Physics 742 Homework 4: scattering 1

For the purposes of these problems, use the following numerical values:

$$\begin{aligned}
m_t &= 174 \text{ GeV} \\
M_H &= 126 \text{ GeV} \\
M_W &= 80.4 \text{ GeV} \\
M_Z &= 91.187 \text{ GeV} \\
m_{b,c,s,d,u} = m_{\tau,\mu,e} &= 0 \\
m_b &= 3.07 \text{ GeV} \quad (\text{when computing } \Gamma_{H\to b\bar{b}}) \\
\alpha_{\rm em} &= \frac{1}{128} \\
\sin^2\theta_W &= 0.2311 \\
\alpha_3 &= 0.118\,. \qquad (1)
\end{aligned}$$

That is, systematically neglect the masses of all the “light” degrees of freedom.

1 Ratio of muon to hadron cross-section

Almost all of the calculation in this problem can be lifted from the book, specifically Eq.(6.21), (6.28), (6.50). Do not repeat these steps (this problem is supposed to be a simple application of results in the book rather than an extensive calculation).

Write an expression for the cross-section for $e^+e^- \to \mu^-\mu^+$ and an expression for the cross-section for $e^+e^- \to u\bar{u}$ an up-antiup pair. In each case neglect the masses of the external states. Plot your result on a log-linear plot, linear in $\sqrt{s}$ from 1 GeV to 200 GeV and logarithmic in σ. Then plot the ratio

$$R \equiv \frac{\sigma_{e^+e^-\to u\bar{u}}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$$

on a linear-linear plot in the same momentum range. Briefly comment on any features in these plots.

2 $e^+e^- \to W^+W^-$

LEP II collided e^+ with e^-, and ran at center of mass energies all the way up to 206 GeV, sufficient to allow the process $e^+e^- \to W^+W^-$ to occur. Draw all 4 tree level (two-vertex) diagrams for the process $e^+e^- \to W^+W^-$. Do *not* evaluate them; just draw what the diagrams are.

Could this process be used to test whether the interactions between weak bosons (γ, Z^0, and $W^{\pm}$) are as predicted in the Standard Model? Is there one of the 4 diagrams which is negligibly small compared to the others?

3 Higgsstrahlung

This should also be a "buddy method" problem.

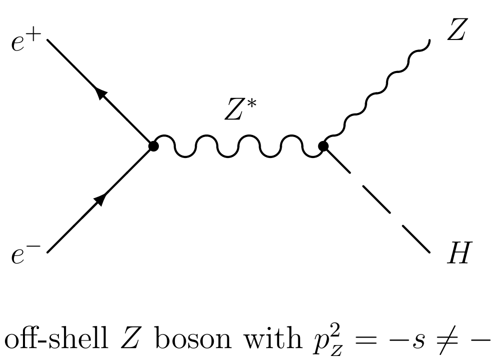
Hadron colliders are actually a poor environment to study the Higgs boson, because the production rate is not that high, they are produced in a confusing environment together with many hadronic particles, and the background rates of other processes are very high. There has recently been intense discussion of studying the Higgs boson in detail by building an e^+e^- collider.

3.1 Resonant production

Quickly compute the partial width and branching fraction for the Higgs boson to decay to an e^+e^- pair. Compare your values to the values for e^+e^- decay of the Z boson. Explain why on-resonance production is not an attractive option. [If a muon collider can be built, on-resonance production is higher by a factor of $(m_\mu^2/m_e^2) \sim 40000$ and resonant production becomes more interesting.]

3.2 Higgsstrahlung matrix element

Consider instead the associated production of a Higgs boson and a Z boson, by a process called "Higgsstrahlung," with Feynman diagram



Here Z^* just means an off-shell Z boson with $p_Z^2 = -s \neq -M_Z^2$.

Calculate the matrix element and write down the final state phase space integration for this process. Do *not* assume that either m_H or m_Z is light compared to $\sqrt{s}$. However you obviously should neglect the electron mass in your expressions.

3.3 Total cross-section

Evaluate the total cross section for this process. This time, you should be able to find a closed expression with no unperformed integrations.

3.4 Optimal energy

Make a plot of the total cross-section as a function of the center of mass energy $\sqrt{s}$, in the range $M_Z + m_H < \sqrt{s} < 400\,\text{GeV}$. What is the best choice or range of values for $\sqrt{s}$?

If an accelerator with a luminosity of $10^{34}/\text{cm}^2\,\text{s}$ operated at this energy for 5×10^7 seconds (5 years of running), how many Higgs bosons would it produce?

4 Matrix element with interference

Consider the scattering of an electron with an *electron*-type neutrino, $e^- \nu_e \to e^- \nu_e$.

Draw the *two* Feynman diagrams which contribute. Write down the contribution of each diagram to the matrix element. Evaluate the spin summed and averaged, squared matrix element. *BE VERY CAREFUL* to compute correctly the *sign* of the interference term between the two Feynman diagrams. Feel free to make the approximation that the Mandelstam variables s, t, u obey $|s|, |t|, |u| \ll M_W^2$ to simplify (a little) the matrix elements and their calculation. Also feel free to neglect the electron mass if it makes the calculation simpler.

Repeat the calculation for the scattering process $e^- \nu_\mu \to e^- \nu_\mu$. This process proceeds via a single diagram.

Without carrying out the final state phase space integration, give a very rough estimate of the ratio of the cross-sections for the two processes.