Medium and Medium-Jet Photons In Perturbation Theory

with Arnold, Yaffe, Gale, Jeon, Turbide, Kovtun, Caron-Huot, Starinets, Aurenche, Gelis ....

hep-ph/0109064, hep-ph/0111107, hep-ph/0211036, hep-ph/0502248, hep-th/0607237, in-progress

- General Story: Current-Current Correlations
- Why Perturbation Theory turns out to be hard
- Why perturbation theory may have big corrections
- Crude upper bound on thermal photon production

Steps of a Heavy Ion Collision

Ions collide, produce q, g and photons "primary"

q, g re-scatter as QGP making photons "QGP thermal, jet-thermal"

q, g hadronize, hadrons re-scatter making photons "Hadron gas thermal"

Hadrons escape, some decay making photons "decay"

Photon re-interaction rare ($\alpha_{\rm EM} \ll 1$): direct info.

Thermal photons *might* tell us about thermalization, QGP properties.

How Photons Get Made

Since $\alpha_{\text{EM}} \ll 1$, work to lowest order:

- assume photon production *Poissonian*
- neglect back-reaction on system cooling insignificant...

Compute single-photon production at $\mathcal{O}(\alpha_{\text{EM}})$

$$2k^0(2\pi)^3 \frac{d\text{Prob}}{d^3k} = \sum_X \text{Tr}\, \rho\, U^\dagger(t)|X,\gamma\rangle\langle X,\gamma|U(t)$$

$U(t)$ time evolution operator, ρ density matrix.

Expand $U(t)$ in EM interaction picture:

$$U(t) = 1 - i \int^t dt' \int d^3x e A^\mu(x,t') J_\mu(x,t') + \mathcal{O}(e^2)$$

A^μ produces the photon. Get assume 4-translation invariance!

$$\frac{d\mathrm{Prob}}{d^3k} = \frac{Vte^2}{(2\pi)^3 2k^0} \int d^4Y e^{-iK\cdot Y} \sum_X \mathrm{Tr}\rho J^\mu(Y)|X\rangle\langle X|J_\mu(0)$$

Vt is spacetime volume – natural to talk about rate

$$\frac{d\Gamma}{d^3k} = \frac{e^2}{(2\pi)^3 2k^0} \int d^4Y e^{-iK\cdot Y} \mathrm{Tr}\rho J^\mu(Y) J_\mu(0)$$

No assumption (yet) about perturbativity.

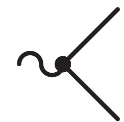
Calculational Approaches

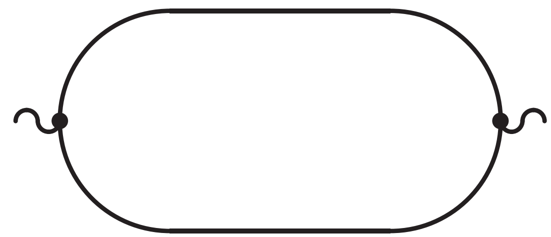
IF ONLY I could compute $\langle J^\mu J_\mu \rangle(K)$ at $\alpha_s = 0.3$ ...
Instead we have

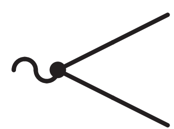
1. Weak-coupling techniques (uncontrolled extrapolation from $\alpha_s < 0.1$)
2. Lattice techniques (uncontrolled analytic continution)
3. Strong-coupling $\mathcal{N}=4$ SYM (Uncontrolled relation to QCD)

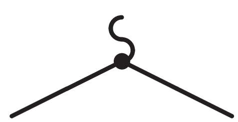
I will discuss 1. and 3.

Perturbative Analysis

$$J^\mu = \sum_{q=uds} e_q \bar{q} \gamma^\mu q :$$ 

Leading diagram: $\langle JJ \rangle =$ 

Timelike K: pair production  kinematically fine

Spacelike K: DIS  also kinematically OK

Lightlike K: on-shell quarks kinematically disallowed!

Photon production starts at 2-loops.

But "wants" to be 1-loop with slightly off-shell quarks

Two leading-order phase-space regions:

- 2-loop with off-shell or soft quark "$2 \leftrightarrow 2$"

  Baier Nakagawa Niegawa Redlich 1992, Kapusta Lichard Seibert 1991

- collinear with self-energies, ladders "Bremsstrahlung"

  Aurenche Gelis Kobes Petitgirard Zaraket 1996-2000; AMY 2001

Off-shell case straightforward except IR piece.

