Equilibration in Nonabelian Gauge Theories

Guy D. Moore, with Aleksi Kurkela, arXiv: 1107.5050

- How to get far from equilibrium in nonabelian gauge theory
- Case 1: isotropic, high occupancy

  Elastic and Inelastic Scattering
- Case 2: isotropic, low occupancy

  showering, LPM effect
- Case 3: anisotropic

  Plasma instabilities, anisotropic daughters, wheels within wheels

Motivation

There are many cases where you meet gauge theories far from equilibrium:

- Cosmology: reheating or preheating (decay products, parametric resonance....)
- Cosmology: phase transitions, *eg.*, electroweak
- Heavy ion collisions (only weakly coupled in ultra-high energy limit)

Cases: homogeneous vs. inhomogeneous, isotropic vs. aniso, high vs. low initial occupancies

I will only consider:

- weak coupling $\alpha \ll 1$, mostly glue (doesn't matter...)
- parametric estimates: $\sim$ not $=$ 55pp, 183eq, 15fig, 619 $\sim$'s!
- homogeneous systems Not as bad as it sounds

Cases:

	High occ.	Low occ.
isotropic weak-aniso strong-aniso		

I will only consider:

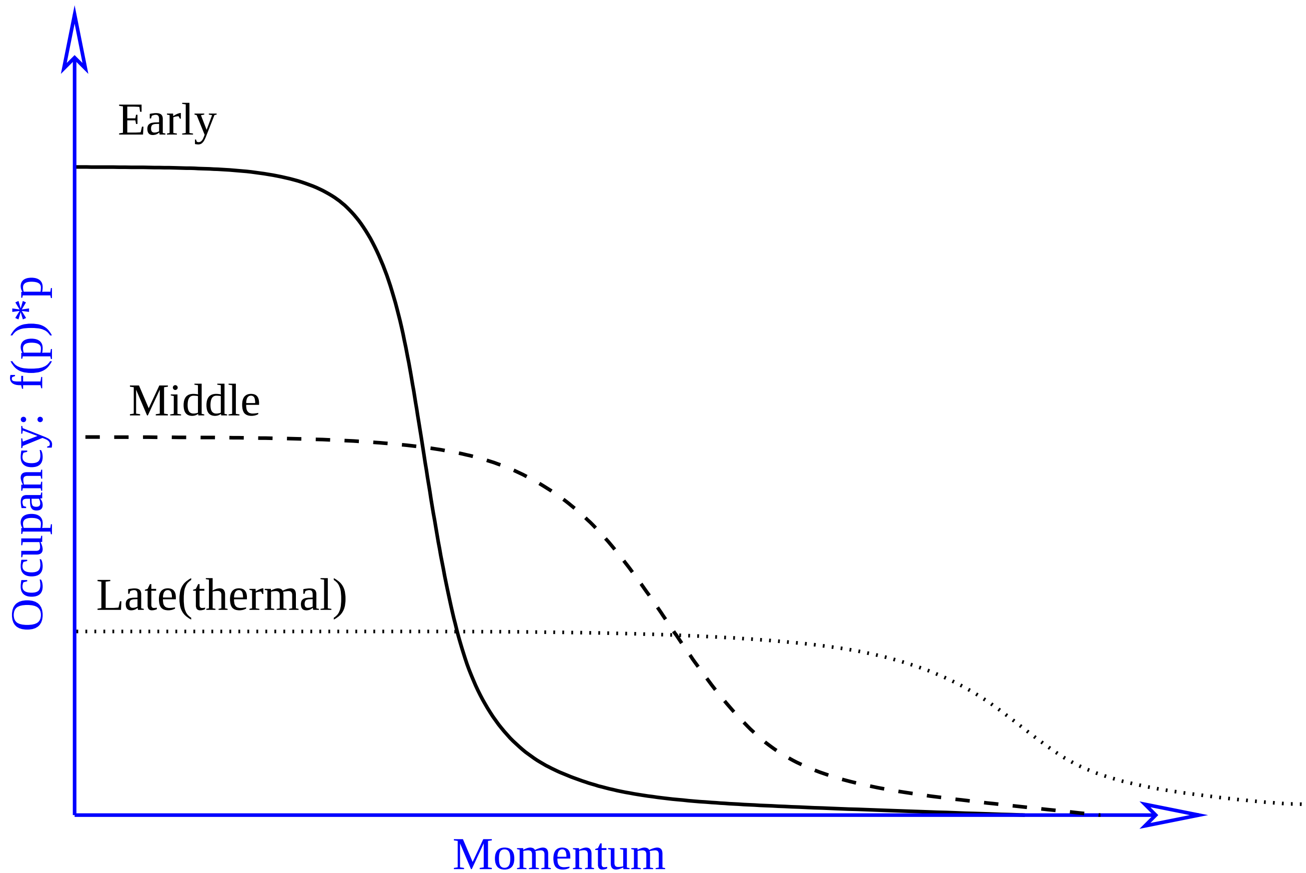
- weak coupling $\alpha \ll 1$, mostly glue (doesn't matter...)
- parametric estimates: $\sim$ not $=$ 55pp, 183eq, 15fig, 619 $\sim$'s!
- homogeneous systems Not as bad as it sounds

Existing Literature:

	High occ.	Low occ.
isotropic	Wrong	well-treated
weak-aniso	—	—
strong-aniso	—	—

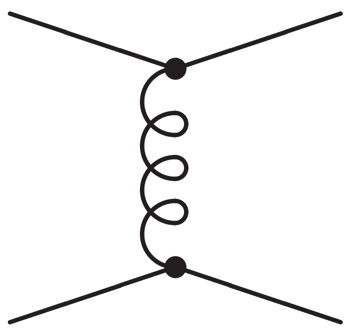
Isotropic, High Occupancy

Initially momentum $p \sim Q$, occupancy $f \sim \alpha^{-c}$:



How do I get to equilibrium $T \sim Q\alpha^{-c/4}$?

Elastic scattering



$$\int_{p,k,p',k'} |\mathcal{M}|^2 \Big(\; f(p)f(k)[1+f(p')][1+f(k')] \\ -f(p')f(k')[1+f(p)][1+f(k)] \Big)$$

Naively: $d\Gamma/d^4x \sim \alpha^2 Q^4 f^4$. Really f^3 (gain$-$loss)

Rate per PARTICLE: $\Gamma \sim \alpha^2 Q f^2 \sim \alpha^{2-2c} Q$

Also not true: Coulombic divergence!

$$\int |\mathcal{M}|^2 \propto \int d^2 q_\perp \left(\frac{1}{q_\perp^2 + m^2} \right)^2 \sim \frac{Q^2}{m^2}$$

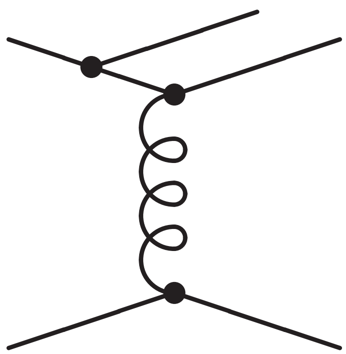
m^2 “screening scale” is a medium effect.

Screening and Splitting

Screening: $m^2 \sim \alpha \int \frac{d^3p}{E_p} f(p) \sim \alpha^{1-c} Q^2$ Note: IR role!

All scatt: $\Gamma_{\rm soft} \sim \alpha^{1-c} Q$. Hard scatt: $\Gamma_{\rm hard} \sim \alpha^{2-2c} Q$.

Soft scattering can cause radiation/absorption!



$$\Gamma \sim \alpha[1+f(r)]\Gamma_{\rm soft} \sim \alpha^{2-2c} Q$$

Radiation as common as hard scatt: part.# changes easily

QCD: No chemical potential, no Bose condensation Blaizot Gelis

Jiao McLerran Venugopalan 1107.5296 miss this point

Approach to Equilibrium

Small-momentum equilibrates faster, $\Gamma(p) \propto p^{-2}$. IR in quasi-equilibrium, $f(p) = T_*/p$ (IR tail of Bose distribution). True if $\Gamma(p)t \gg 1$. Cut-off at $p_{\rm max}$ where $\Gamma(p_{\rm max})t \sim 1$. Above $p_{\rm max}$, occupancy falls off fast.

