Physics 742 Homework 6

DO ONE of these two problems.
DO IT BY YOURSELF without discussing with other students.
This is your chance to see if you can solve these problems alone!!!

1 Top quark production at hadron colliders

This assignment forces you to work with the concepts involved in parton distribution functions.

1.1 Tree level cross-sections

At lowest order and at tree level, top quark pairs are produced by two processes: $gg \to t\bar{t}$ and $q\bar{q} \to t\bar{t}$. Here g is a gluon and $q, \bar{q}$ are any other quark flavor and its antiparticle.

The differential cross-section for each process is tabulated in the book, Eq. (9.62) and Eq. (9.63). Verify the expression for the $q\bar{q} \to q'\bar{q}'$ process (q is any light quark, q' is the top quark). The integral over this differential cross-section is presented in Eq. (9.64) in the book. Perform the integration over Mandelstam t for this process. Note that the range of $\hat{t}$ which is kinematically available, and the relation between $\hat{s}$, $\hat{t}$, and $\hat{u}$, are modified from the massless-particle case because of the top quark mass.

Use the expressions for the $gg \to q'\bar{q}'$ matrix element and total cross-section given in the book. Do not try to compute them.

1.2 Total production rate

Consider the collision of a proton with an antiproton. Recall that the parton distribution functions $q(x), \bar{q}(x), g(x)$ are defined as the probability density of finding a quark, antiquark, or gluon in a proton containing momentum fraction x.

If a proton contains a quark of momentum fraction x and an antiproton contains an antiquark of momentum fraction x'; and if the squared center of mass energy of the proton-antiproton system is s, what is the Mandelstam $\hat{s}$ for quark-antiquark system?

Write an expression in terms of a double integral (integrals over x, x') for the total cross-section for

1. a q in the proton and a $\bar{q}$ in the antiproton to annihilate into a $t\bar{t}$ pair;
2. a $\bar{q}$ in the proton and a q in the antiproton to annihilate into a $t\bar{t}$ pair;

3. a g in the proton and a g in the antiproton to annihilate into a $t\bar{t}$ pair.

Repeat for the case of a pp (proton-proton) collision. Hint: the $\bar{q}$ parton distribution in an antiproton equals the q distribution in the proton, and vice versa. The gluon distributions are the same.

Your expressions should depend only on s, m_t, and the parton distribution functions $u(x), \bar{u}(x), d(x), \bar{d}(x), \ldots, g(x)$. Compare them with Eq. (9.65) in the book.

1.3 Actual evaluation

There is a table of parton distribution functions linked to the webpage next to this problem. Use it and make a plot of the partial rate of $t\bar{t}$ production by each of the three processes mentioned above, as a function of the center of mass energy $\sqrt{s}$, in the range from 500 GeV to 20 TeV. Put curves for pp and for $p\bar{p}$ on the same plot. Make it a log-log plot!

Now answer the following questions:

- If the Tevatron (initial CM energy 1.8 TeV) was intended as a top quark discovery machine, why was it worth their while to make it a $p\bar{p}$ machine rather than a pp machine?

- How much did the energy boost from 1.8 TeV to 1.96 TeV increase the $t\bar{t}$ production rate at the Tevatron?

- The LHC has operated, so far, at 7 TeV and 8 TeV, and will (hopefully) reach an CM energy of 14 TeV.

  In terms of the LHC's ability to produce top quarks, does it make much difference that it is a pp machine rather than $p\bar{p}$? How much did the top-production rate improve when the LHC stepped from 7 TeV, to 8 TeV? How much will it improve when it goes from 8 TeV to its design energy of 14 TeV? Assume no change in luminosity [though in fact the luminosity improves as the machine goes to higher energy].

2 Pascos-Wolfenstein Ratio

This problem is, in my opinion, easier than the other one, and it has no numerical component. Try it!

Consider the scattering of high energy muon neutrinos off of nuclei. This experiment has recently been performed with a very high energy and intense beam, in the NuTeV experiment at the Fermilab Tevatron in Illinois. By a "high energy beam," I mean the energy is much higher than the proton mass, $p_\nu^0 \gg 1\text{GeV}$, but the center of mass energy is safely less than the W boson mass, $p_\nu^0 m_p \ll M_W^2$. Therefore, you are free to either use W boson propagators in the large M_W^2 approximation, or the Fermi effective theory, in this problem. Their beam could be switched between almost pure ν_μ and almost pure $\bar{\nu}_\mu$; their goal was to measure $\sin^2\theta_W$. This problem will examine how this can be extracted in a clean way by scattering neutrinos from nuclei.

