Photons and Transport at NLO

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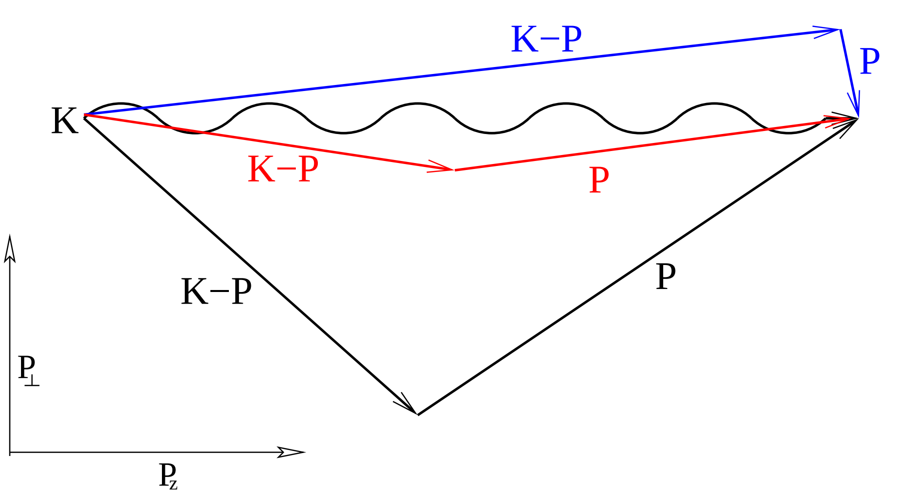
- Photon calculation "guts:" emergence of condensates
- Photon results
- Condensates from the lattice?
- $\hat{q}_{||}$ and transport
- Viscosity and diffusion: the complication

Phase space again

$$\gamma \text{ produc: } \sum_{\psi_f} \overbrace{\langle \psi_i | A^\mu \bar{\psi}\gamma_\mu \psi | \psi_f \rangle}^{\overline{\mathcal{M}}} \overbrace{\langle \psi_f | A^\nu \bar{\psi}\gamma_\nu \psi | \psi_i \rangle}^{\mathcal{M}}$$

In $\mathcal{M}$, $\psi, \bar{\psi}$ momenta $p, k-p$ must add to k of photon:



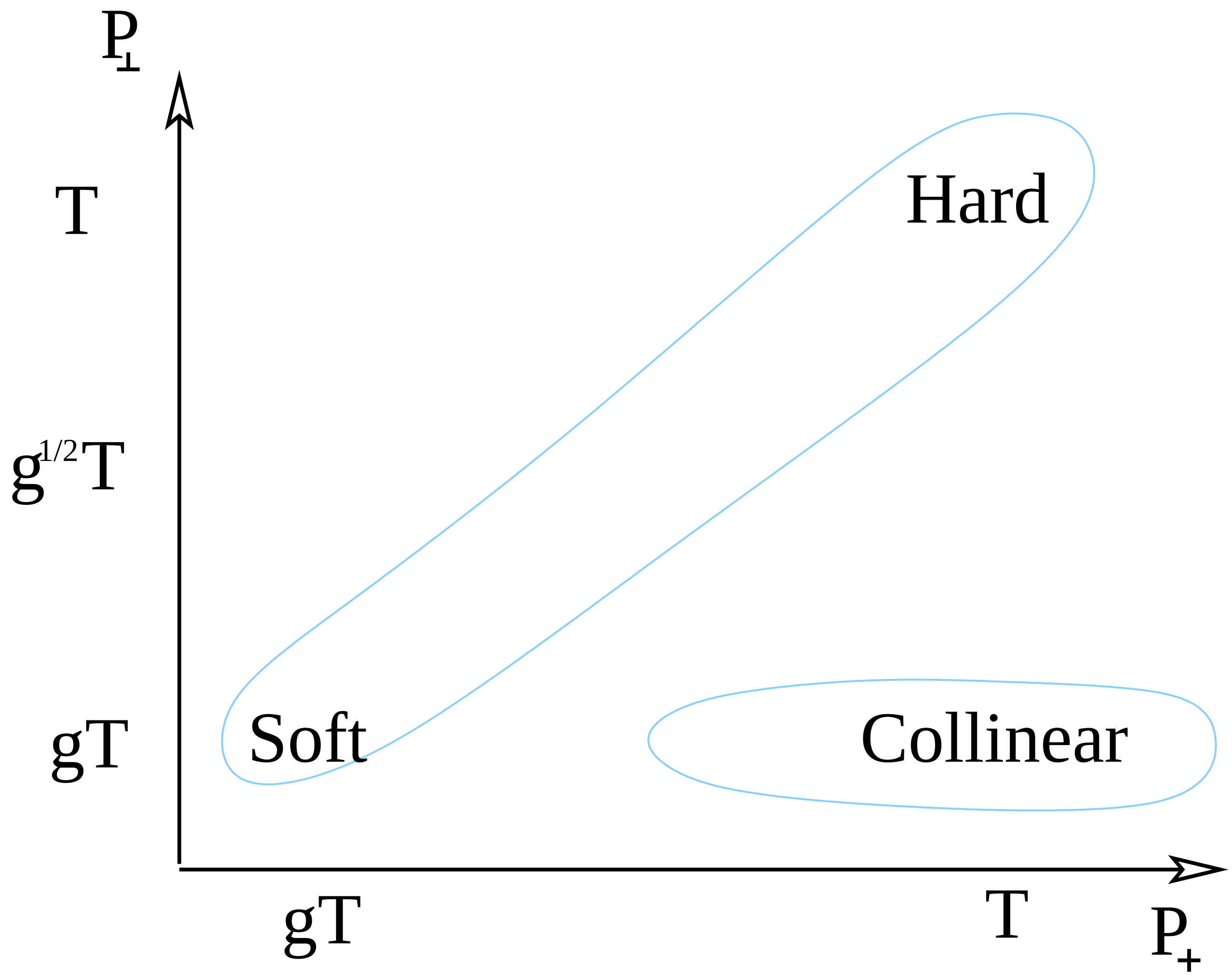
Black: way off-shell, but big phase space

