Dark Matter Part III: Axions

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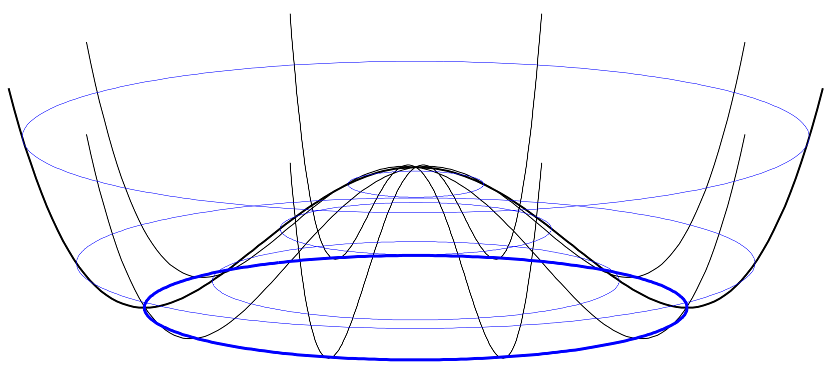
- Moduli as a class of DM candidates
- The $\mathbf{T}$ problem of QCD
- Axions as an elegant solution and moduli field
- Misalignment vs field dynamics and strings
- Astrophysical constraints
- Searching for axions

Moduli field

Consider $\mathcal{C}$ scalar field with symmetry-breaking potential

$$-\mathcal{L}(\varphi)/\sqrt{-g} = g^{\mu\nu}\partial_\mu\varphi^*\partial_\nu\varphi + \frac{m^2}{8f^2}\left(2\varphi^*\varphi - f^2\right)^2 .$$



If Universe starts with $\varphi = \frac{f}{\sqrt{2}}e^{i\theta}$ that value will persist

Breaking symmetry explicitly

Suppose a very slight
breaking of
symmetry:

Initial value undergoes (Hubble) damped oscillations
Before osc. start: acts like cosmological constant
Late times: acts like matter – dark matter!

Dependence on m and f

Dark matter density depends on three variables:

- Mass m. Larger value: $V \propto m^2$: more energy
  Oscillations start sooner $t_{\rm start} \propto m^{-1}$
  Final DM energy density scales as $m^{1/2}$ at fixed f

- Vacuum value f. Larger value: more energy
  Change f at fixed m: $\varepsilon \propto f^2$, $t_{\rm start} \propto f^0$.

- Initial angle: $\varepsilon \propto \theta^2_{\rm init}$ except $\theta \simeq \pi$: anharmonicity

Is this model realistic?

In string, SUSY theories, often many such fields
"Vacuum manifold" not always circle: can be more complex

Nomenclature

- Field couples to curvature R or dimension-4 operators: evolution causes fundamental "constants" to evolve
  "Moduli field"

- Field couples to $E\&M$ through $\theta\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$:
  "Axion-Like Particle" We'll come back to this

Initial conditions: Inflation

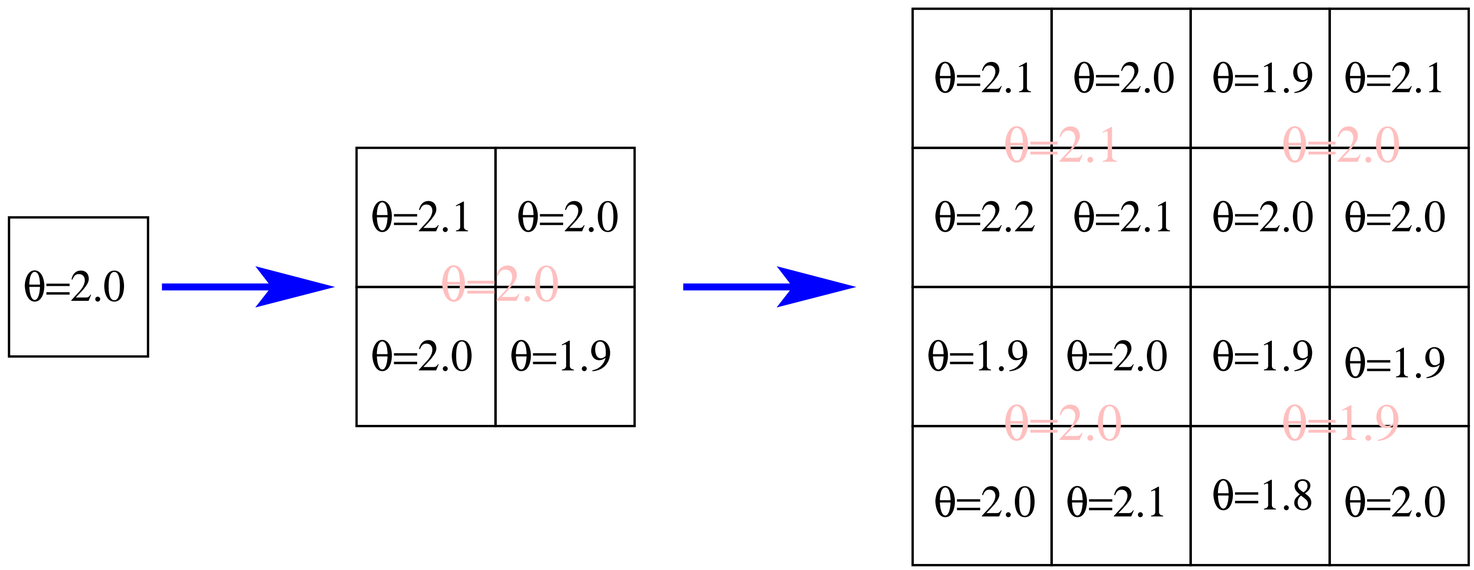
Inflation: (nearly) constant energy density (scalar potential)
Exponential growth in Universe scale factor

$$H^2 = \left(\frac{da}{adt}\right)^2 = \frac{8\pi G_N}{3}\varepsilon \quad \Rightarrow \quad a = e^{t\sqrt{3/(8\pi G_N \varepsilon)}}$$

Region of space $\rightarrow$ region of $2\times$ scale
$\rightarrow$ region $2\times$ ($2\times$ scale) $\rightarrow$ region $2\times$ ($2\times$ ($2\times$ scale)) ...
Value of θ in starting region $\rightarrow$ value in *huge* domain
Inflation: 60 e-folds of growth or more.

Stretching of Quantum Fluctuations

During inflation, each e-fold, field φ vacuum fluctuations on scale $x \sim H^{-1}$ get "frozen", $\Delta\varphi \sim H/2\pi$:



Generates *Gaussian, nearly scale-invariant* fluct. in θ

Value of θ: pre- or post-inflation?

These fluctuations sound good! But might be bad!

- Not *adiabatic*. In DM alone "Isocurvature"
- Size $\Delta\theta/\theta \sim H/2\pi f$. If $\gg 10^{-6}$, ruled out!

Alternative: reheat to high T: $\varphi \to 0$ via thermal effects
As T falls, $T < f$, symmetry breaks.
θ picks *random independent* value in each causal region,
leading to statistically uniform random start

Behavior $\neq$ incoherent sum of all θ-values . . .

Change gears: **T**-symmetry in QED and QCD

T symmetry: "when you run a movie backwards, the *microphysics* is correct."

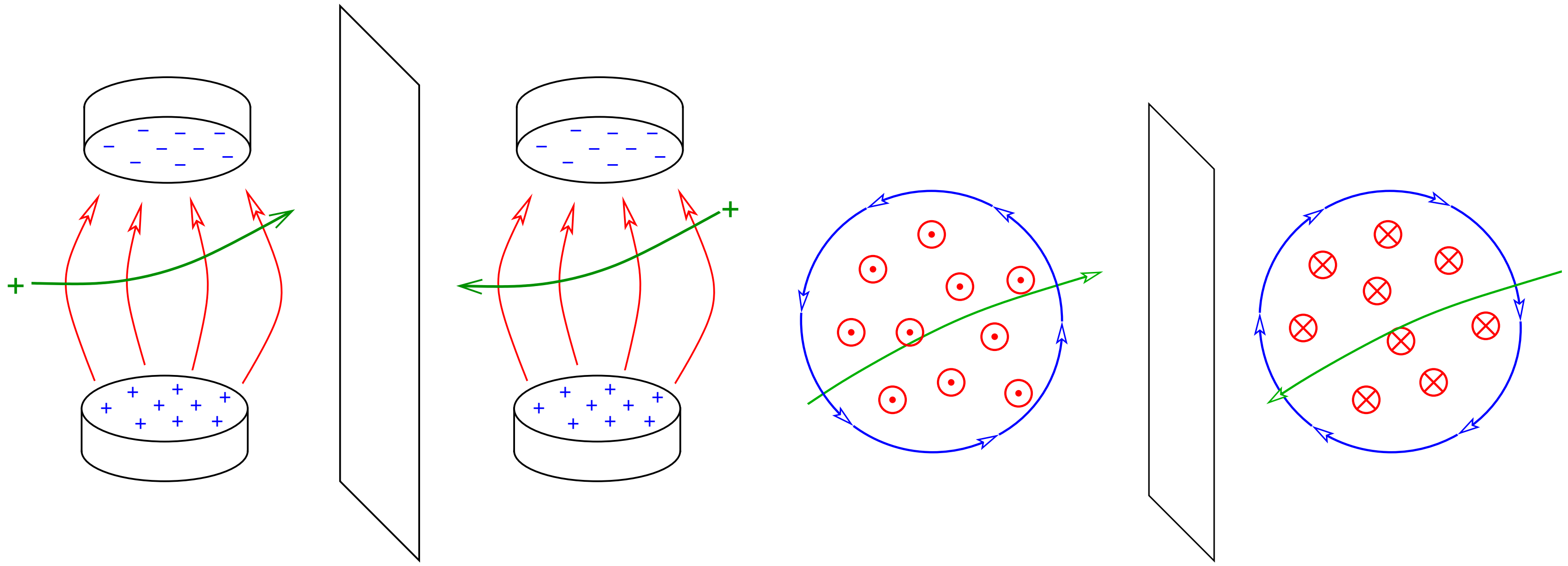
Statistical mechanics breaks **T**.
But microphysics very nearly obeys it!