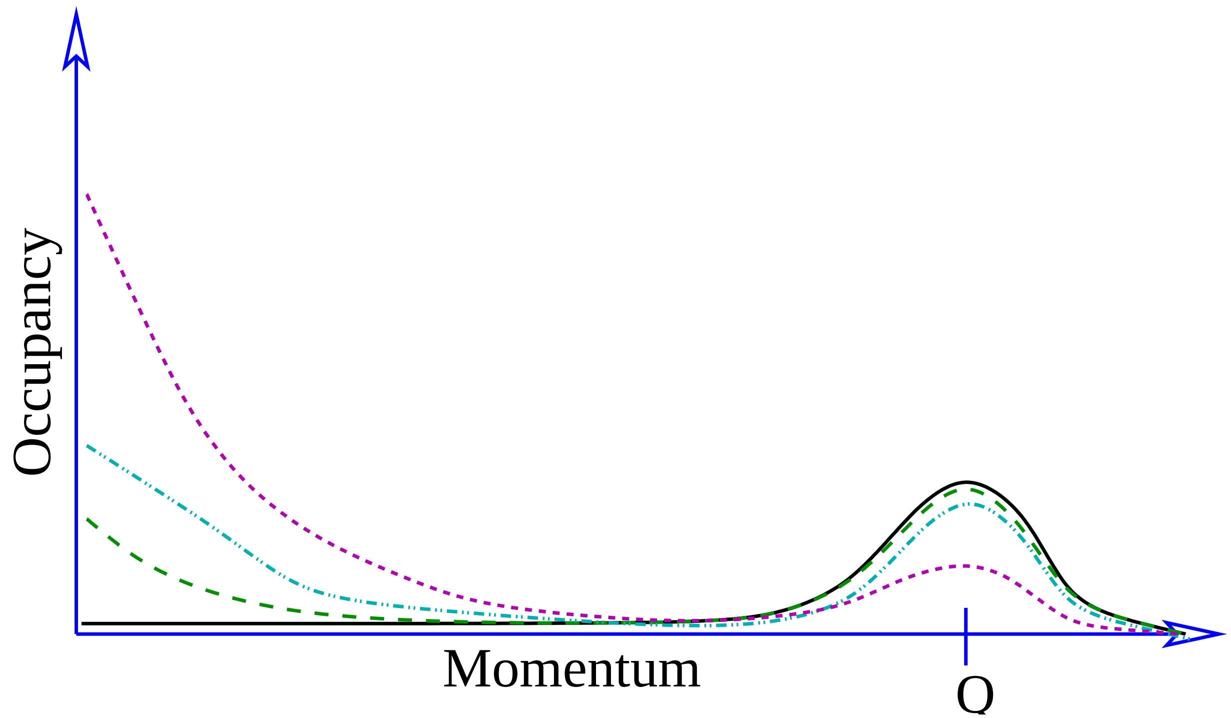
Energy density $\varepsilon \sim \alpha^{-c} Q^4 \sim p_{\rm max}^3 T_*$ is conserved. $\Gamma \sim \alpha^2 T_*^2 / p_{\rm max}$. Solve self-consistently:

$$p_{\rm max} \sim \alpha^{\frac{2-2c}{7}} Q^{\frac{8}{7}} t^{\frac{1}{7}} , \quad f \sim \alpha^{\frac{-8+c}{7}} (Qt)^{\frac{-4}{7}}$$

$$t_{\rm eq} \sim \alpha^{-2+\frac{c}{4}} Q^{-1} \sim \alpha^{-2} T_{\rm final} \quad \text{(of course!)}$$

Small initial occupancy

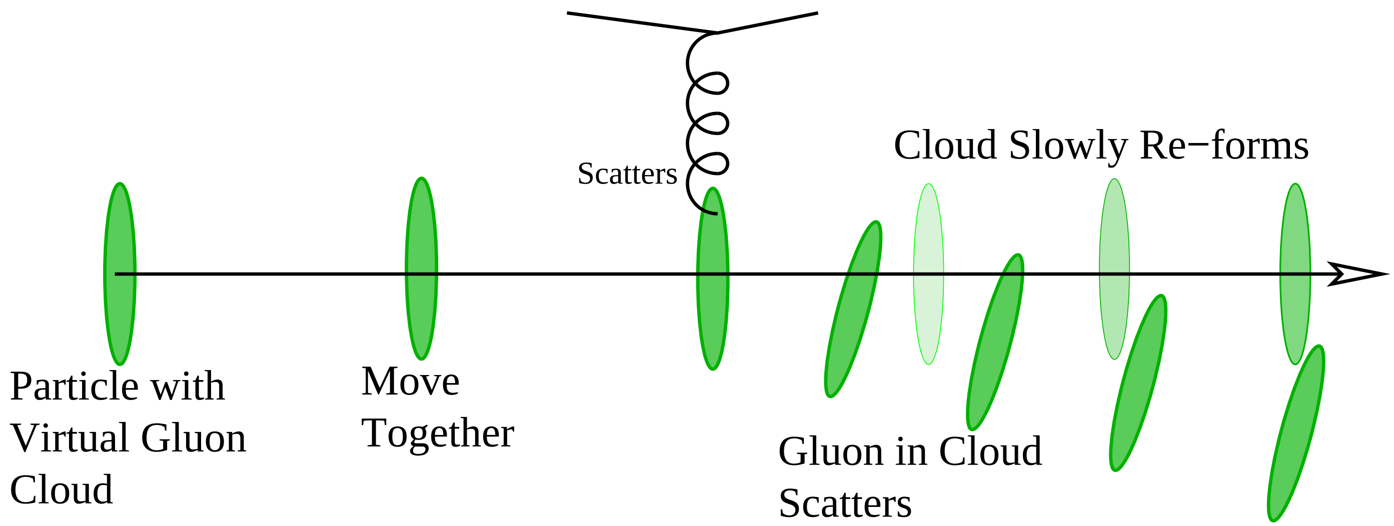
Just the same thing backwards??? NO!



Build soft particle distribution

which “eats” hard excitations via radiation cascade

Radiation, LPM effect 1

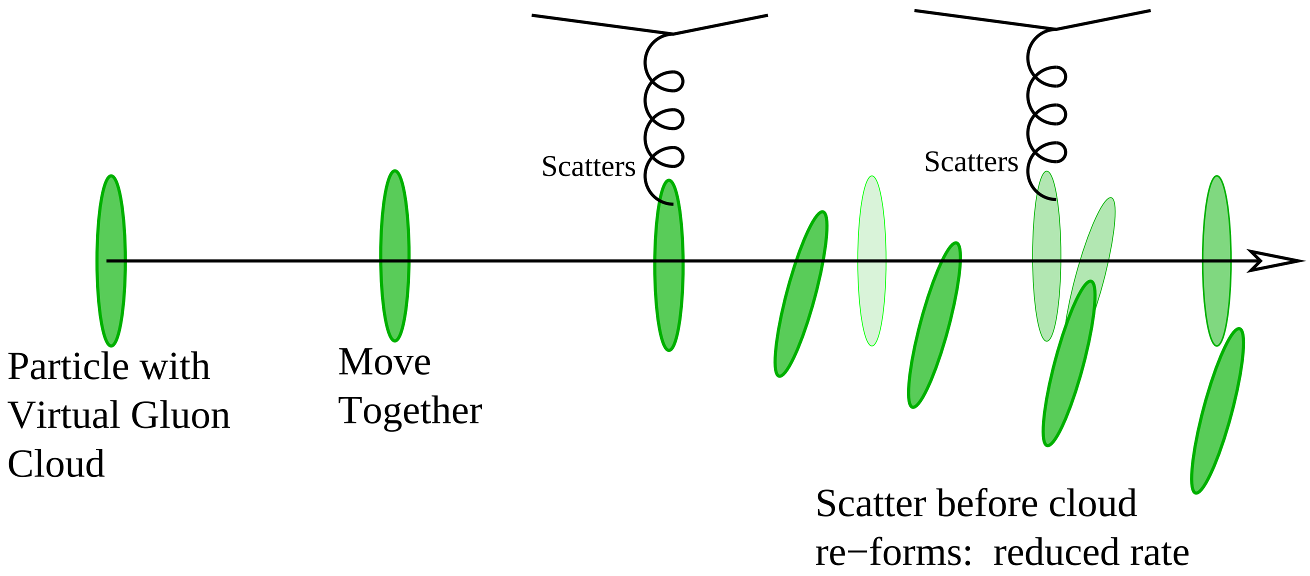


Scattering knocks gluon from virtual cloud.