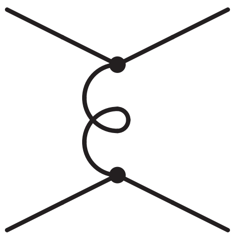
Important but I will skip it.

Let's look at collinear almost on-shell case

Bremsstrahlung

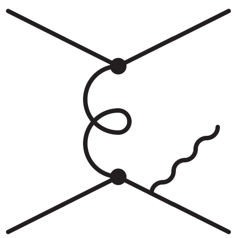
Two pieces of well-known physics:

- Scattering rate is IR (Coulomb) divergent



$$\sim \alpha_s^2 \int \frac{s d^2 q_\perp}{(q_\perp^2 + m^2)^2} \sim \alpha_s \quad \text{as } m^2 \sim \alpha_s T^2$$

- All scatterings incl. soft have $\mathcal{O}(\alpha_{\rm EM})$ chance of ISR/FSR



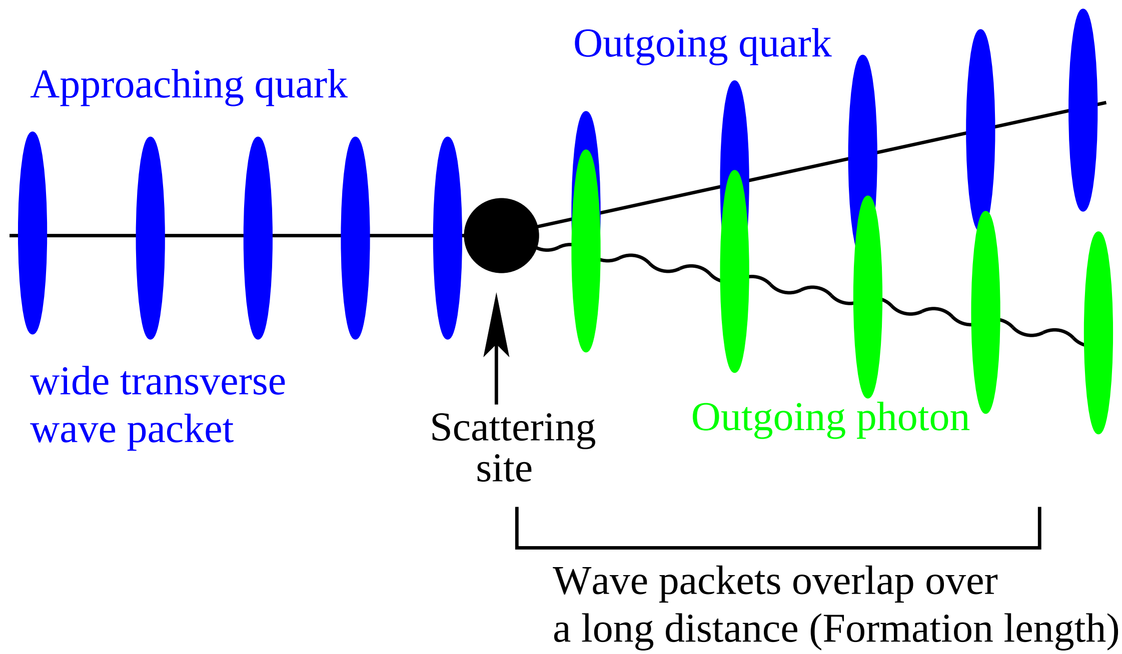
Hence rate of brem is $\mathcal{O}(\alpha_s \alpha_{\rm EM})$ like $2 \leftrightarrow 2$. Aurenche *et al*