Weak physics breaks **T**, but only through very small CKM effects. Observed in handful of experiments, all involving neutral meson oscillation.

No evidence for **T** viol in E&M or Strong interactions.

$\mathbf{T}$ in E&M

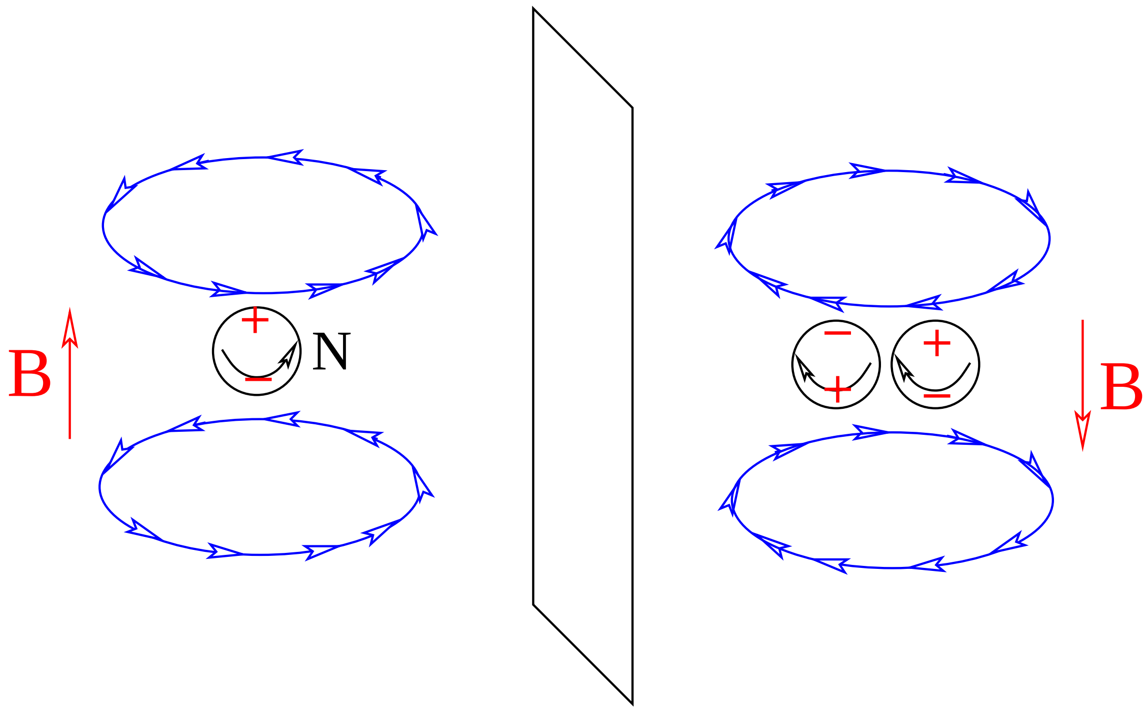
How do E, B fields change when you run movie backwards?



Q's unchanged, but J's flip. E same, but B flips.

Looking for $\mathbf{T}$: Neutron EDM

Put neutron in $\vec{B}$ field – spin lines up with $\vec{B}$.



Is there an Electric Dipole Moment (EDM) aligned with spin?

If so: looks different when movie runs backwards, $\mathbf{T}$ viol!

$\mathbf{T}$ and the E&M Action

Action $S \Rightarrow$ all physics. Local field thy: $S = \int \mathcal{L}\, d^4x$.
$\mathcal{L}$ a singlet (gauge symm) and spacetime scalar (Lorentz):

$$\mathcal{L} = \frac{\vec{B}^2 - \vec{E}^2}{2e^2} + \frac{\Theta}{4\pi^2}\vec{E}\cdot\vec{B} + \text{(electrons...)}$$

$\mathbf{T}$ flip: $\vec{E} \to \vec{E}$ and $\vec{B} \to -\vec{B}$:
$(B^2 - E^2) \to (B^2 - E^2)$ **BUT** $E \cdot B \to -E \cdot B$.

$$\mathcal{L} \xrightarrow{T} \frac{\vec{B}^2 - \vec{E}^2}{2e^2} - \frac{\Theta}{4\pi^2}\vec{E}\cdot\vec{B} + \text{(electrons...)}$$

Nonvanishing Θ is a $\mathbf{T}$ violation!

E&M $\mathbf{T}$ violation is Illusory!

The $\Theta \vec{E} \cdot \vec{B}$ term has no *consequences*!

$$\vec{E} \cdot \vec{B} = \frac{1}{4} \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta} = \partial^{\mu} K_{\mu} \,, \quad K^{\mu} \equiv \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} A^{\nu} F^{\alpha\beta}$$

I can integrate $\vec{E} \cdot \vec{B}$ to a boundary term.
Vanishes if $F^{\alpha\beta}$ vanishes on boundary. Alternately, EOM:

$$0 = \partial_{\mu} \left(\frac{1}{e^2} F^{\mu\nu} + \frac{\Theta}{8\pi^2} \epsilon^{\mu\nu\alpha\beta} \partial_{\alpha} A_{\beta} \right)$$

Second term zero by antisymmetry (**if** Θ constant)

QCD and its Lagrangian

QCD is like 8 copies of E&M, but with non-linearities:

$$\text{Field strength}: \quad G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu + g f_{abc} G_b^\mu G_c^\nu \,,$$

g: coupling. $a = 1 \ldots 8$. f_{abc} "structure constants"

$$S = \int dt \int d^3x \left(\frac{\vec{E}_a^2 - \vec{B}_a^2}{2g^2} + \frac{\Theta}{8\pi^2} \vec{E}_a \cdot \vec{B}_a \right)$$

where $\vec{E}_a \cdot \vec{B}_a$ still a total derivative:

$$\vec{E}_a \cdot \vec{B}_a = \partial^\mu K_\mu \,, \qquad 2K_\mu = \epsilon_{\mu\nu\alpha\beta} \left(G_a^\nu G_a^{\alpha\beta} + \frac{g f_{abc}}{3} G_a^\nu G_b^\alpha G_c^\beta \right)$$

Last term *need not* vanish on boundary even if $\vec{E}_a = 0 = \vec{B}_a$ there!
It's always $8\pi^2 N_I$ with N_I integer. So Θ mod 2π has *physical consequences*
G. 't Hooft, PRL 37, 8(1976); R. Jackiw and C. Rebbi, PRL 37, 172 (1976);

Callan Dashen and Gross, Phys Lett 63B, 334 (1976)

Neutron EDM and Θ

Theory: Neutron electric dipole moment should exist,

$$d_n = -3.8 \times 10^{-16}\, e\,\text{cm} \times \Theta$$

so long as Θ is not zero! Guo *et al*, arXiv:1502.02295, assumes Θ, modulo 2π, is small

Experiment: Consistent with zero! Baker *et al* (Grenoble), arXiv:hep-ex/0602020

$$|d_n| < 2.9 \times 10^{-26}\, e\,\text{cm}$$

Either $|\Theta| < 10^{-10}$ by (coincidence? accident?) or there is something deep going on here.

Θ from UV physics

Consider heavy Dirac quark $[Q^\alpha \; q_{\dot\alpha}]$ Two Weyl spinors
Q^α is 3, q^α is $\bar{3}$. Lagrangian:

$$\mathcal{L}(Q,q) = \frac{1}{2}\bar{Q}\not{D}Q + \frac{1}{2}\bar{q}\not{D}q + m q_\alpha Q^\alpha + m^* q^{\dot\alpha} Q_{\dot\alpha}$$

Mass m is in general complex.
Rotate $m = |m|e^{i\theta} \to |m|$ by rotating Q but not q.
Such a chiral rotation generates shift, $\Theta \to \Theta + \theta$.

