Physics 742 Homework 3: Decays

1 Top quark decay

Compute the decay width and lifetime of the top quark. Get the right answer without factor of 2 type mistakes and without looking it up anywhere. Do this as follows.

Choose a partner from the students enrolled in the class (or making an effort to do the homework). You are NOT to do this problem by yourself.

The problem will be broken up into a number of parts. Do each part by yourself; then meet your partner and confer on the answer. If you differ, walk through your treatments and figure out who made the mistake. When you agree on the answer and feel semi-certain you have it right, separate again and do the next part by yourself; then meet again, and so on. This is the way people actually solve real problems without making algebraic mistakes.

In conducting this problem, use the following values:

$$
\begin{aligned}
m_t &= 174 \text{ GeV} \\
M_W &= 80.4 \text{ GeV} \\
M_Z &= 91.187 \text{ GeV} \\
m_{b,c,s,d,u} = m_{\tau,\mu,e} &= 0 \\
\alpha_{\text{em}} &= \frac{1}{128} \\
\sin^2\Theta_W &= 0.2311
\end{aligned}
$$

That is, take the other 5 quarks to be massless, but do not neglect the masses of the gauge bosons.

1.1 Interaction Hamiltonian

Determine what the correct final state is, and what term or terms in the interaction Hamiltonian will be relevant in the computation of the decay width.

1.2 Matrix element

Write down the matrix element. Then square it and sum over final and average over initial spins.

1.3 Reducing $\mathcal{M}^2$

Perform traces, Lorentz algebra and so forth on the squared matrix element, and express the width as an integral over a final state phase space, with an integrand of Lorentz invariant contractions of the relevant 4-momenta.

1.4 Final integration

Perform the phase space integration. Write an analytic expression, neglecting only the masses of the 5 other quarks (that is, treating M_Z/m_t and M_W/m_t as order 1). Substitute numerical values.

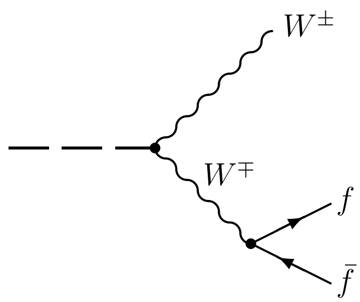
Compute the lifetime from the width. Hint: $\hbar c = 197.32696\,\text{MeV Fermi}$.

2 $H \to WW$

The dominant Higgs boson decay mechanism is $H \to b\bar{b}$. However, the decay width is very small due to the very small b quark mass. Therefore, other decay mechanisms which are formally higher order may nevertheless compete with $\mathcal{H} \to b\bar{b}$, and may be easier to observe in a hadron collider. Therefore, consider the decay

$$H \to W f \bar{f}\,, \tag{1}$$

via the diagram



where f and $\bar{f}$ are a pair of fermions which could result from the decay of a $W^{\mp}$ (that is, for W^- they are $e^-\bar{\nu}_e$, $\mu^-\bar{\nu}_\mu$, $\tau^-\bar{\nu}_\tau$, $d\bar{u}$, $s\bar{c}$).

I strongly recommend that you do this problem by the same "buddy method" used in the first problem, but you are free to do it by yourself if you prefer. Systematically ignore fermion masses throughout this problem; but you obviously cannot neglect the W boson mass M_W.

2.1 Matrix element

Argue that exactly half the width, via this process, will be from the case with a W^+ in the final state, and half from the case with a W^-. (Is there a symmetry at play here?) Having made this argument, concentrate on the partial width with a W^+ appearing in the final state. Remember to multiply by 2 at the end of the problem.

Write down the matrix element for this process, before summing on the external state spins and polarizations. Call the incoming momentum p_H, and the outgoing momenta p_W, p_f, and $p_{\bar{f}}$, and call the polarization state of the external W boson, ϵ. You may find convenient to define and use $q \equiv p_f + p_{\bar{f}} = p_H - p_W$ in expressing $\mathcal{M}$ and in manipulating it in later steps.

2.2 Squared matrix element

Evaluate the final spin and polarization summed, squared matrix element. Carry out all Dirac traces. DO NOT try to simplify further by contracting the fermion momenta onto the W propagators–you will just have to undo that work in a moment.

2.3 Integration on fermionic momenta

Write down the width as an integral over final state momenta, of the squared matrix element.

Leaving the p_W integral for last, carry out the integration over the final state fermionic momenta. Use the same tricks you learned from the book; plagarize as much as possible. This will replace the expression involving p_f and $p_{\bar{f}}$ with an expression involving q.

2.4 Total width

Express the total width as a single integral. Perform this integral numerically for the choice $M_H = 126$ GeV, $M_W = 80.4$ GeV to find the partial width for this decay mechanism. Compare this to the partial width you find for the decay $H \to b\bar{b}$, using $m_b = 3.07$ GeV.

Now repeat the calculation (this should not be too hard!) for the decay $H \to Zf\bar{f}$, where $f\bar{f}$ is a pair of fermions which can arise in the decay of a Z boson. What is the partial width, and what is the partial width *restricting* to the case that $f\bar{f}$ are $\mu\bar{\mu}$ *and* that the Z boson also decays to $\mu\bar{\mu}$? This final state is one of the two channels in which the Higgs boson was discovered; although the partial width you find is quite small, the final state is very distinctive, relatively easy to measure, and allows a clean energy determination.