Why ISR/FSR is Efficient

Photon emerges at a small $\mathcal{O}(g)$ angle, $p_\perp \sim gT$

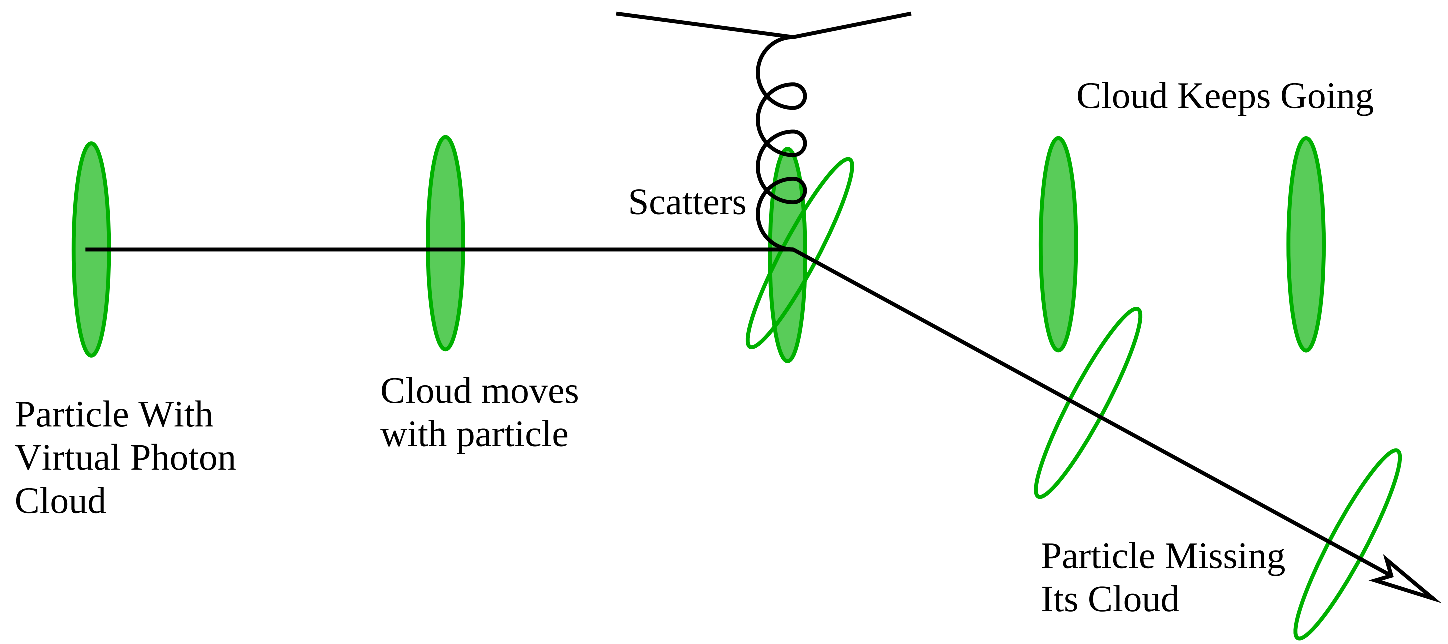
Transverse extent large $\Delta x_\perp \sim p_\perp^{-1} \sim 1/(gT)$

Time to separate from quark is long, $t \sim \Delta x_\perp/\theta \sim 1/(g^2 T)$



Emission *coherent* over $1/g^2T$ timescale.

Another way to think about Bremsstrahlung:



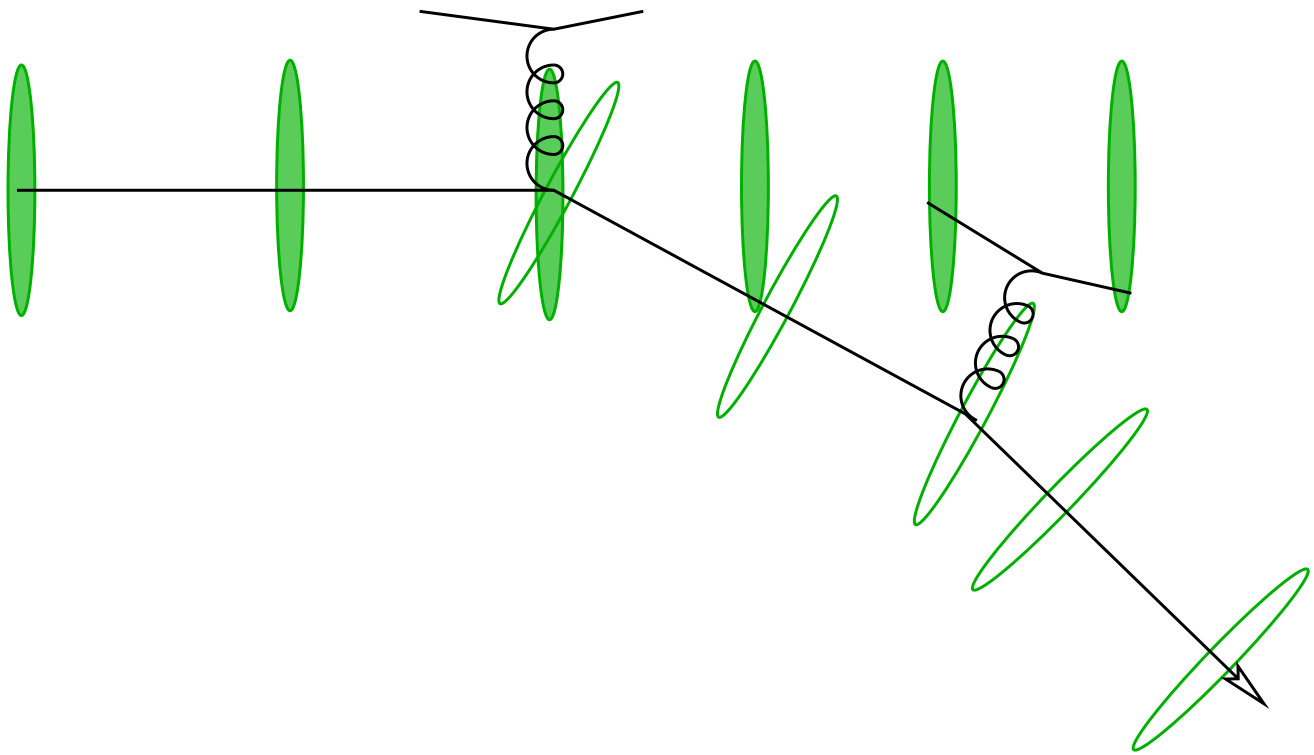
Cloud is $\mathcal{O}(\alpha)$ photons/$d\ln(k)d\ln(\theta)$.