Phase in mass of heavy quark becomes part of Θ_{QCD}.

Axion

Give Q^α, q^α different (global) U(1) charges (so $m = 0$)
Introduce complex φ with U(1) charge: can now write

$$\mathcal{L}_{\varphi q Q} = y \varphi q_\alpha Q^\alpha + y \varphi^* q^{\dot{\alpha}} Q_{\dot{\alpha}}$$

Symmetry-breaking potential for φ:

$$\mathcal{L}_\varphi = \mathcal{L}_{\varphi q Q} + \partial_\mu \varphi^* \partial^\mu \varphi + \frac{m^2}{8 f_a^2} \left(2\varphi^* \varphi - f^2 \right)^2$$

Phase $\varphi = e^{i\theta_A} f_a$ becomes part of Θ: $\Theta_{\text{eff}} = \Theta + \theta_A$ or

$$\mathcal{L}_\varphi = \partial_\mu \varphi^* \partial^\mu \varphi + V(\varphi^* \varphi) + \theta_A \frac{G^a_{\mu\nu} \tilde{G}^{\mu\nu a}}{32\pi^2}$$ [dim-5]

Kim PRL 43 103 (1979): Shifman Vainstein Zakharov NPB 147 385 (1979)

How the axion works

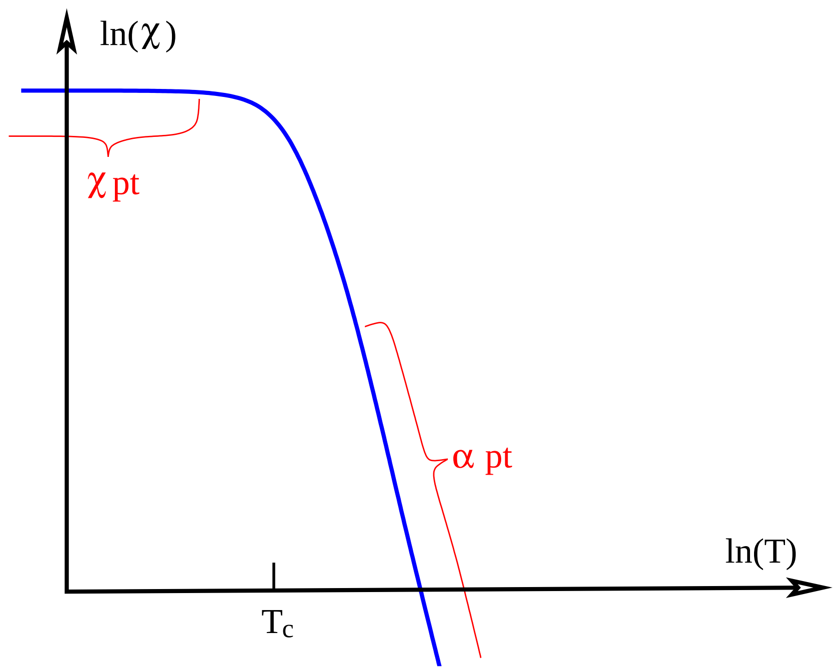
φ, therefore θ_A, can evolve. What value is (free) energetically preferred? $W = \Omega V_{\rm eff}(\varphi) = -T\ln(Z_{\rm Eucl})$, so

$$\begin{aligned} V_{\rm eff}(\theta_A) &= -\frac{T}{\Omega}\ln \int \mathcal{D}(G_\mu \bar\psi\psi)\, {\rm Det}\,(\not{D}+m) e^{-\int \frac{G^2_{\mu\nu}}{4g^2}} \times e^{i(\Theta+\theta_A)\int \frac{G\tilde{G}}{32\pi^2}} \\ &\simeq \chi(T)(1-\cos[\Theta+\theta_A])\,, \\ \chi(T) &= \left\langle \int d^4x \frac{G\tilde{G}(x)}{32\pi^2}\frac{G\tilde{G}(0)}{32\pi^2} \right\rangle_\beta \end{aligned}$$

Nontrivial $\Theta + \theta_A$ ($\mathbf{T}$-violation) $\rightarrow$ phase cancellation $V_{\rm eff}$ minimized when $\Theta_{\rm eff} = 0 \rightarrow \mathbf{T}$ valid.

Peccei Quinn PRL 38, 1440 (1977);
J. E. Kim, PRL 43, 103 (1979); Shifman Vainshtein and Zakharov, NPB 166, 493 (1980)

$\chi(T)$: what we expect



Low T: χ-pt works.

$\chi \simeq \frac{m_u m_d}{(m_u+m_d)^2} m_\pi^2 f_\pi^2$

Hi T: standard

pert-thy works(??)

Low T: $\chi(T \ll T_c) = (76 \pm 1\ \mathrm{MeV})^4$ Cortona *et al*, arXiv:1511.02867

High T: $\chi(T \gg T_c) \propto T^{-8}$ Gross Pisarski Yaffe Rev.Mod.Phys.53,43(1981)

Recent lattice results arXiv:1606.07494: $\chi(T)$ to high T

This time the potential tilts...

This is just like moduli
field from before!
But now potential tilt is T,
time dependent

Final osc. frequency = axion mass: $m_a^2 = \chi / f_a^2$

Dynamics

if θ_A starts uniform, DM density depends on:
f_a and $\theta_{A,\mathrm{init}}$. no prediction for f_a
Problems with isocurvature pert. as discussed.

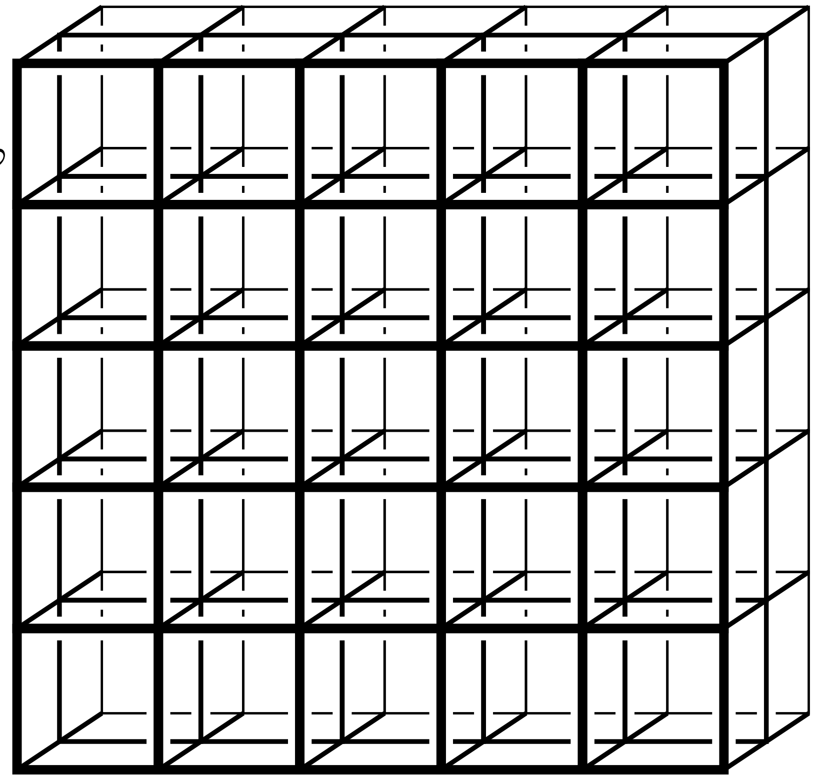
but if θ_A starts randomly different different places,
statistically known starting conditions
DM density depends on f_a *alone*, and solvable dynamics
Solve field dynamics: DM density $\Rightarrow$ prediction for f_a

Visinelli Gondolo arXiv:1403.4594

Solving space-inhomogeneous case

Put the Lagrangian

$$\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi + \frac{\lambda}{8}(2 - \varphi^* \varphi)^2 - \chi(t)\,\mathrm{Re}\,\varphi$$



as classical field thy.

on **real-time lattice**,

$\varphi(t=0)$ random,

Hubble drag,

Count axions at end.

