Physics 731 Homework 5

Due 7 March 2007

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1. Starting with the superfield Lagrangian for abelian gauge theories given in class, find the supersymmetric QED Lagrangian by coupling 2 massive chiral superfields to one vector superfield. The chiral superfields have opposite U(1) charge:

$$\Phi_+ \;\to\; \Phi'_+ = e^{-ie\Lambda}\,\Phi_+\,, \qquad \Phi_- \;\to\; \Phi'_- = e^{ie\Lambda}\,\Phi_- \tag{1}$$

   and the (real) mass matrix has the following components

$$m_{+-} = m_{-+} = \frac{m}{2}\,, \qquad m_{++} = m_{--} = 0\,. \tag{2}$$

2. Expand the Lagrangian in component fields. Work in Wess Zumino gauge and use

$$\begin{aligned}
\Phi_\pm(x,\theta,\bar\theta) &= A_\pm(x) + \sqrt{2}\,\theta\psi_\pm(x) + \theta\theta\,F_\pm(x) + i\theta\sigma^\mu\bar\theta\,\partial_\mu A_\pm(x) \\
&\quad -\frac{i}{\sqrt{2}}\,\theta\theta\,\partial_\mu\psi_\pm(x)\sigma^\mu\bar\theta + \frac{1}{4}\,\theta\theta\bar\theta\bar\theta\,\Box A_\pm(x) \\
\overline{\Phi}_\pm(x,\theta,\bar\theta) &= \bar A_\pm(x) + \sqrt{2}\,\bar\theta\bar\psi_\pm(x) + \bar\theta\bar\theta\,\bar F_\pm(x) - i\theta\sigma^\mu\bar\theta\,\partial_\mu\bar A_\pm(x) \\
&\quad +\frac{i}{\sqrt{2}}\,\bar\theta\bar\theta\theta\,\sigma^\mu\partial_\mu\bar\psi_\pm(x) + \frac{1}{4}\,\theta\theta\bar\theta\bar\theta\,\Box\bar A_\pm(x) \\
V(x,\theta,\bar\theta) &= -\theta\sigma^\mu\bar\theta\,v_\mu(x) + i\theta\theta\bar\theta\bar\lambda(x) - i\bar\theta\bar\theta\theta\lambda(x) + \frac{1}{2}\,\theta\theta\bar\theta\bar\theta\,D(x) \\
W_\alpha(x,\theta,\bar\theta) &= -i\lambda_\alpha(x) + \theta_\alpha\,D(x) - \frac{i}{2}\,(\sigma^\mu\bar\sigma^\nu\theta)_\alpha\,v_{\mu\nu}(x) + \theta\theta\,(\sigma^\mu\partial_\mu\bar\lambda(x))_\alpha \\
\overline{W}_{\dot\alpha}(x,\theta,\bar\theta) &= i\bar\lambda_{\dot\alpha}(x) + \bar\theta_{\dot\alpha}\,D(x) + \frac{i}{2}\,(\bar\sigma^\mu\sigma^\nu\bar\theta)^{\dot\beta}\epsilon_{\dot\alpha\dot\beta}\,v_{\mu\nu}(x) - \bar\theta\bar\theta\,(\bar\sigma^\mu\partial_\mu\lambda(x))^{\dot\beta}\epsilon_{\dot\alpha\dot\beta}
\end{aligned}$$

   with implied contraction of spinor indices, *i.e.* $\theta\theta\bar\theta\bar\theta = \theta^\alpha\theta_\alpha\bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha}$ and $(\sigma^\mu\bar\sigma^\nu\theta)_\alpha = (\sigma^\mu)_{\alpha\dot\alpha}(\bar\sigma^\nu)^{\dot\alpha\beta}\theta_\beta$ *etc.*

3. What would have gone wrong if we had started with just one chiral multiplet? [You may not get this if you don't know about anomalies.]

4. Show that the mass term for the scalar $A_\pm(x)$ takes the canonical form after eliminating the auxiliary $F_\pm(x)$. What terms arise from eliminating the auxiliary $D(x)$?

5. Identify the minimal coupling terms in the Lagrangian. Would you have guessed the non–minimal couplings if you had been asked to construct super–QED from an electron, a photon and their superpartners? Hopefully this makes you appreciate the superfield formulation; writing down a Lagrangian in superfields gives you the supersymmetric interaction terms "for free."

Hint: You might find the following identities useful

$$\begin{aligned}
\theta^\alpha\theta^\beta &= -\frac{1}{2}\,\epsilon^{\alpha\beta}\,\theta^2\,, \qquad \theta_\alpha\theta_\beta = \frac{1}{2}\,\epsilon_{\alpha\beta}\,\theta^2 && (3)\\
\bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} &= \frac{1}{2}\,\epsilon^{\dot\alpha\dot\beta}\,\bar\theta^2\,, \qquad \bar\theta_{\dot\alpha}\bar\theta_{\dot\beta} = -\frac{1}{2}\,\epsilon_{\dot\alpha\dot\beta}\,\bar\theta^2 && (4)\\
Tr\,(\sigma^\mu\bar\sigma^\nu) &= -2\eta^{\mu\nu} && (5)\\
Tr\,(\bar\sigma^\mu\sigma^\nu\bar\sigma^\rho\sigma^\sigma) &= 2(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - i\epsilon^{\mu\nu\rho\sigma}) && (6)\\
(\bar\psi\bar\sigma^\mu\lambda) &= -\lambda\sigma^\mu\bar\psi && (7)
\end{aligned}$$