Cloud re-forms once knocked-out gluon physically separates

$$t_{\rm form} \sim \frac{\textsf{trans. size}}{\theta} \sim \frac{1/p_\perp}{p_\perp/p} \sim \frac{p}{p_\perp^2}$$

LPM effect 2



Frequent or soft scatterings: cloud hasn't re-formed!

At most one emission "chance" per $t_{\rm form}$. "LPM effect"

$$p_\perp^2 \equiv \hat{q} t_{\rm form} \quad \Rightarrow \quad \Gamma_{\rm emit}(p) \sim \alpha t_{\rm form}^{-1} \sim \frac{\alpha\sqrt{\hat{q}}}{\sqrt{p}}$$

QED: Landau Pomeranchuk Migdal '53,'54. QCD: Baier Dokshitzer Mueller Peigné Schiff hep-ph/9607355

LPM and You

Phase space, kinematics *etc.*: $\Gamma(p) \propto 1/p$.

Number produced $\sim dp/p$ log-distributed in p.

With LPM: $\sim dp/p^{\frac{3}{2}}$ IR dominated. Radiated energy $\sim dp/\sqrt{p}$ hard dom.

More soft than hard daughters produced. And they also:

- screen with efficiency $\propto 1/E$. m^2 strongly IR dom.
- cause scattering with efficiency $\propto [1+f]$. Also IR dom.

Once enough daughters accumulate, they dominate physics!

long before they dominate energy density!

Sequence of events

- Bath of soft excitations form by radiation
- Bath numerous enough to dominate screen+scattering
- Bath thermalizes with itself
- Bath induces radiative breakup of hard excitations

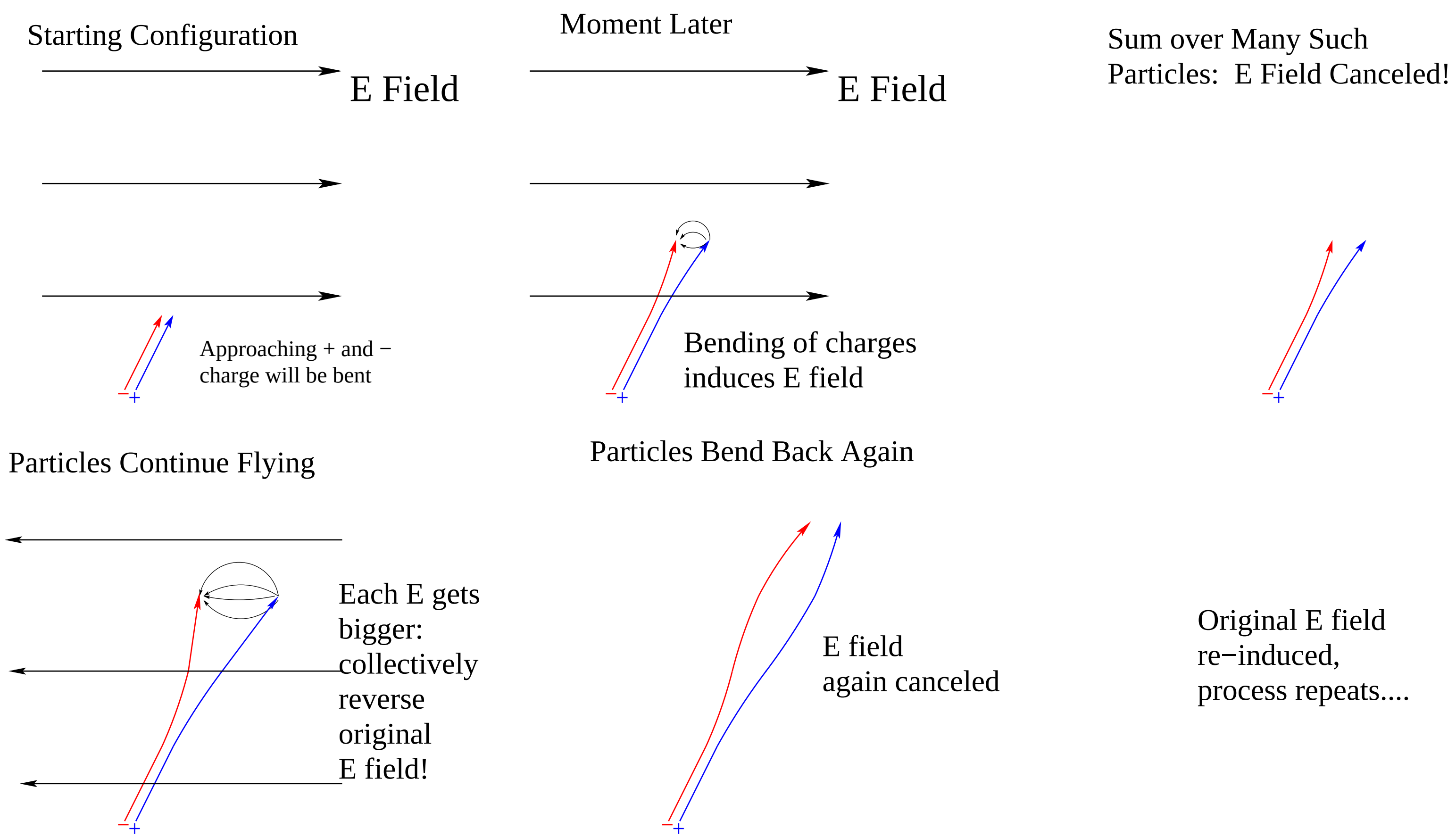
Equilibration finishes in time for $p \sim Q$ excitation to lose energy in bath with $T \sim \varepsilon^{\frac{1}{4}}$ (that is, $T \sim \alpha^{-c/4} Q$)

This is $t \sim \alpha^{-2} T^{-1} \sqrt{(Q/T)} \sim \alpha^{-2-3c/8} Q$

Baier Mueller Schiff Son hep-ph/0009237 basically got this right.

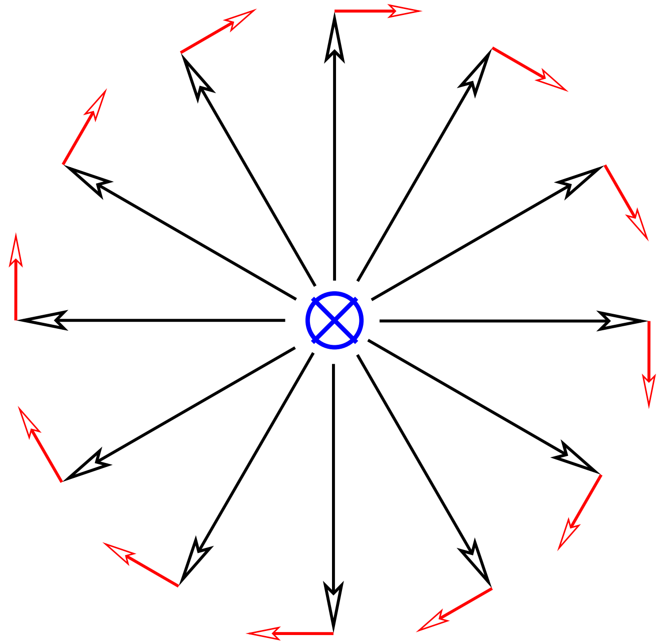
Anisotropic systems

Discuss screening in more depth. Electric fields:



E-field: plasma oscillations are like adding m^2. "stabilizing" (make fields oscillate)

B-field quite different:
for B out-of-board,
isotropic dist. just gets
rotated – not changed.