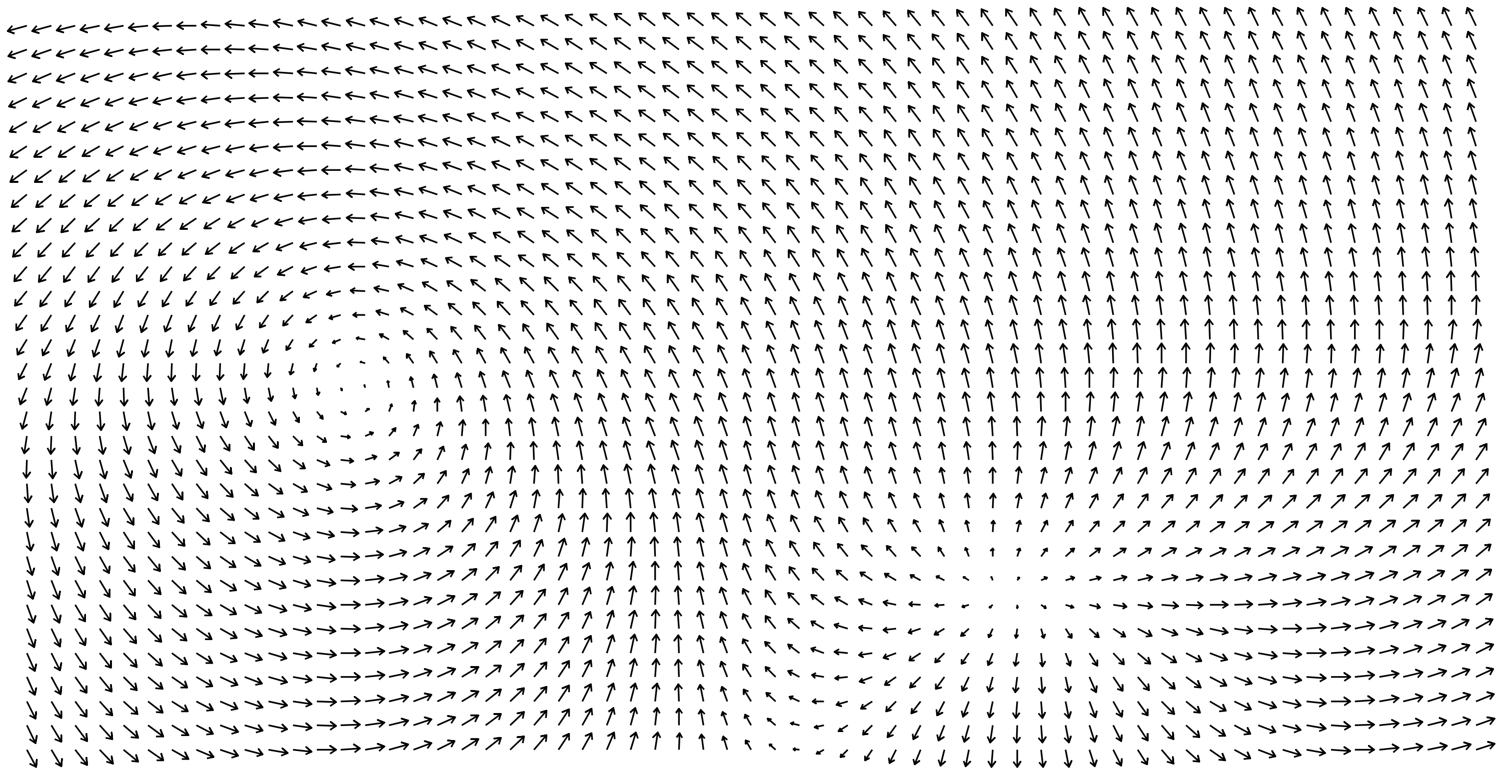
Nonperturbative approach.

$\chi(T)$ from Borsanyi *et al* 1606.07494

Axions and Topology I

φ is a complex number – plot as a 2D arrow.

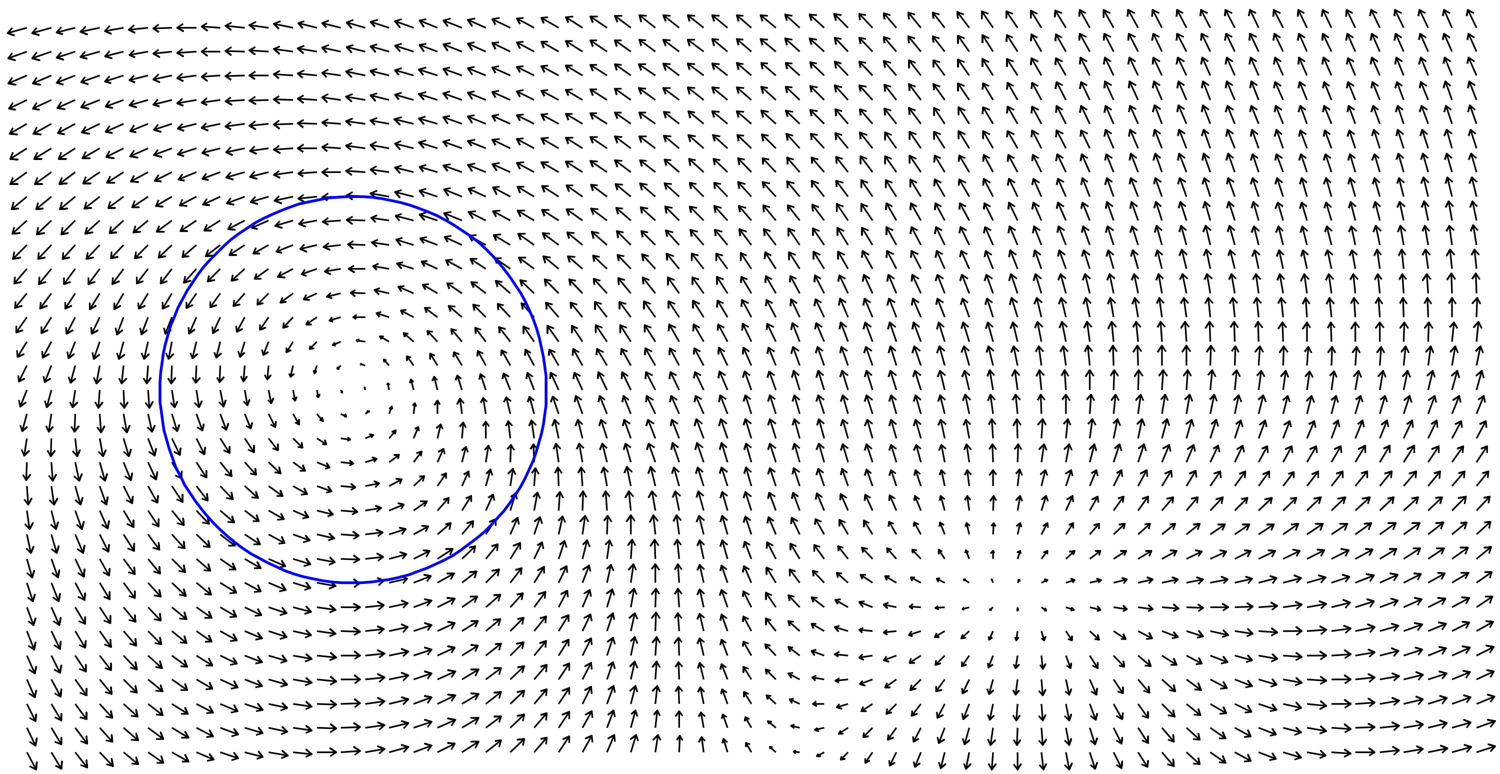
Axion field: a field of arrows. 2D slice for instance:



Field generically has vortices Davis, PLB180 225 (1986)