Blue: less phase sp, but soft enhancement

Red: both can be almost on-shell.

Call these regions Hard, Soft, and Collinear.

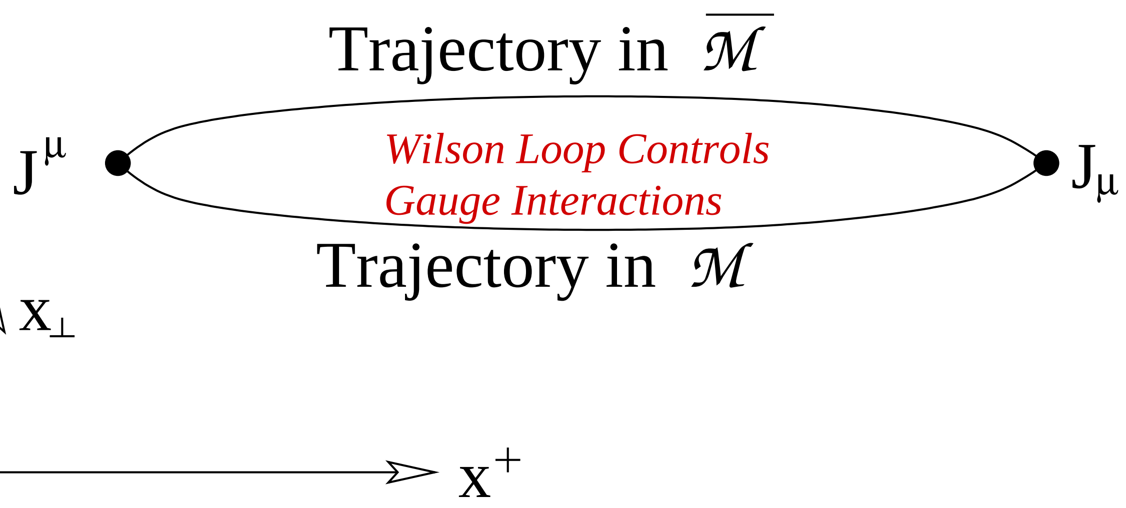
The $P_\perp$, P_+ plane:



Collinear case

Since $P, K - P$ collinear, move in approx. same direction.

J^μ in $\mathcal{M}$ and J_μ in $\overline{\mathcal{M}}$ not at same x-point.
Collinear $\Rightarrow$ almost on-shell $\Rightarrow$ can have large x separation;
$x^- \ll x_\perp \ll x^+$:



Involves *condensate* $\mathcal{C}(x_\perp)$.