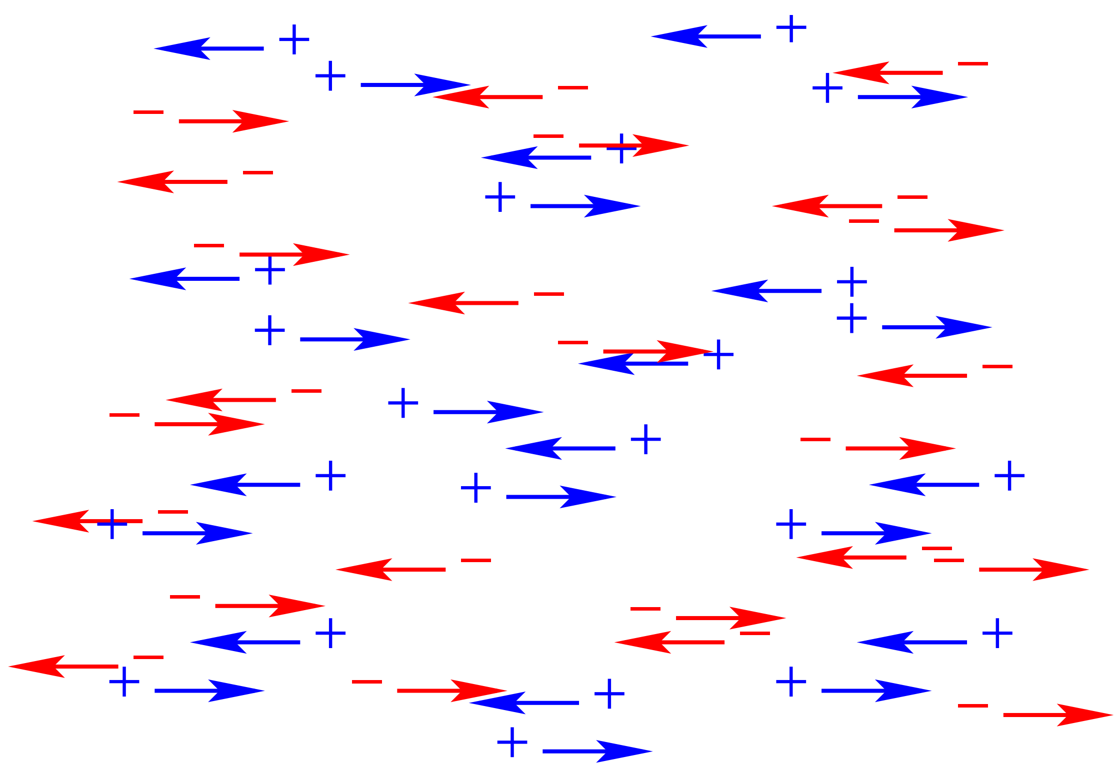


No current induced – no effect on B. $m^2 = 0$ for B!

Isotropic: Electric, but not magnetic, fields screened.

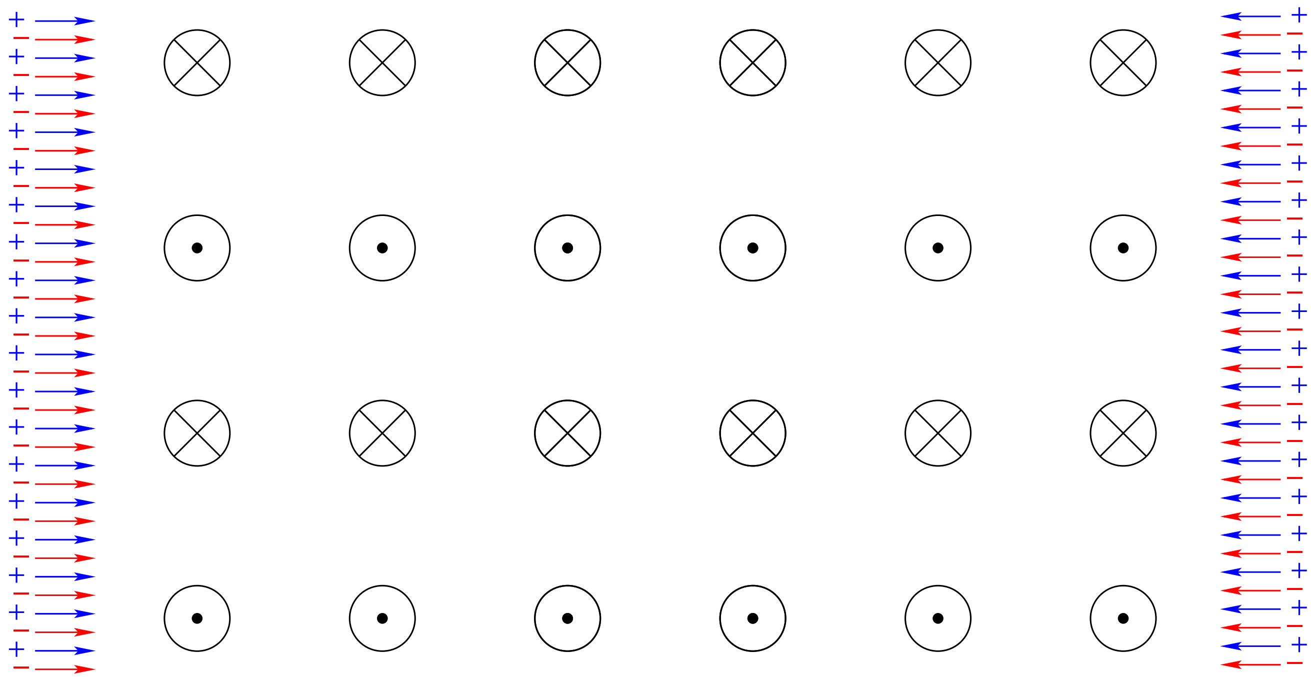
Anisotropic medium: Instabilities!

Consider maximum anisotropy: all particles move only in z direction:



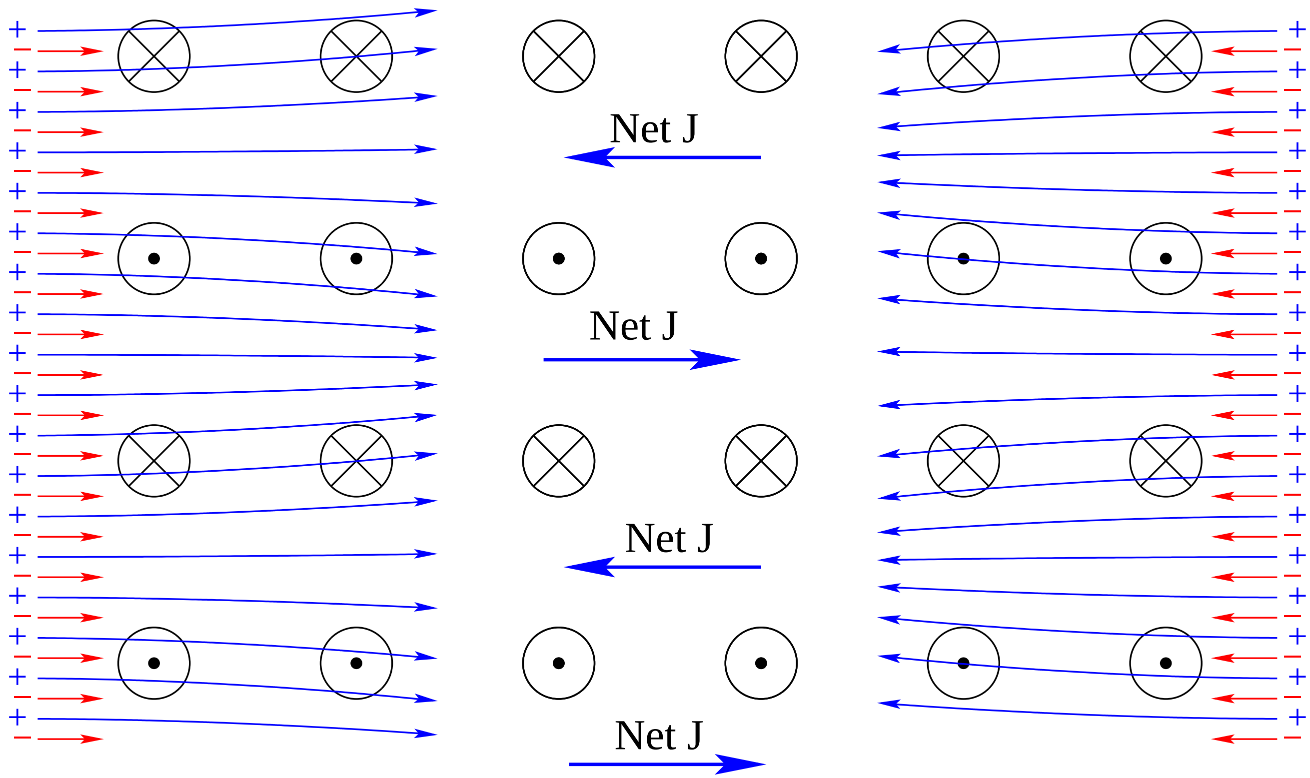
Magnetic field growth!

Consider the effects of a seed magnetic field $\hat{B} \cdot \hat{p} = 0$ and $\hat{k} \cdot \hat{p} = 0$



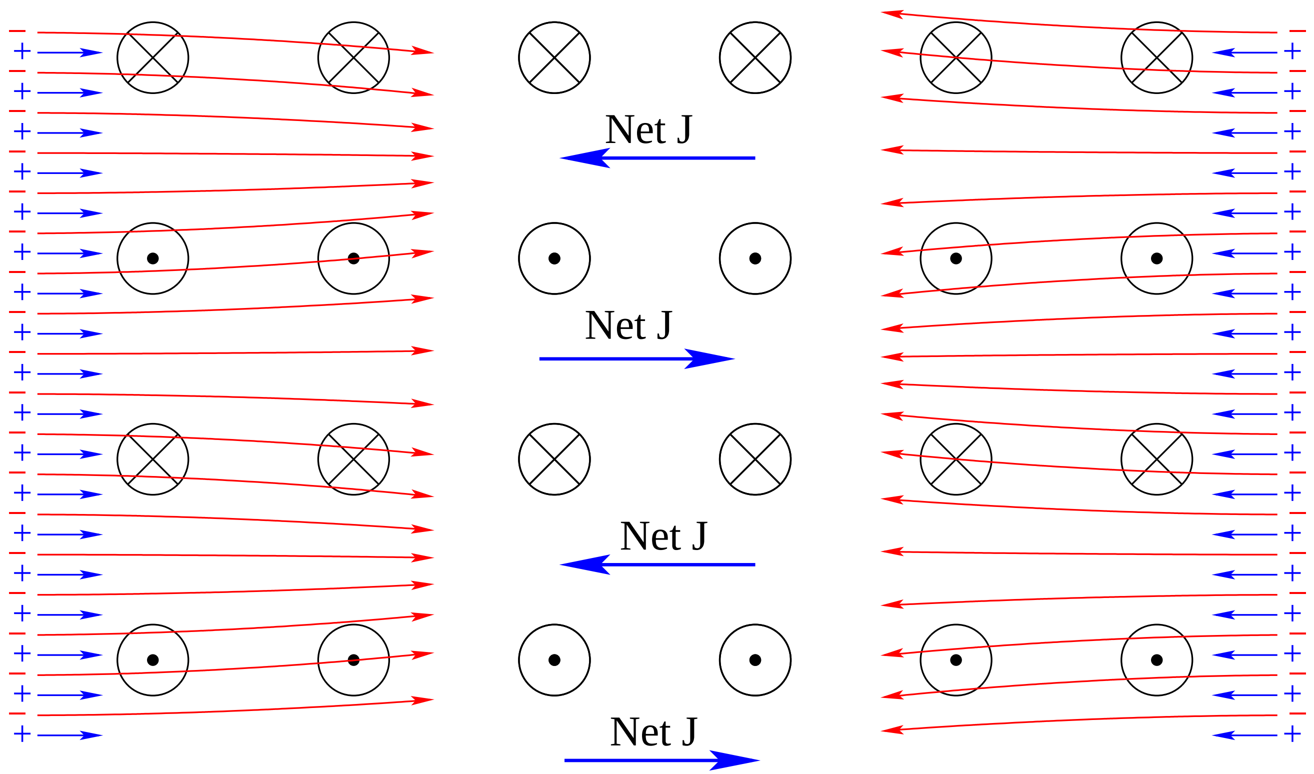
How do the particles deflect?

Positive charges:



No net ρ. Net current is induced as indicated.

Negative charges: same-sign current contribution



Induced B *adds* to seed B. Exponential Weibel instability.

Guesstimate of growth rate:

Force on particle $\mathbf{F} \sim g\mathbf{B}$. Velocity change $\mathbf{v} \sim \mathbf{F}t/p$

Deflection: $\Delta x \sim \mathbf{v}t \sim \mathbf{F}t^2/p \sim gBt^2/p$

Concentration: $k\Delta x$. Current per particle: $g(k\Delta x)$.

Current: $J \sim \int d^3p f(p)\, g(k\Delta x) \sim \int \frac{d^3p}{p} f(p) g^2\, k\mathbf{B}t^2$

That is, $J \sim m^2 t^2 k\mathbf{B}$

Current matters when $J \sim \nabla \times B \sim kB$, which is $m^2 t^2 \sim 1$.

Growth rate must be $\Gamma \sim m$.

Growth occurs *iff* particles stay in same-sign B for $t \gtrsim 1/m$.
(Otherwise J never builds up.)

Weak anisotropy

Define angular distribution $\Omega(\mathbf{v})$:

$$\Omega(\mathbf{v}) \equiv \int \frac{d^3\mathbf{p}}{E} f(\mathbf{p}) \delta(\hat{\mathbf{p}} - \mathbf{v})$$

Weak anisotropy means $\Omega(\mathbf{v}) = \Omega + \epsilon Y_{20} + \dots$

Large isotropic part plus small anisotropic extra.

Only aniso. bit causes instability. $m_{\mathrm{eff}}^2 \sim \epsilon m^2$.

Particles must be in same-sign B for $t \sim 1/m\sqrt{\epsilon}$.