Axions and Topology II

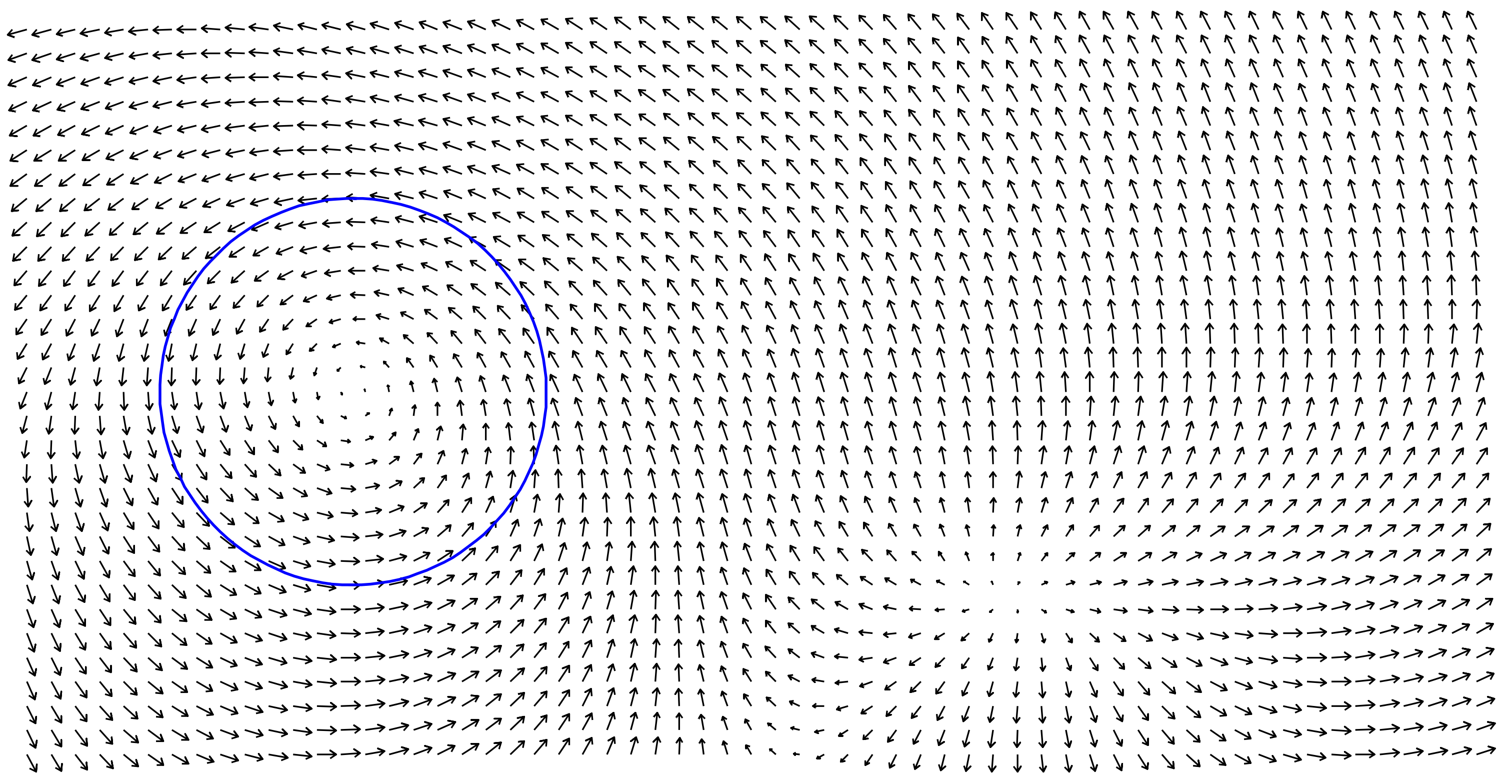
As you circle (anti)vortex, angle θ_A varies by $(-)2\pi$.



Continuity: angle θ_A *must* be undefined somewhere inside the circle. $\varphi = 0$ somewhere. Center of vortex.

Axions and Topology III

As you circle vortex, angle θ_A varies by 2π.



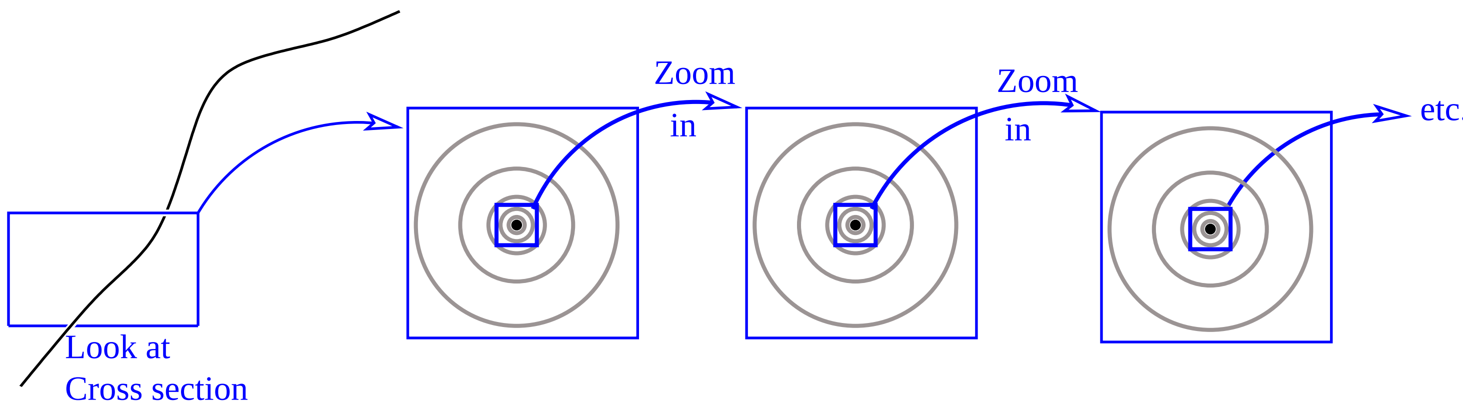
2D slice of a 3D picture: these "vortices" are 1D line structures.

Domain walls

2D slice of evolution, When the potential tilts:

Layers of String Energy

$$E_{\rm str} = \int dz \int d\phi \int r\,dr\,\left(\nabla\phi^*\nabla\phi \simeq f_a^2/2r^2\right) \simeq \pi\ell f_a^2 \int_{\sim f_a^{-1}}^{\sim H^{-1}} \frac{r\,dr}{r^2}$$



Series of "sheaths" around string:

equal energy in each $\times 2$ scale, 10^{30} scale range! $\ln(10^{30}) \simeq 70$.

Log-large string tension $T_{\rm str} = \pi f_a^2 \ln(10^{30}) \equiv \pi f_a^2 \kappa$

Not reproduced by numerics (separation/core $\sim$ 400)

Getting string tension correct MATTERS!

String dynamics are controlled by:

- String tension and inertia: $\propto \kappa\pi f_a^2$ FACTOR of κ
- String radiation and inter-string interactions: $\propto \pi f_a^2$ NO factor of κ

Relative importance of these effects,
and string energy, are κ dependent

We really need to get this physics right!

An effective description

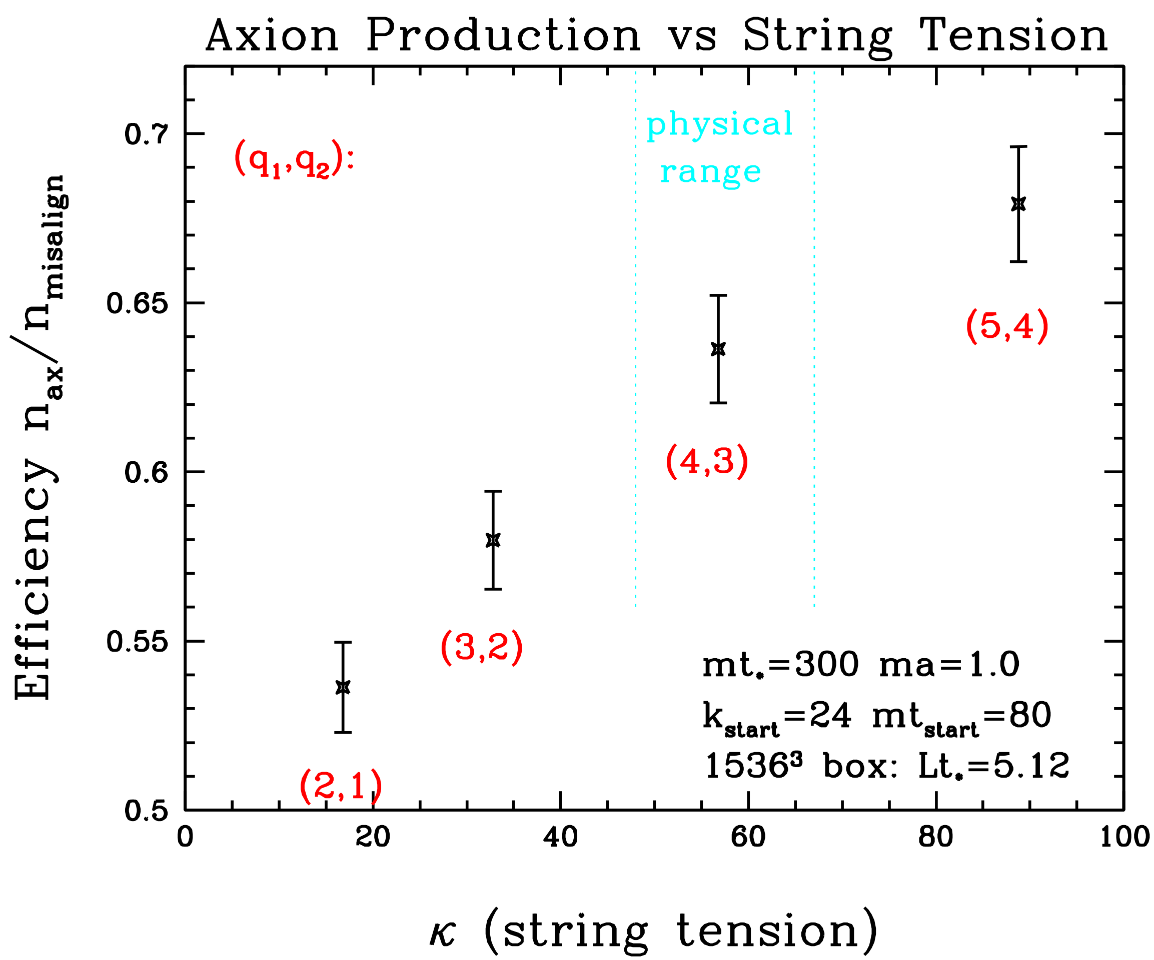
The important axion-production physics is:

- Only long-range (light) degree of freedom is axion
- Axion strings: thin cores with high tension $T \simeq 70 \times \pi f_a^2$
- Correct string-field interactions.

