\mathcal{U}_m &= U^{(u)}_{mn} P_L\,\mathcal{U}'_n\,, & P_R\,U_m &= V^{(u)}_{mn} P_R\,U'_n\,, \\ P_L\,\mathcal{D}_m &= U^{(d)}_{mn} P_L\,\mathcal{D}'_n\,, & P_R\,D_n &= V^{(d)}_{mn} P_R\,D'_n\,, \end{aligned} \tag{2.61}$$

where the matrices $U^{(e)}, U^{(u)}, U^{(d)}, V^{(e)}, V^{(u)}, V^{(d)}$ act on the generation indices (*e.g.* connect e to μ to τ) and must be unitary in order to preserve the canonical form for the kinetic terms.

As argued in Subsection 1.3.2, it is always possible to choose $U^{(e)} = V^{(e)*}, U^{(u)} = V^{(u)*}, U^{(d)} = V^{(d)*}$ and then choose $U^{(e)}$ to ensure that the new mass matrices are diagonal:

$$U^{(e)\dagger} f V^{(e)} = V^{(e)^T} f V^{(e)} = \text{ diag }(f_e, f_u, f_\tau)\,, \tag{2.62}$$

with f_e, f_μ, f_τ real and non-negative. The same may be done for $V^{(u)^T} g V^{(u)}$ and $V^{(d)^T} h V^{(d)}$. The resulting mass terms then become (dropping the primes on the new fields)

$$\mathcal{L} = -\frac{1}{\sqrt{2}} v\left[f_m\bar{\mathcal{E}}_m P_R E_m + g_m\bar{\mathcal{U}}_m P_R U_m + h_m\bar{\mathcal{D}}_m P_R D_m + \text{h.c.}\right]. \tag{2.63}$$

This has a simple expression in terms of the Dirac spinors, e_m, d_m and u_m, defined as

$$\begin{aligned} e_m &\equiv P_L\,\mathcal{E}_m + P_R\,E_m\,, \\ d_m &\equiv P_L\,\mathcal{D}_m + P_R\,D_m\,, \\ u_m &\equiv P_L\,\mathcal{U}_m + P_R\,U_m\,. \end{aligned} \tag{2.64}$$

To see this, use

$$\begin{aligned} \bar{\mathcal{E}}_m P_R\,E_m + \text{ h.c.} &= \bar{\mathcal{E}}_m P_R\,E_m + \bar{\mathcal{E}}_m P_L\,E_m \\ &= \bar{\mathcal{E}}_m P_R\,E_m + \bar{E}_m P_L\,\mathcal{E}_m \\ &= \bar{e}_m P_R\,e_m + \bar{e}_m P_L\,e_m \\ &= \bar{e}_m e_m\,. \end{aligned} \tag{2.65}$$

(The derivation of the identities used here was the subject of problem 1 of chapter 1.)

In terms of these Dirac spinors, the final form for the mass terms is

$$\mathcal{L} = -\frac{1}{\sqrt{2}} v(f_m\bar{e}_m e_m + g_m\bar{u}_m u_m + h_m\bar{d}_m d_m)\,, \tag{2.66}$$

which, when compared to the standard mass term, $-m\bar{\psi}\psi$, gives the fermion masses as:

$$m^{(e)}_n = \frac{1}{\sqrt{2}} f_n v, \qquad m^{(u)}_n = \frac{1}{\sqrt{2}} g_n v, \qquad m^{(d)}_n = \frac{1}{\sqrt{2}} h_n v. \tag{2.67}$$

Notice that there is a separate Yukawa parameter, f_n, for every independent mass, m_n, so there are no mass formulae along the lines of Eq. (2.52) for the fermions. The numerical values of these fermion masses are presented in Appendix A.

Note that no mass term for the neutrinos is generated. If only renormalizable interactions and the minimal field content of the standard model are included, then this is exactly true, not just at the semiclassical level. A neutrino mass could appear if we extended the theory to include right handed neutrinos N_m, because this would allow another Yukawa matrix between L and N. However, nothing forbids a mass term $m_m\bar{N}_m N_m$ for such right handed neutrinos. One interpretation of the recent evidence for neutrino masses is that such right handed neutrinos exist but their mass is very heavy. This is discussed in more detail in Chapter 10 and in problem 2.3.

2.3.4 Hadrons

What we have just presented is the *perturbative* spectrum, that is, the spectrum assuming all interactions are weak. As we will discuss in Section 7.4, this is a valid approximation except for the $SU_c(3)$ ("strong") interactions, which become strong at scales of order 500 MeV. The result is that quarks and gluons do not appear as actual particles of the spectrum. Rather, the particles we observe are bound states of quarks and gluons, in appropriate combinations to be color singlets. Such bound states are called *hadrons*.