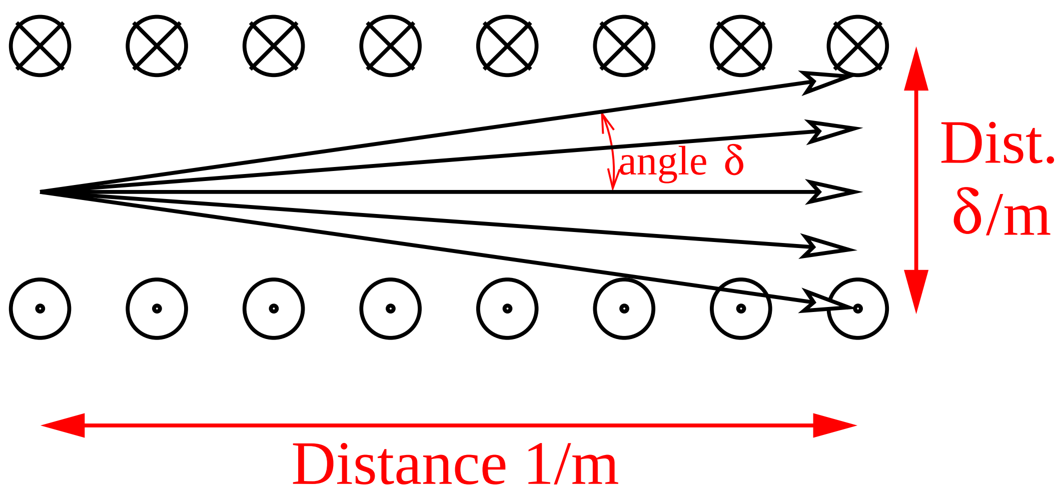
Hence unstable $\mathbf{k}$ have $|\mathbf{k}| \sim \sqrt{\epsilon} m$.

Growth rate $\Gamma \sim \sqrt{\epsilon} m$. Actually $\epsilon^{\frac{3}{2}} m$ due to E-screening.

Strong anisotropy

What happens when $\Omega(\mathbf{v})$ peaked in narrow angle range?

$\Omega(\mathbf{v})$ small unless $|v_z| < \delta \ll 1$?

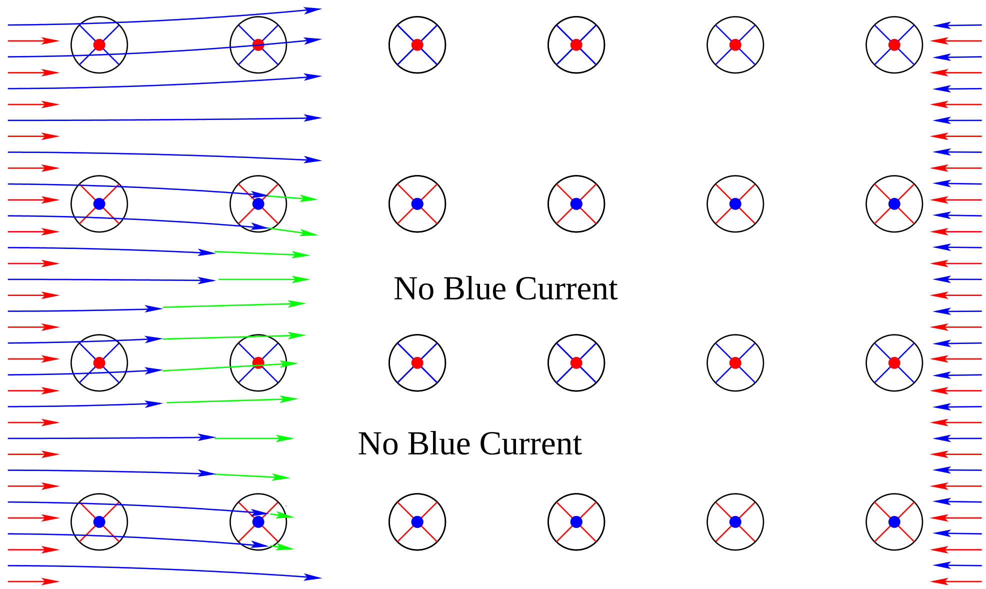


Narrow z-spacing of B's (large k_z) still allowed!

Instability for $\mathbf{k} \sim (m, m, m/\delta)$, growth $\Gamma \sim m$.

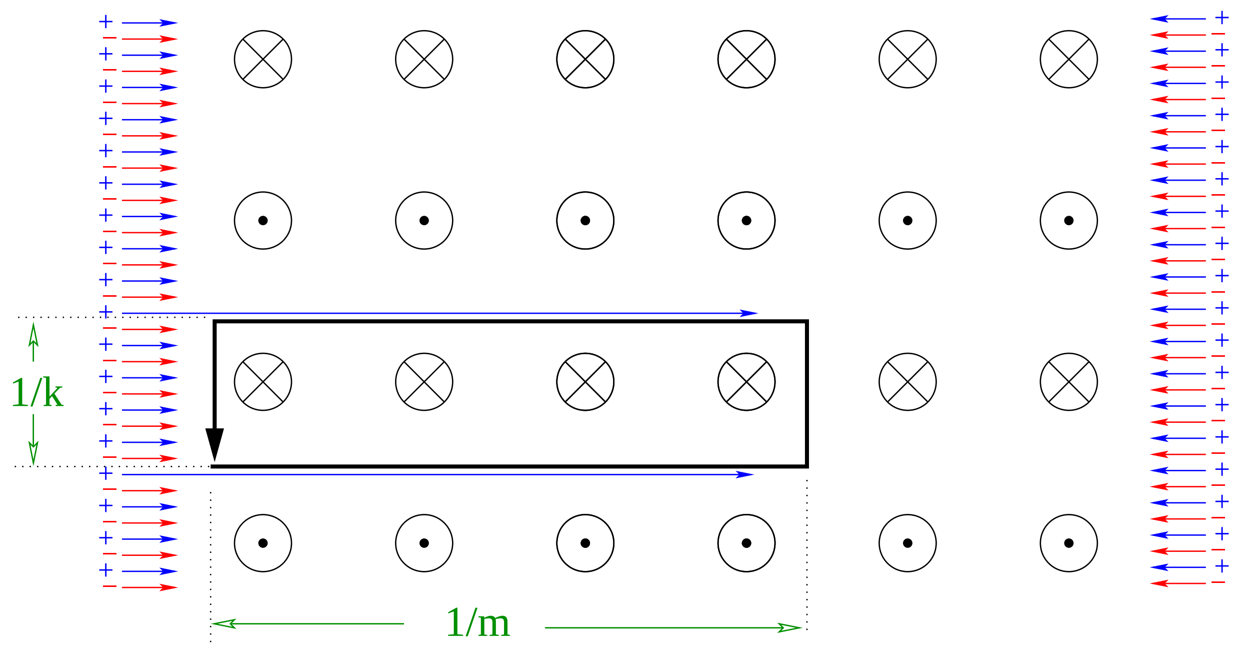
What limits B field growth?

Color randomization!



B growing in all colors, many k at once. Large B: Wilson lines so $\neq \mathbf{1}$ that color rotation happens. Growth cut-off if color-coherence shorter than $1/m$ [$1/(m\sqrt{\epsilon})$ weak-aniso]