Our approach Klaer and Moore, arXiv:1707.05566,1708.07521: find field content which reproduces this same effective description, plus modes which can be made arbitrarily heavy in small-a limit

Higher tension = higher initial density, longer lasting, hardier loops

Results



Axions production mildly string-tension dependent

Results

- $10\times$ string tension leads so $3\times$ network density but
- only 30% more axions than with axion-only simulation,
- **Fewer** (78%) axions than $\theta_{A\,\mathrm{init}}$-averaged misalignment
- Axionic string networks are *very bad* at making axions
- Results in less axion production.
  Must be compensated by ligher axion mass.

Put it all together

Axion production: $n_{\rm ax}(T = T_*) = (13 \pm 2) H(T_*) f_a^2$

Hubble law: $H^2 = \dfrac{8\pi\varepsilon}{3m_{\rm pl}^2}\,,$

Equation of state: $\varepsilon = \dfrac{\pi^2 T^4 g_*}{30}\,, \quad s = \dfrac{4\varepsilon}{3T}\,, \quad g_*(1{\rm GeV}) \simeq 73$

Susceptibility: $\chi(T) \simeq \left(\dfrac{1\,{\rm GeV}}{T}\right)^{7.6} (1.02(35) \times 10^{-11}{\rm GeV}^4)$

Dark matter: $\dfrac{\rho}{s} = 0.39\,{\rm eV}$

One finds $T_* = 1.54\,{\rm GeV}$ and $m_a = 26.2 \pm 3.4\,\mu{\rm eV}$

Summary so far

- If the QCD axion exists (solving Θ **T** problem)
- If the axion is the Dark Matter
- If its "symmetry breaks" after inflation

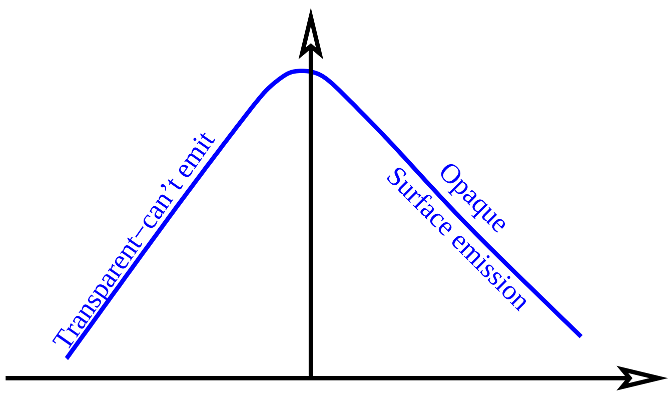
Then it has $m = 26.2 \pm 3.4\,\mu\mathrm{eV}$ ($\simeq 6\,\mathrm{Ghz}$)

So, what are constraints and detection prospects?

Next: constraints and detection strategy

- What are astrophysical constraints?
- Searching for dark-matter axions
- Planned next-generation detectors
- More general context – Axion-Like Particles

Astrophysics constrains axions



Energy emission suppressed at very small coupling but also at large coupling where only cold surface emits

Supernovae actually opaque to ν. Could cool faster through axion emission. Contradicts ν pulse from SN1987A

If axion coupling $> \nu$-coupling, limits from red giants, white dwarfs, normal stars ...

Axion ruled out for $m_a \geq \mathrm{few} \times 10^{-3}\,\mathrm{eV}$

Looking for DM axions today

Axion field still fluctuating today. Energy density is:

$$\varepsilon_{\rm DM} = \frac{\chi(0)}{2}\theta^2_{\rm max} \quad \Rightarrow \quad \frac{(76\,{\rm MeV})^4}{2}\theta^2_{\rm max} = \frac{0.3\,{\rm GeV}}{{\rm cm}^3}$$

$$\theta_{\rm max} = 4 \times 10^{-19}$$

Expected $\theta_A G_{\mu\nu}\tilde{G}^{\mu\nu}$ effect: neutron EDM oscillates by $10^{-34} e{\rm cm}$ at $6\,{\rm Ghz}$

Never going to see that! Need to consider other couplings

Other couplings of the axion

Symmetries allow several other couplings to axions:

$$
\begin{aligned}
-\mathcal{L}_{\rm int} &= g_{a\gamma}\theta_A \vec{E}\cdot\vec{B} + \sum_{\psi} \partial_\mu \theta_A \, \bar{\psi}_i \gamma^\mu \left(A_{ij} + B_{ij}\gamma^5\right)\psi_j \\
g_{a\gamma} &= \frac{\alpha_{\rm EM}}{2\pi f_a}(1.92 - C)
\end{aligned}
$$

The $\bar{\psi}\gamma^\mu\psi$ couplings are to fermions.
The $\vec{E}\cdot\vec{B}$ is coupling to electromagnetism.
Coefficient: 1.92 from *mixing between* θ_A, π^0 *at QCD scale*
C is (model-dependent) UV coupling of θ_A to EM

Nice review article: Redondo et al arXiv:1801.08127

Looking for DM axions with EM coupling

EOM for EM in presence of axion and dielectric

$$\begin{aligned}\mathcal{L} &= \frac{\kappa}{2}E^2 - \frac{1}{2}B^2 + g_{a\gamma}\theta_A \vec{E}\cdot\vec{B} \\ \partial_0 \frac{\partial\mathcal{L}}{\partial\partial_0 A_i} &= -\partial_j \frac{\partial\mathcal{L}}{\partial_j A_i} \\ \kappa\partial_0 E_i &= -g_{a\gamma}\partial_0\theta_A\, B_i + (\nabla\times B)_i\end{aligned}$$

$g_{a\gamma}\partial_0\theta_A\, B$ contributes like oscillating current

But $g_{a\gamma} \sim 10^{-14}\,\mathrm{GeV}^{-1}$ and $\partial_0\theta_A \sim (6\,\mathrm{Ghz}\times 4\times 10^{-19})$

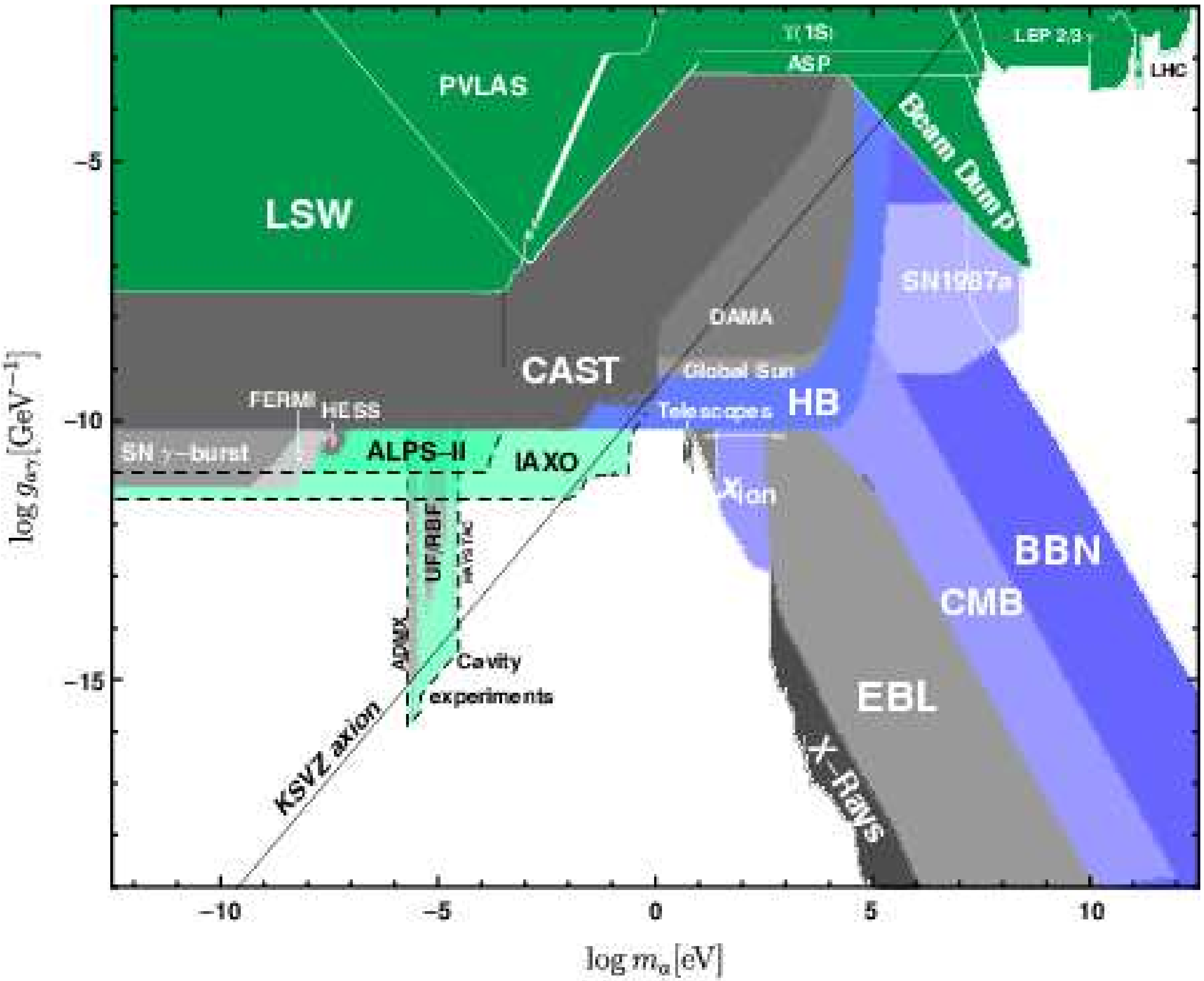
Axion? Or Axion-Like Particle?