Nontrivial analysis AMY hep-ph/0109064,hep-ph/0111107 (see Peter's talk?)

$$\frac{dN_\gamma}{d^3\mathbf{k}d^4x} = \frac{\alpha_{\rm EM}}{\pi^2 k}\int_{-k/2}^{\infty}\frac{dp^+}{2\pi}\frac{n_f(k{+}p)\,[1{-}n_f(p)]}{2[p\,(p{+}k)]^2}\left[p^2+(p{+}k)^2\right]\times$$
$$\times \lim_{\mathbf{x}_\perp\to 0} 2\,\mathrm{Re}\,\partial_{\mathbf{x}_\perp}\mathbf{f}(x_\perp)$$
$$2\nabla_\perp\delta^2(x_\perp) = \left[\mathcal{C}(x_\perp)+\frac{ik}{2p^+(k{+}p^+)}(m_\infty^2+\nabla_{x_\perp}^2)\right]\mathbf{f}(x_\perp)$$

To evaluate this at NLO I need:

- $\mathcal{C}(x_\perp)$ at NLO [Condensates!!]
- small $p^+ \sim gT$ behavior: $\lim_{p^+\ll T}$ [integrand] $\to (p^+)^0$
- higher-order-in-Eikonal corrections

Some condensates are Euclidean!

$\mathcal{C}(x_\perp)$: Wilson loop with space-separated lightlike lines. All points at spacelike or lightlike separation.

Soft contribution is *Euclidean*!! S. Caron-Huot, 0811.1603

Calculate it with $simple$ perturbation theory (EQCD)

Calculate it on the lattice?!

NLO corrections to $\mathcal{C}(x_\perp)$ computed. NNLO would be nonperturbative; but may be possible via lattice.

How Things Get Euclidean S. Caron-Huot

Consider correlator $G^<(x^0, \mathbf{x})$ with $x^z > |x^0|$. Fourier representation

$$G^<(x^0, \mathbf{x}) = \int d\omega \int dp_z d^2p_\perp e^{i(x^z p^z + \mathbf{x}_\perp \cdot \mathbf{p}_\perp - \omega x^0)} G^<(\omega, p_z, p_\perp)$$

Use $G^<(\omega, \mathbf{p}) = n_b(\omega)(G_R(\omega, \mathbf{p}) - G_A(\omega, \mathbf{p}))$ and define $\tilde{p}^z = p^z - (t/x^z)\omega$:

$$G^< = \int d\omega \int d\tilde{p}^z d^2p_\perp e^{i(x^z \tilde{p}^z + \mathbf{x}_\perp \cdot \mathbf{p}_\perp)} n_b(\omega) \left(G_R(\omega, \tilde{p}^z + \omega \frac{x^0}{x^z}, \mathbf{p}_\perp) - G_A \right)$$

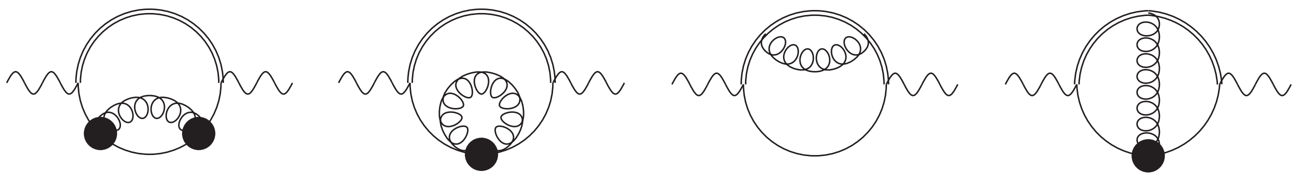
Perform ω integral: upper half-plane for G_R, lower for G_A, pick up poles from n_b:

$$G^<(x^0, \mathbf{x}) = T \sum_{\omega_n = 2\pi n T} \int dp^z d^2p_\perp e^{i\mathbf{p}\cdot\mathbf{x}} G_E(\omega_n, p_z + i\omega_n(x^0/x^z), p_\perp)$$

Large separations: $n \neq 0$ exponentially small. $n = 0$ contrib. is x^0 independent!

Soft momenta

Diagrams:



Cut diagrams: hard momentum is on-shell, $p^- = 0$.