Proper gauge invariant version

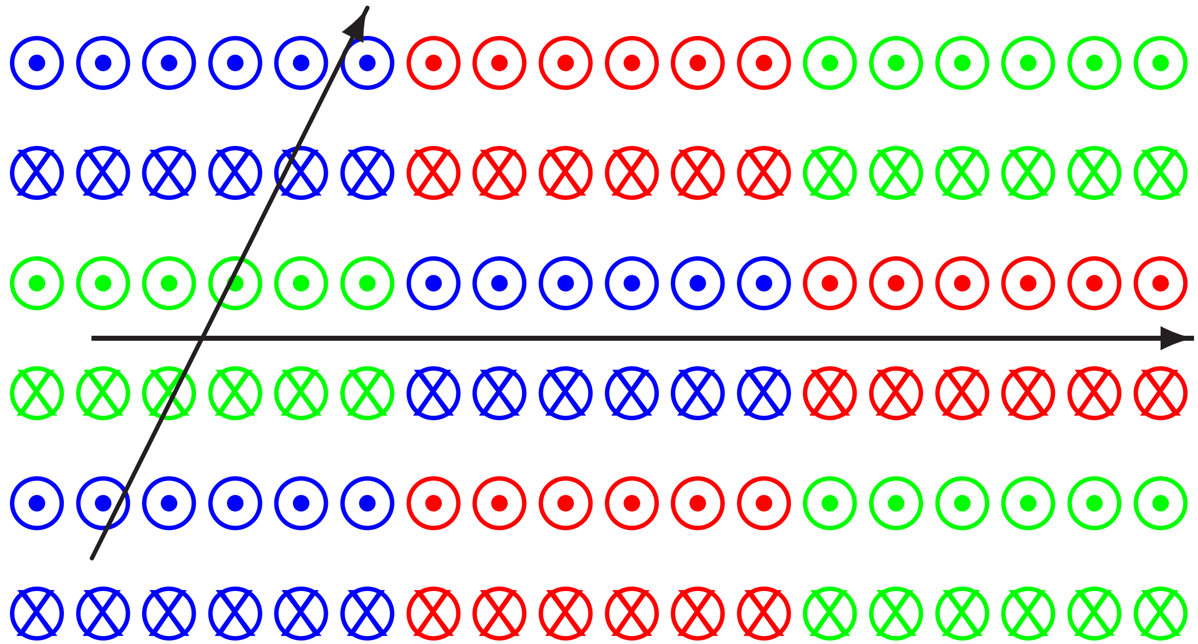


Wilson loop must contain $\mathcal{O}(1)$ phase. Requires $B \sim km/g$.

Weak aniso: $B \sim \epsilon m^2/g$. Strong aniso: $B \sim m^2/\delta$.

Determining $\hat{q}$

How much to particles get kicked around by B?



$\hat{q} = (\Delta p)^2/t \sim (gB\Delta t)^2/\Delta t \sim g^2 B^2 \Delta t$.

Weak aniso: $\hat{q} \sim (m\sqrt{\epsilon})^3 \sim \epsilon^{\frac{3}{2}} m^3$.

Strong aniso: $\hat{q} \sim (m/\delta)^2 m \sim m^3 \delta^{-2}$ OR $\sim (m/\delta)^2 (\delta m) \sim m^3 \delta^{-1}$.

What Instabilities Do

Plasma instabilities bend particle momenta, randomizing p distribution. Also induce (LPM suppr.) radiation.

Possibilities:

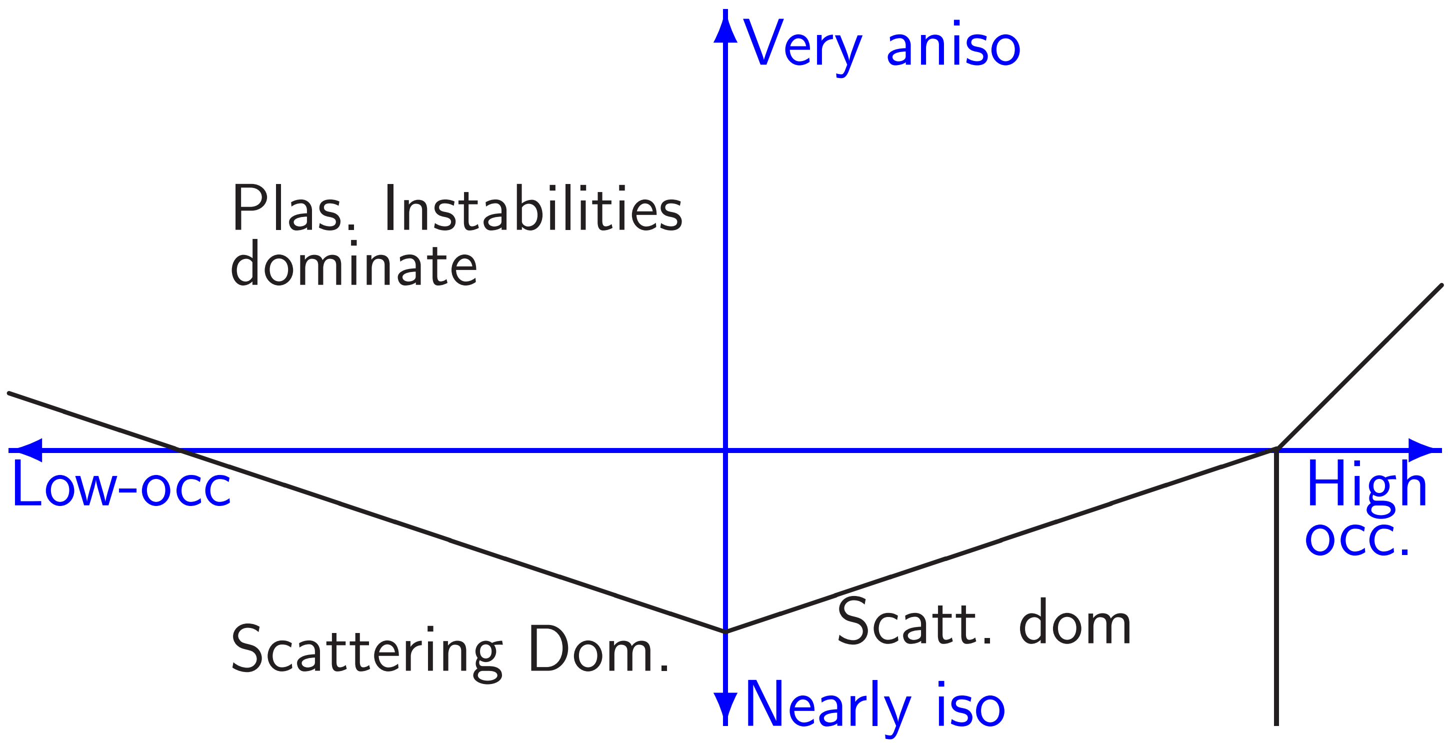
- Direction change dominated by ordinary scatt.
- Plasma instabilities primarily randomize directions
- Plasma instabilities induce merging (inv. radiation)
- Plasma instabilities cause radiation of daughters, which form a bath, go on to dominate physics

Occupancy-Anisotropy Plane

Consider system with one characteristic p-scale Q

Call anisotropy δ or ϵ with $\delta \equiv \alpha^{d}$ or $\epsilon \equiv \alpha^{-d}$

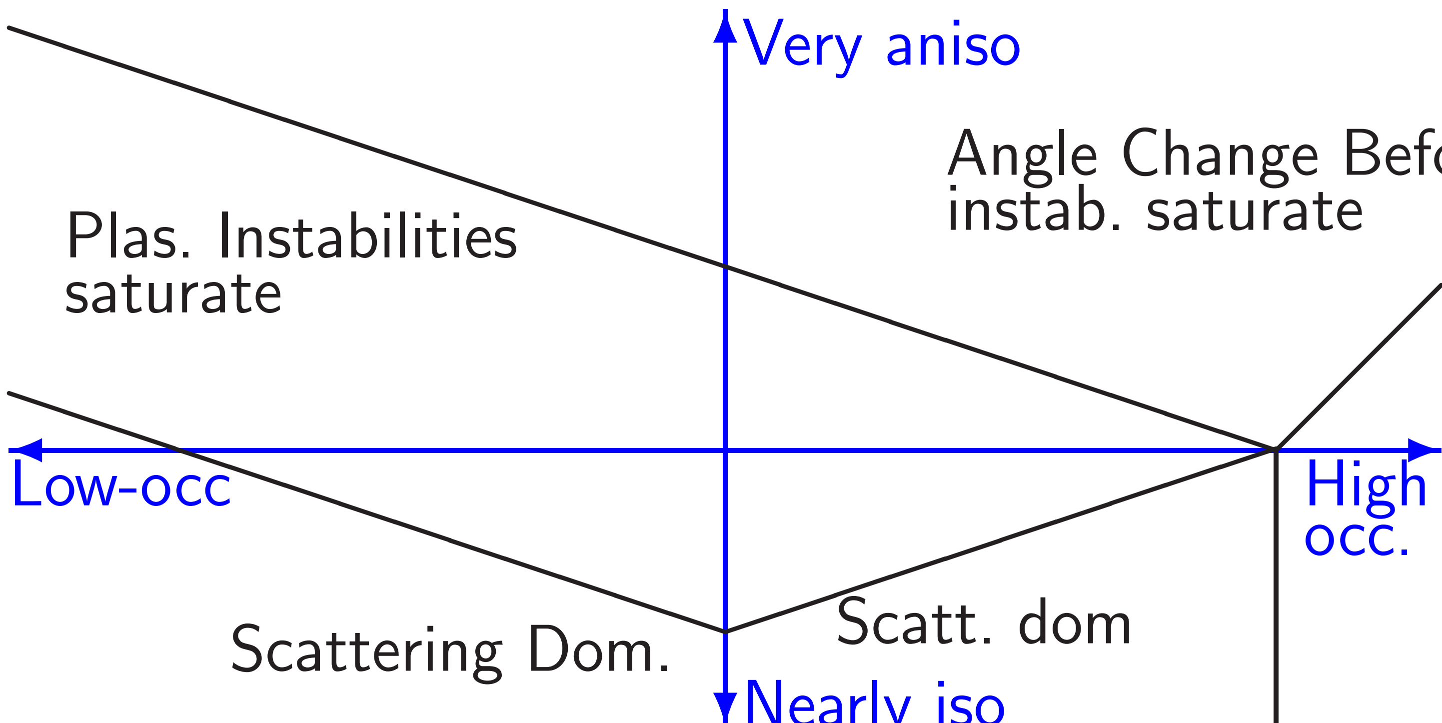
Call typical occupancy $f(p \sim Q, p_z = 0) \sim \alpha^{-c}$.