If I look for axion-EM coupling I am sensitive to more general class of particles, the ALPs

- Any very light pseudoscalar ϕ with coupling $\mathcal{L} = g_{a\gamma}\phi\vec{E}\cdot\vec{B}$
- May have nothing to do with Θ, QCD. No need for relation between $m, g_{a\gamma}$
- Less well motivated than QCD axion. But "generically arise in string theory"

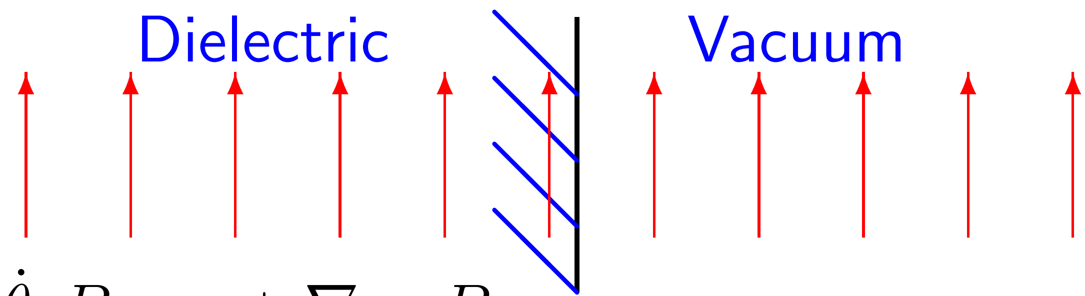
Wider mass-coupling parameter space can be searched

Existing and projected constraints



How to turn axions into microwaves

Consider dielectric-vacuum interface, B tangent to surface



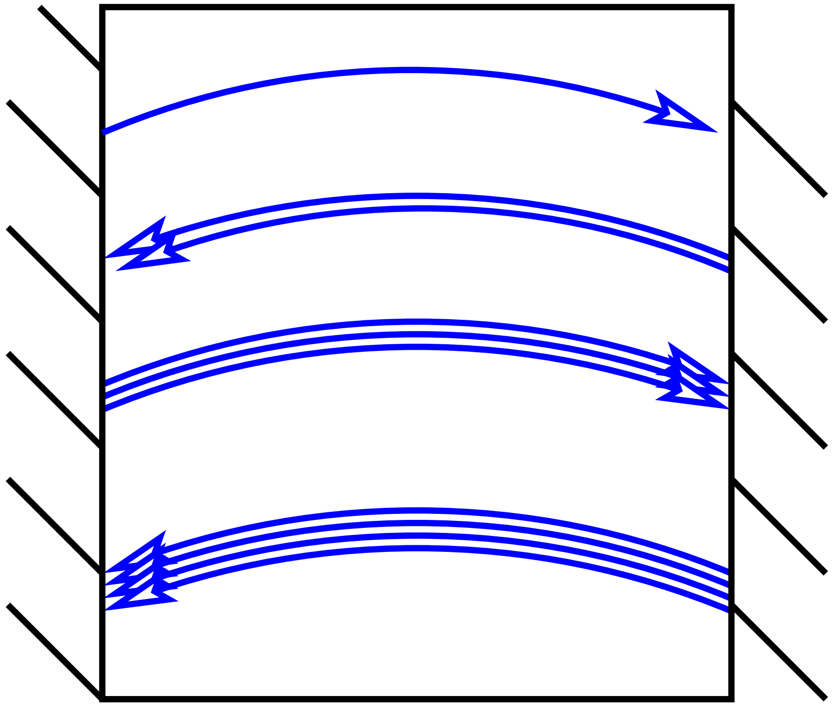
$$\dot{E}_{\parallel} = \frac{-g_{a\gamma}\dot{\theta}_A B_{\text{static}} + \nabla \times B}{\kappa} \qquad \dot{E}_{\parallel} = -\gamma_{a\gamma}\dot{\theta}_A B_{\text{static}} + \nabla \times B$$

As usual, $E_{\parallel}$ must match at boundary

Impossible without $\nabla \times B$ traveling wave component!

Interface, bathed in B-field, emits microwaves with $\omega = m_a$

Resonant cavity detection



Conductor: $\kappa \simeq i\infty$

Separate by $\lambda/2$: resonant growth of wave

Power produced enhanced by Q. Noise reduced $1/Q$

But must probe Q-times as many resonant frequencies

Ability to search enhanced by one factor of Q

Resonance search

Resonant search: enormously enhanced signal/noise
Problem: don't know the mass!

Build apparatus with *tuneable* resonance
Sweep through all possible resonance frequencies

If you see a "hint" – go back and integrate on it
If you find something – integrate as long as you want

Any detection will always have a S/N of $\sim \sqrt{Q} \gg 1$.
Absolutely no false-positive "hints"

Resonant cavity approach

Resonant cavities: ADMX PRL104 041301, HAYSTAC arXiv:1611.07123, CULTASK arXiv:1707.05925

- Challenge: cavity with tuneable resonant frequency
- Challenge: low T_{noise}, high resonant Q

$$\text{Power} = 7\times 10^{-23}\,\text{W}\left(\frac{Q}{10^5}\right)\left(\frac{\mu\text{eV}}{m_a}\right)\left(\frac{g_{a\gamma}}{2\times 10^{-16}\text{GeV}^{-1}}\right)^2\left(\frac{B_e}{8\text{T}}\right)^2\frac{V}{200\,\text{l}}\times\mathcal{O}(1)$$

- Problem: $\ell = \lambda/2$ gets small at large m_a: $V \propto \ell^3$

Most results at few μeV. Future: to $25\,\mu\text{eV}$

Alternative: MADMAX

Parallel dielectric plates

B field: microwaves..