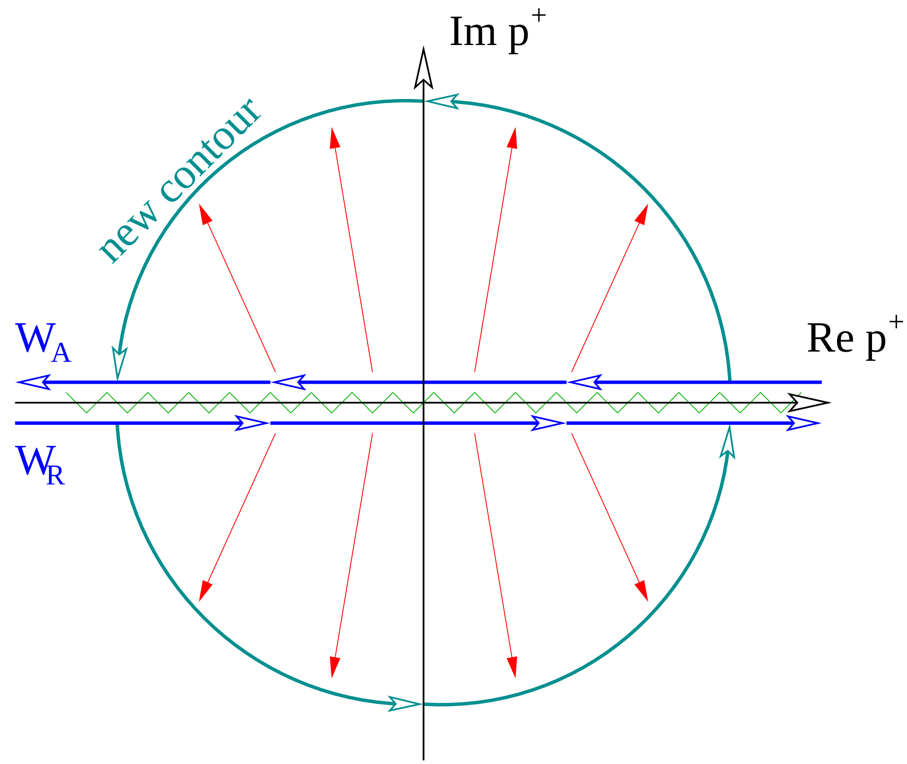
Write out Q, remaining P integrals and use KMS:

$$\int_{\sim gT} d^2p_\perp dp^+ \int_{\sim gT} d^4Q n_b(k^0)(G_{\rm R} - G_{\rm A})$$

$G_{\rm R}$: retarded function of sum of all 4 diagrams' guts.

Momentum p^+ is **null**. Any R/A function is analytic in upper/lower half plane for time-like **or null** p-variable.
Analytically continue in p^+!!

Deform p^+ contour into complex plane



Now $p^+ \gg p_\perp, Q$. (On mass-shell) Expand in $p^+ \gg p_\perp, Q$

$$G_{\rm R}[\text{4 diagrams}] = C_0 (p^+)^0 + C_1 (p^+)^{-1} + \ldots$$

C_0 is on-shell width, gives linear in p^+ divergence.

C_1 is on-shell dispersion correction, dp^+/p^+ gives const.

We can do this continuation because the J^μ correlators are null-separated. It becomes simple because null-separated correlators **are** simple.

- C_0 **term:** arises at NLO. **equals** the small-p^+ limit of the collinear calculation. completes treatment of that region.