2.1 Diagrams

Draw the leading order Feynman diagrams for

- $\nu_\mu q \to \nu_\mu q$
- $\nu_\mu q \to \mu^- q'$
- $\nu_\mu \bar{q} \to \nu_\mu \bar{q}$
- $\nu_\mu \bar{q} \to \mu^- \bar{q}'$

For the cases where the final state contains a muon, also write what quark species q, q' can be.

2.2 Matrix elements

Calculate the matrix element squared for the process $\nu_\mu d \to \nu_\mu d$. Use crossing to find the matrix element for $\bar{\nu}_\mu d \to \bar{\nu}_\mu d$. Replace the values of g_V and g_A (or g_L and g_R) you used to find the same expressions for up quarks.

Find the squared matrix element for the process $\nu_\mu q \to \mu^- q'$, and find the squared matrix element for the process $\bar{\nu}_\mu q' \to \mu^+ q$. What are q, q'?

Do not perform the phase space integral to find the total cross-section. But *do* determine how the total cross-section depends on the center-of-mass energy – that is, as s is increased, the cross-section increases or decreases as what power of s?

2.3 Parton distribution functions

The probability of finding an up-quark in a proton, carrying momentum fraction x of the proton's momentum, is $u(x)dx$. The probability for down quark, up antiquark, *etc.* are defined similarly as $d(x), \bar{u}(x)$, *etc.* A neutron is the same as a proton but with the exchange $u \leftrightarrow d$.

If a neutrino of energy E strikes a proton, mass m_p, at rest, what is the Mandelstamm s of the collision? What is s for the neutrino to hit a quark carrying a momentum fraction x of the proton's momentum?

Call the total cross-section for neutrino-quark scattering at a given value of s $\sigma_{\nu_\mu q\to\nu_\mu q}(s)$. (Don't evaluate this quantity yet.) Using your result for how this quantity scales with s, write an expression for the total cross-section that a neutrino will hit *any* of the up quarks in a proton:

$$\sigma_{\nu_\mu u\to\nu_\mu u} = \sigma_{\nu_\mu u\to\nu_\mu u}(s)\int dx x^m u(x)$$

which is an integral over x of the cross-section to hit an up quark of momentum fraction x. What is the exponent m?

This result means that the scattering cross-section for the neutrino to hit *any* up-quark in the proton equals the scattering cross-section from an up-quark with the FULL energy of the proton, times the indicated weighted integral over the distribution of up-quarks in the proton.

Write similar expressions for the probability to scatter from $d, \bar{u}, \bar{d}$ in a proton, and for charged scattering. Again, DO NOT compute the cross-sections $\sigma_{\nu d\to\nu d}$.

2.4 Paschos-Wolfenstein ratio

Suppose the neutrinos scatter on a material which is made of an even admixture of neutrons and protons – for instance, Deuterium, D. For a moment, neglect the $\bar{u}$ and $\bar{d}$ content of the nucleons and take them to be made purely of up and down quarks, with $u(x) = d(x)$ (because there are the same number of neutrons and protons). Use the expressions from the last subsection to evaluate the *Paschos-Wolfenstein ratio*,

$$\frac{\sigma_{\nu_\mu D\to\nu_\mu X} - \sigma_{\bar{\nu}_\mu D\to\bar{\nu}_\mu X}}{\sigma_{\nu_\mu D\to\mu X} - \sigma_{\bar{\nu}_\mu D\to\bar{\mu} X}}. \tag{1}$$

To do so, note that the $\bar{\nu}q$ scattering rate is the same as the $\nu\bar{q}$ scattering rate by CP symmetry.

You should find that "almost all the details" cancel out when you take the differences and ratios. So all the things you have postponed or avoided computing, turn out not to matter! In fact, the ratio reduces to a pure number depending only on $\sin^2\theta_W$ (via g_L and g_A of up and down quarks, *etc*).

Now besides up and down quarks, protons and neutrons also contain some $\bar{u}$ and $\bar{d}$. Show that taking the difference between neutrinos and anti-neutrinos, as shown above, means that the numerator and denominator in the ratio each depend, not on $u(x)$ and $\bar{u}(x)$ separately, but only on the difference $u(x) - \bar{u}(x)$. But again, the details cancel in the ratio, which the simple function of $\sin^2\theta_W$ found above. Also, even if protons and neutrons contain some admixture of s and $\bar{s}$ quarks, provided that $s(x) = \bar{s}(x)$, this detail also cancels in the difference and ratio.

Therefore the Paschos-Wolfenstein ratio is a very clean way to measure $\sin^2\theta_W$ experimentally.