Angle change

Angle change matters when $\Delta\theta > \delta$ $(d > 0)$ $or > 1$ $(d < 0)$

Can happen before or after plas. inst. finish growing:



Radiated Daughters

Plas. instabilities raise rate of soft radiation.

Radiated daughters are born anisotropic.

Can have their own plasma instabilities! (¡Ay Caramba!)

Driven to isotropy by plas. instabilities, scattering, their own plas. instabilities.

Become important when they dominate scattering – typically by having their own plasma instabilities.

Merging is anisotropic and can also be important!

My complication had a complication

Physics sensitive to occupancy and anisotropy:

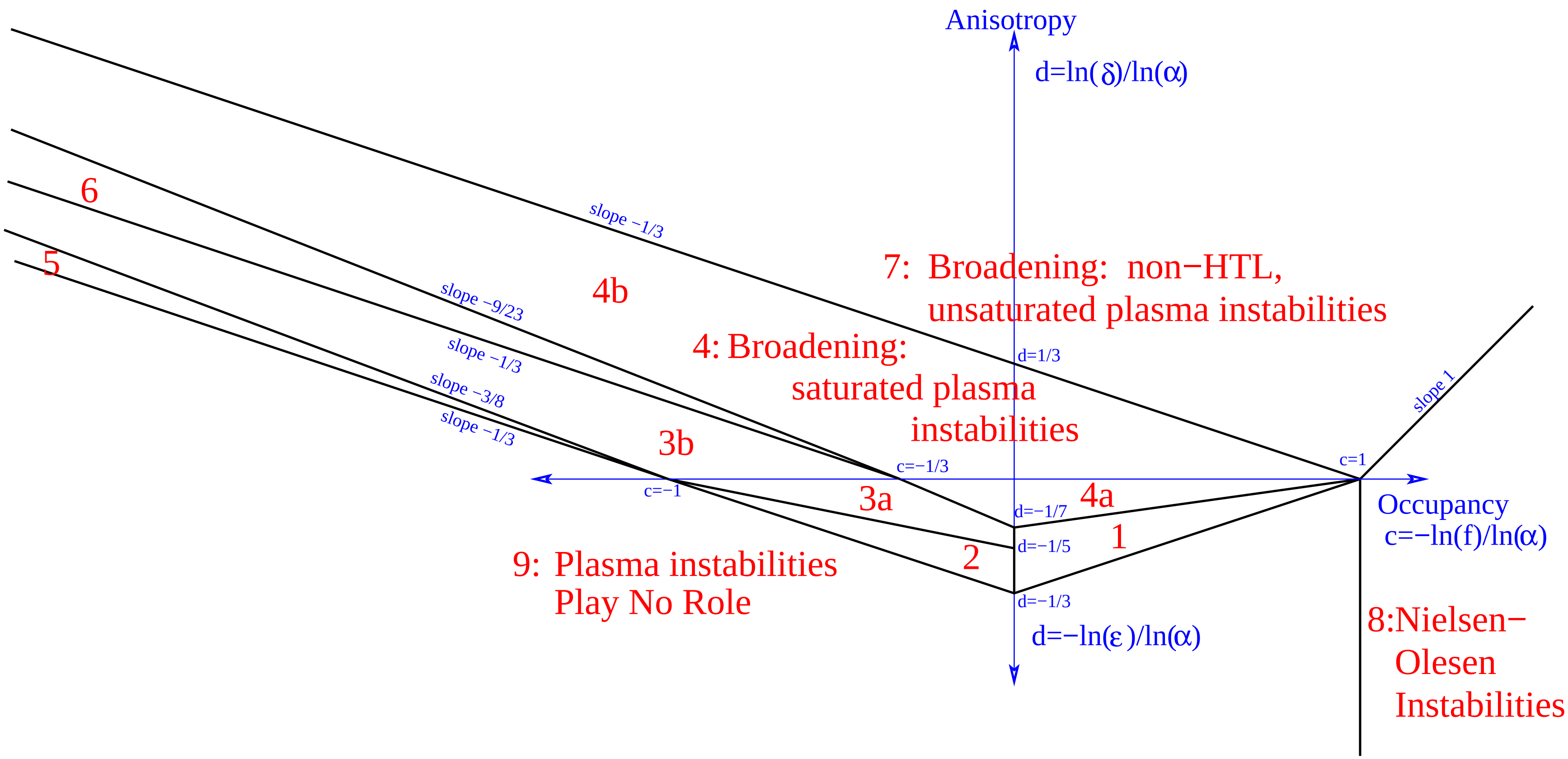
- More anisotropic: plasma instabilities more effective.
- Low occupancy: easier for daughters to become important.

When considering daughters, 3 scales which evolve with time:

- Scale $k_{\rm re-join}$ where daughters so numerous that re-merging onto hard modes occurs. Scales with time as $t^{2/5}$
- Scale $k_{\rm iso}$ where daughters' directions randomized: scales as $t^{1/2}$
- Scale $k_{\rm split}$ where daughters split again into lower-momentum "grand-daughters". Scales as t^1.

30 pages, 349 $\sim$'s later....

Anisotropic Case: Summary



1: merging dominated. 2,3: noneq. daughters. 5: daughters before instabilities saturate. 6: almost-thermal daughters. 10(not shown): thermal daughters

How fast *can* things thermalize?

Minimum time estimate:

- Assume $\frac{1}{2}$ energy has gone into a nearly-thermal bath
- Ask how long it takes starting-particles, $p \sim Q$, to lose energy and join this bath. (LPM suppressed radiation?)

High occupancy: always find $t_{\rm eq} \sim \alpha^{-2} T^{-1}$.

Low occupancy, isotropic: $\alpha^{-2} T^{-1} \sqrt{Q/T}$.

Anisotropic: instab. hasten splitting, $t_{\rm eq} \sim \alpha^{\frac{-13}{7}} Q^{\frac{5}{7}} T_{\rm final}^{\frac{-12}{7}}$.

Conclusions and Questions

- Isotropic: physics of elastic and inelastic scattering
- Low occupancy: bath of daughters becomes dominant
- Anisotropic: plasma instabilities drive dynamics.
  Daughters cause own instabilities Wheels within Wheels

To do:

- Apply to some interesting problems One completed! arXiv:1108.4684
- What estimates can I make quantitative?
- Is "min. equilibration time" estimate always right?