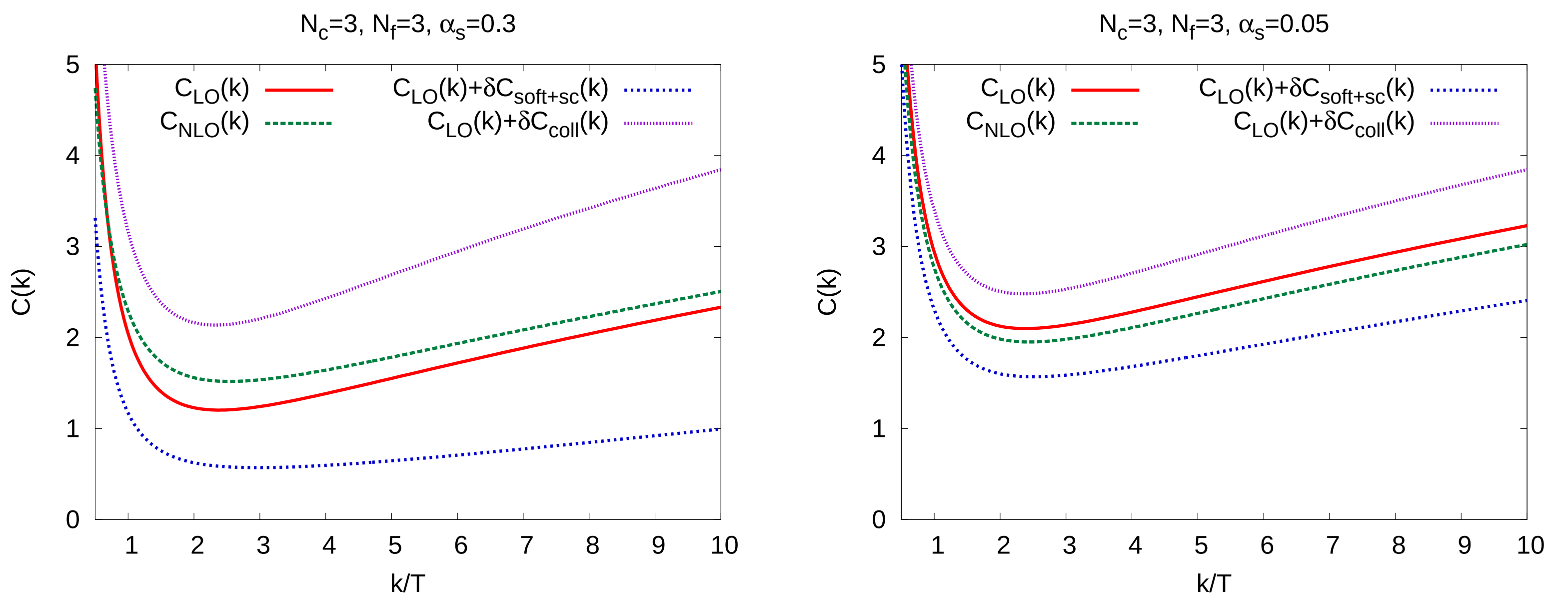
- C_1 **term:** real dispersion-correction. Really simple:

$$\gamma\text{-rate} \propto \int \frac{d^2p_\perp}{(2\pi)^2} \frac{m_\infty^2}{p_\perp^2 + m_\infty^2}$$

  where m_∞^2 is dispersion correction. Has leading-order piece (hard modes) and subleading piece (dispersion correction of soft modes). *both are known*.

Remaining region–similar story. Null-separation physics, all condensates.

Summing it up: two corrections



Upward correction: more scattering at NLO.

Downward correction: fewer soft gluons, less dispersion corr.

Numerical conspiracy: effects nearly cancel [Accidental!]