Emission from each plate

adds coherently if optical

dist between plates $= \lambda$

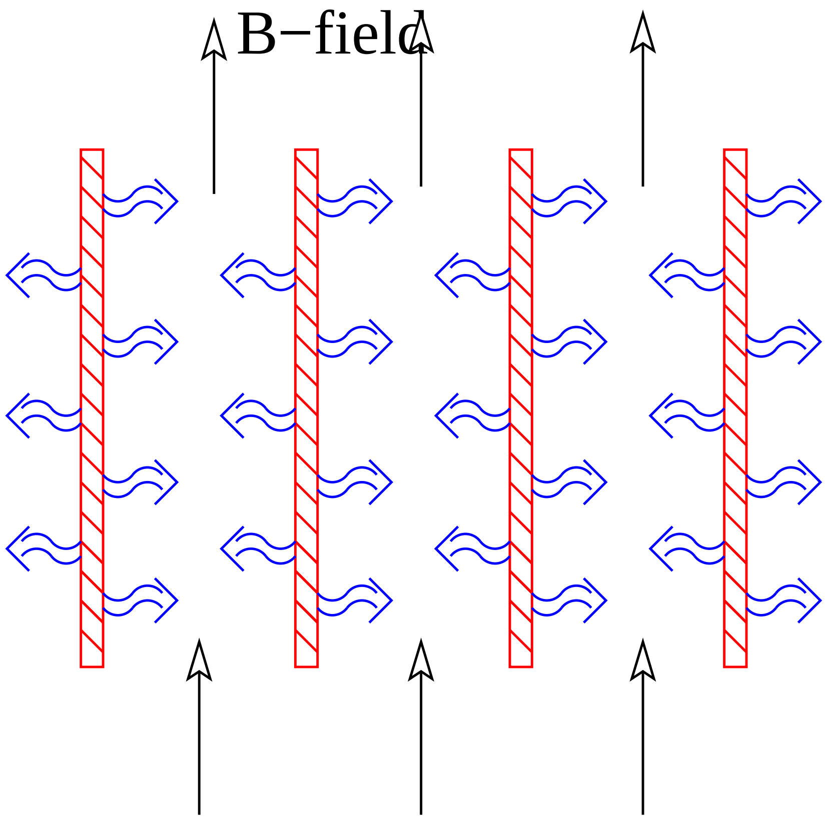


Plate separations can be actuated to tune sensitive λ

Loses Q but gains volume $\sim \mathrm{m}^3$ arXiv:1901.07401

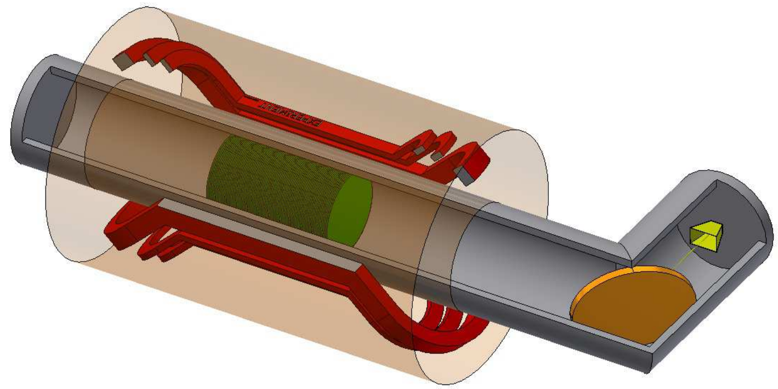
MADMAX

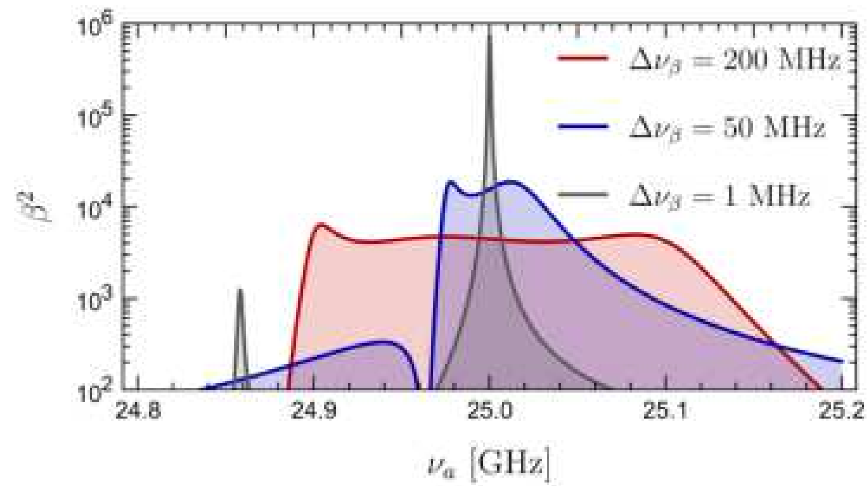
Design: dielectric disks in magnet, optics and detector

Only detector cryogenic

Magnet: design challenge

"Boost factor" plays role of Q. Choice of disk separations: high and narrow or tophat-like





MADMAX: plans

Magnet design underway: cost $\sim 20M$Euro

Designed for axion sensitivity in $40\mu\mathrm{eV}$ to $400\mu\mathrm{eV}$

I'm trying to talk them into going down to $25\mu\mathrm{eV}$

Hopefully built and on-line in ~ 5 years

Conclusions

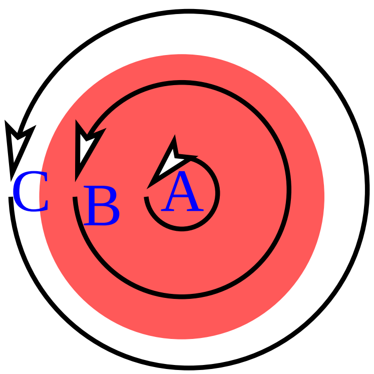
- "Moduli field" or "Axion-like Particle" coherently produced in early Universe, interesting DM candidate
- $\mathbf{T}$ symmetry in QCD motivates Axion particle
- Symmetry breaking after inflation $\rightarrow$ complex field dynamics
- Predictive for axion mass
- Axions hard, not impossible to find

Pursue multiple plausible DM candidates.

Abelian Higgs Model: Tension-Only Strings

$$\mathcal{L}(\varphi, A_\mu) = \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + (D_\mu \varphi)^*(D^\mu \varphi) + \frac{\lambda}{8}\left(2\varphi^*\varphi - f_a^2\right)^2$$

with $D_\mu = \partial_\mu - ieA_\mu$ covariant derivative



$$\oint \partial_\phi \varphi \, d\phi = 2\pi f_a \quad \text{but}$$

$$\oint D_\phi \varphi \, d\phi = (2\pi - B_{\rm encl}) f_a$$

A: full $\nabla\varphi$ energy.

B: partial. C: cancels.

Outside string, B compensates $\nabla\varphi$.

Finite tension $T \simeq \pi f_a^2$. No long-range interactions.

Abelian Higgs model

- Network of strings with tension $T \simeq \pi f_a^2$
- Only massive fields (Higgs, massive vector) outside cores
- No long-range interactions between strings
- Leads to dense networks, $\sim 8\times$ denser than...
- Look just like what we want "string cores" to look like

Trick: global strings, local cores

Hybrid theory with A_μ and two scalars

$$\begin{aligned}\mathcal{L}(\varphi_1, \varphi_2, A_\mu) \;=\; & \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\ & + \frac{\lambda}{8}\left[(2\varphi_1^*\varphi_1 - f^2)^2 + (2\varphi_2^*\varphi_2 - f^2)^2\right] \\ & + |(\partial_\mu - iq_1 e A_\mu)\varphi_1|^2 + |(\partial_\mu - iq_2 e A_\mu)\varphi_2|^2\end{aligned}$$

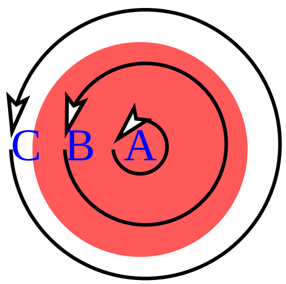
Pick $q_1 \neq q_2$, say, $q_1 = 4$, $q_2 = 3$.

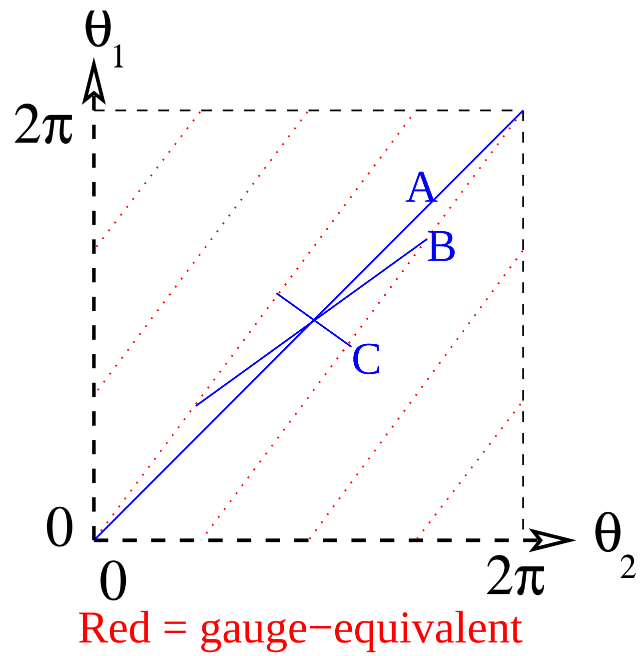
Two rotation symmetries, $\varphi_1 \to e^{i\theta_1}\varphi_1$, $\varphi_2 \to e^{i\theta_2}\varphi_2$

$q_1\theta_1 + q_2\theta_2$ gauged, $q_2\theta_1 - q_1\theta_2$ global (Axion)

Two scalars, one gauge field

String where *each* scalar winds by 2π:





B-field *almost* compensates gradients outside string.

$f_a^2 = f^2/(q_1^2{+}q_2^2)$.

$$T \simeq 2\pi f^2\,, \qquad \frac{dF}{dl} = \frac{f^2}{(q_1^2+q_2^2)r}\,, \qquad \kappa_{\rm eff} = 2(q_1^2+q_2^2)\,.$$

Two scalars, one gauge field

- Strings have Abelian-Higgs core $\rightarrow$ **Tension**
- Outside core: $q_1\theta_2 - q_2\theta_1 =$ **Axions**
- Ratio of tension to f_a tunable:
  can get string tension right!
- Bad news: extra (very heavy) DOF
  - $*$ Can change string interactions, cusps
  - $*$ Can propagate off strings

Is this model "right"?

NO! But neither is lattice gauge theory, or chiral perturbation theory, or ... Need *limiting procedure*

Limit $a \ll 1/m_{\mathrm{heavy}} \ll t_{\mathrm{tilt}}$: right physics. Extrapolate.
Convergence now polynomial, not logarithmic.