Physics 731 Homework 2

Due 24 January 2007

1 Dilations

Suppose that we extend the spacetime symmetry group by the *dilation* (aka *dilatation*) operator D, which commutes with the Lorentz generators and has

$$[P_\mu, D] = iP_\mu \ . \tag{1}$$

Evaluate $[D, Q_\alpha]$. To do this, first show that $\Big[D\ ,\ Q_\alpha\Big] = icQ_\alpha$ and then calculate c **assuming that it is real**.
Note: D is only a valid symmetry (possibly) if all single-particle states are massless. There is, in fact, a further extension of the (super)Poincaré algebra to a (super)conformal algebra with additional vector (and spinor) generators.

2 Fermion Number

For shorthand, define $M_{12} \equiv J$. (Here $M_{\mu\nu}$ is an alternative notation for $J_{\mu\nu}$, the generator of rotations.) The operator $\exp(2\pi iJ)$ is a fermion number operator in that fermionic states have an eigenvalue of -1 and bosons $+1$ (because spinors come back to themselves only up to a sign under a 2π rotation).
Using the superalgebra, show

$$\exp(2\pi iJ)\, Q = -Q \exp(2\pi iJ) \ . \tag{2}$$

Then use this relation to show that $\mathrm{tr}\exp(2\pi iJ)$ vanishes when the trace is over a supersymmetry particle multiplet (*not* the vacuum).

3 $\mathcal{N} = 3$ versus $\mathcal{N} = 4$

Show that the vector particle multiplet (with a highest spin of 1) of $\mathcal{N} = 3$ supersymmetry is the same as that for $\mathcal{N} = 4$ supersymmetry when CPT is taken into account. This is why globally supersymmetric theories with $\mathcal{N} = 3$ are the same as $\mathcal{N} = 4$ at the renormalizable level.
Next, show that the gravity multiplets (highest spin of 2) are different for $\mathcal{N} = 3$ and $\mathcal{N} = 4$ supersymmetry.

4 Gauge Symmetry Breaking

Consider $\mathcal{N} = 1$ supersymmetry. By examining the massless and massive spin 1 vector multiplets, argue that a vector multiplet "eats" an entire chiral multiplet's worth of states when the gauge symmetry is broken. That is, argue that the number and spin of the states in a single massive spin-1 vector multiplet is the same as total of the states in a massless vector multiplet plus a massless chiral multiplet.