Main lesson

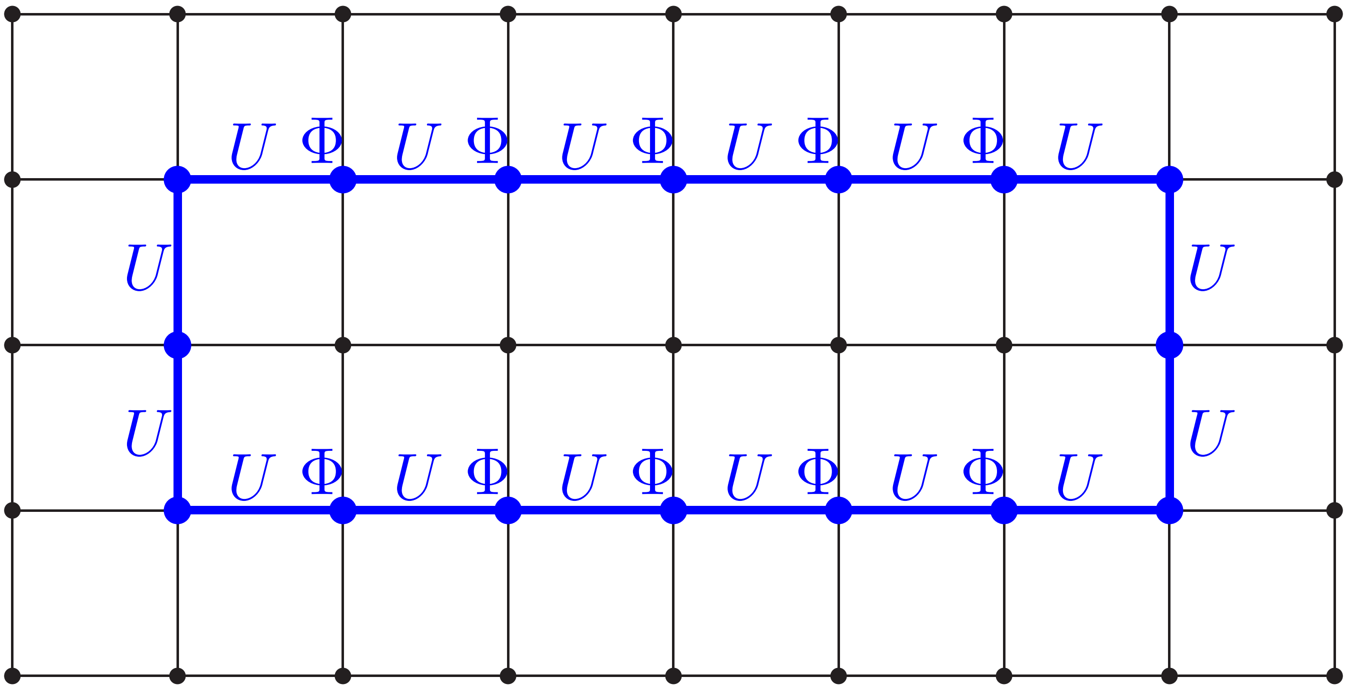
All the sticky IR physics shows up in a few condensates.
Some are dispersion corrections – physically simple.
Some are Euclidean – get directly on the lattice.

Bad news: $\mathcal{O}(g)$ corrections big even for $\alpha_s = 0.1$ or $1000\,T_c$.

Good news: A few condensates. Determine them nonperturbatively, maybe get down to $5\,T_c$?

Get them on the lattice?

$\mathcal{C}(x_\perp)$ on the lattice



Short side: $x_\perp$ Wilson line $\exp \int iA_\perp \cdot x_\perp \Rightarrow U_\perp U_\perp \ldots$

Long side: x^+ Wilson line $\exp \int i(A^z + A^0) dz \Rightarrow U_z e^{a\Phi} U_z e^{a\Phi} U_z \ldots$

The latter is a new beast. Lattice renormalization properties?

Under investigation.

The two $\hat{q}$s

One thing which arises in the calculation is $\hat{q}_\perp$,

$$\hat{q}_\perp \equiv \int \frac{d^2 q_\perp}{(2\pi)^2} q_\perp^2 \, \mathcal{C}(q_\perp) = \lim_{x_\perp \to 0} \nabla^2_{x_\perp} \mathcal{C}(x_\perp)$$

$\perp$-momentum diffusion. Reduces to

$$\hat{q}_\perp = \frac{g^2 C_R}{d_A} \int_{-\infty}^{\infty} dx^+ F^a_{+\perp}(0,0) U_{ab}(0,0;x^+,0) F^b_{+\perp}(x^+,0)$$

a transverse-force-force correlator.

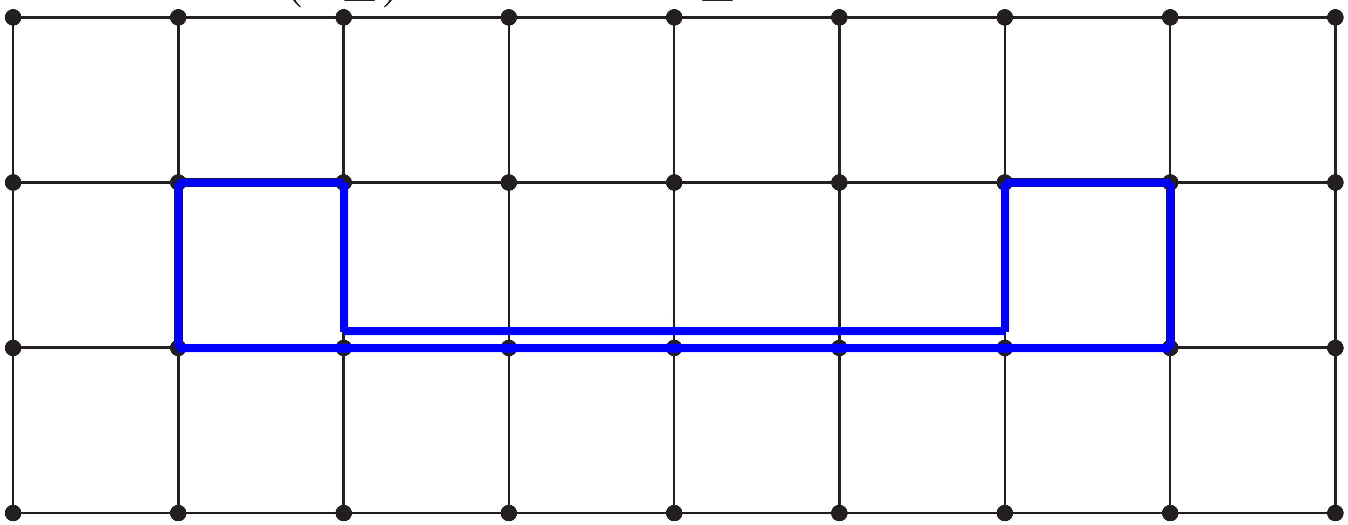
Can also define its cousin (not needed)

$$\hat{q}_\| = \frac{g^2 C_R}{d_A} \int_{-\infty}^{\infty} dx^+ F^a_{+-}(0,0) U_{ab}(0,0;x^+,0) F^b_{+-}(x^+,0)$$

correlator of force $along$ direction of motion.

$\hat{q}_\perp$ on the lattice

$\hat{q}$ is a limit of $\mathcal{C}(x_\perp)$ at small $x_\perp$:



plus Φ-difference contribution. Much more UV sensitive:

- Leading-Order: quadratic divergent cancel if well-designed
- NLO (1-loop): linear divergence, requires matching
- NNLO (2-loop): log divergence, requires matching

And $\hat{q}_{\parallel}$?

Transverse force – "bumps" on Wilson line are to the side.
Longitudinal force – "bump" in x^+, x^- plane.
time direction; *not* all spacelike-separated.

But contour deformation method still works.
Related to hard dispersion-correction of *gluons*

$$\hat{q}_{\parallel} \sim \int \frac{d^2 p_\perp}{(2\pi)^2} \frac{m^2_{\infty,g}}{p^2_\perp + m^2_{\infty,g}}$$

With some matching, useful ingredient in other transport coeff. and in jet medium-modification

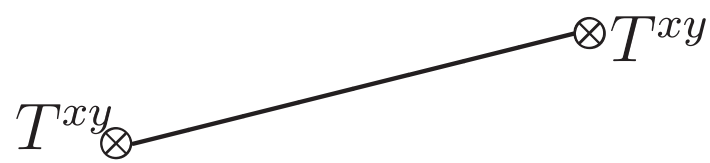
Other transport coefficients?

We want Baryon Diffusion D and (especially) shear η!

Both controlled by high-energy $E =$ several T particles

Lightlike correlators should again dominate:

T^{xy} disturbs a particle

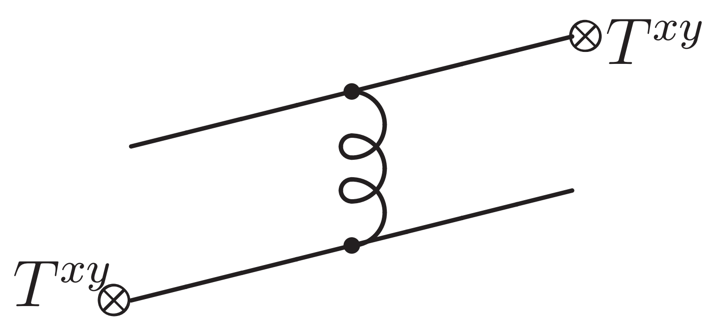


It induces a T^{xy}

NLO effects arise along particle's lightlike trajectory.

Problem: transfer of stress to someone else

Lines are timelike separated



There are $\mathcal{O}(g)$ corrections

Conclusions

- NLO corrections to transport are *large but simple*
- Need a few *condensates* at lightlike-separated points
- Most can be extracted from the lattice
- Shear and diffusion will be harder. Stay tuned