Cloud without quark: physical photons (ISR)

Quark without cloud: physical photons (FSR)

The LPM Effect

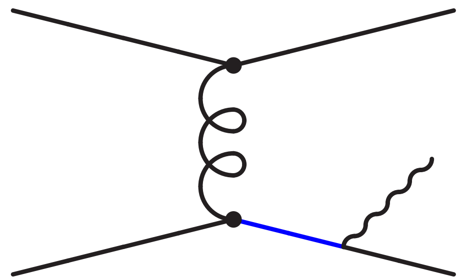
Long-timescale coherence is sensitive to re-scattering



Scattering before cloud "re-forms" does NOT make more radiation. Radiation rate limited by cloud re-formation.

Diagrammatic description

When computing square of diagram

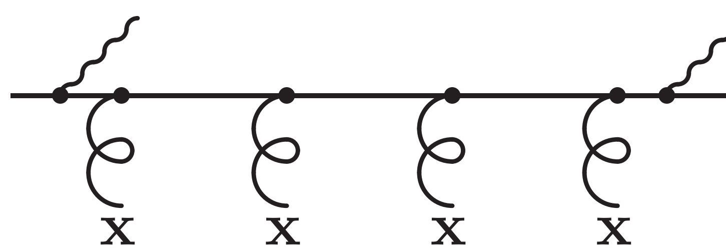


propagator in blue is $\mathcal{O}(g^2T)$ off-shell.

Time separation between vertices $\sim 1/(g^2T)$,

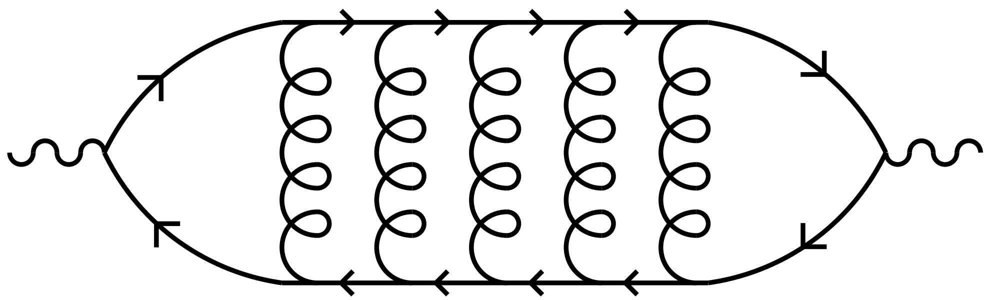
which is $\sim$ inter-scattering spacing.

Different scattering events interfere, γ emission can be separated by several scatterings:

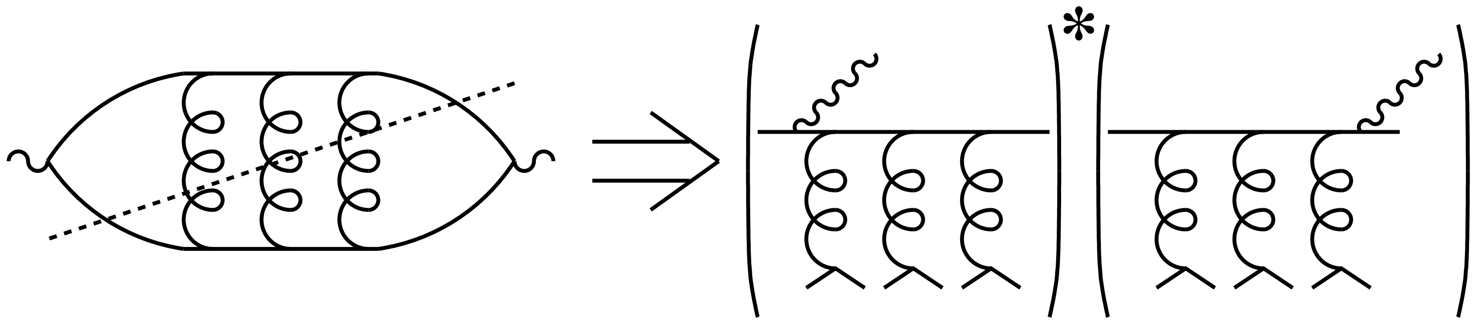


In terms of $\langle JJ \rangle$ correlator diagram,

corresponds to

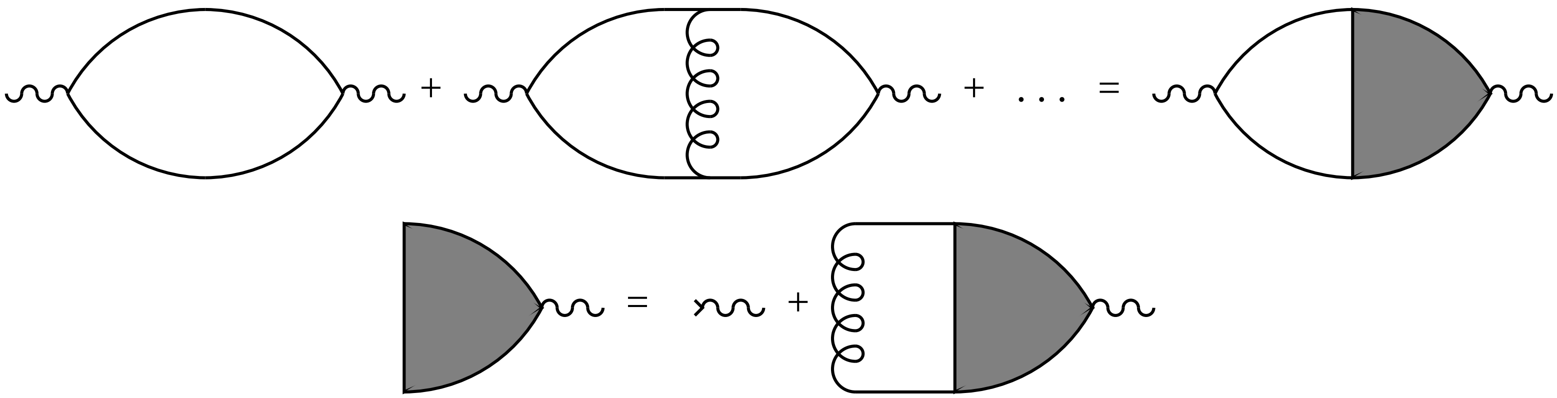


physically corresponding to



Resummation of Diagrams

Diagrams may be resummed by defining a dressed vertex,



determined by an integral equation (second line).

Emission rate from thermal QGP (3 light flavors) is AMY

$$\frac{dN_\gamma}{d^3\mathbf{k}d^4x} = \frac{2\alpha_{\rm EM}}{4\pi^2 k}\int_{-\infty}^{\infty}\frac{dp}{2\pi}\int\frac{d^2\mathbf{p}_\perp}{(2\pi)^2}\,\frac{n_f(k{+}p)\,[1{-}n_f(p)]}{2[p\,(p{+}k)]^2}\times$$

$$\times\left[p^2+(p{+}k)^2\right]\mathrm{Re}\left\{2\mathbf{p}_\perp\cdot\mathbf{f}(\mathbf{p}_\perp;p,k)\right\}$$

$$2\mathbf{p}_\perp = i\delta E\,\mathbf{f}(\mathbf{p}_\perp;p,k)+\frac{2\pi}{3}g_{\rm s}^2\int\frac{d^2q_\perp}{(2\pi)^2}\frac{m_{\rm D}^2\,T}{q_\perp^2\,(m_{\rm D}^2{+}q_\perp^2)}\times$$

$$\times\left[\,\mathbf{f}(\mathbf{p}_\perp;p,k)-\mathbf{f}(\mathbf{q}{+}\mathbf{p}_\perp;p,k)\right],$$

$$\delta E = \frac{\mathbf{p}_\perp^2+m_\infty^2}{2}\;\frac{k}{p(k{+}p)}$$

Note, (2) is implicit and must be solved numerically.

To Clarify

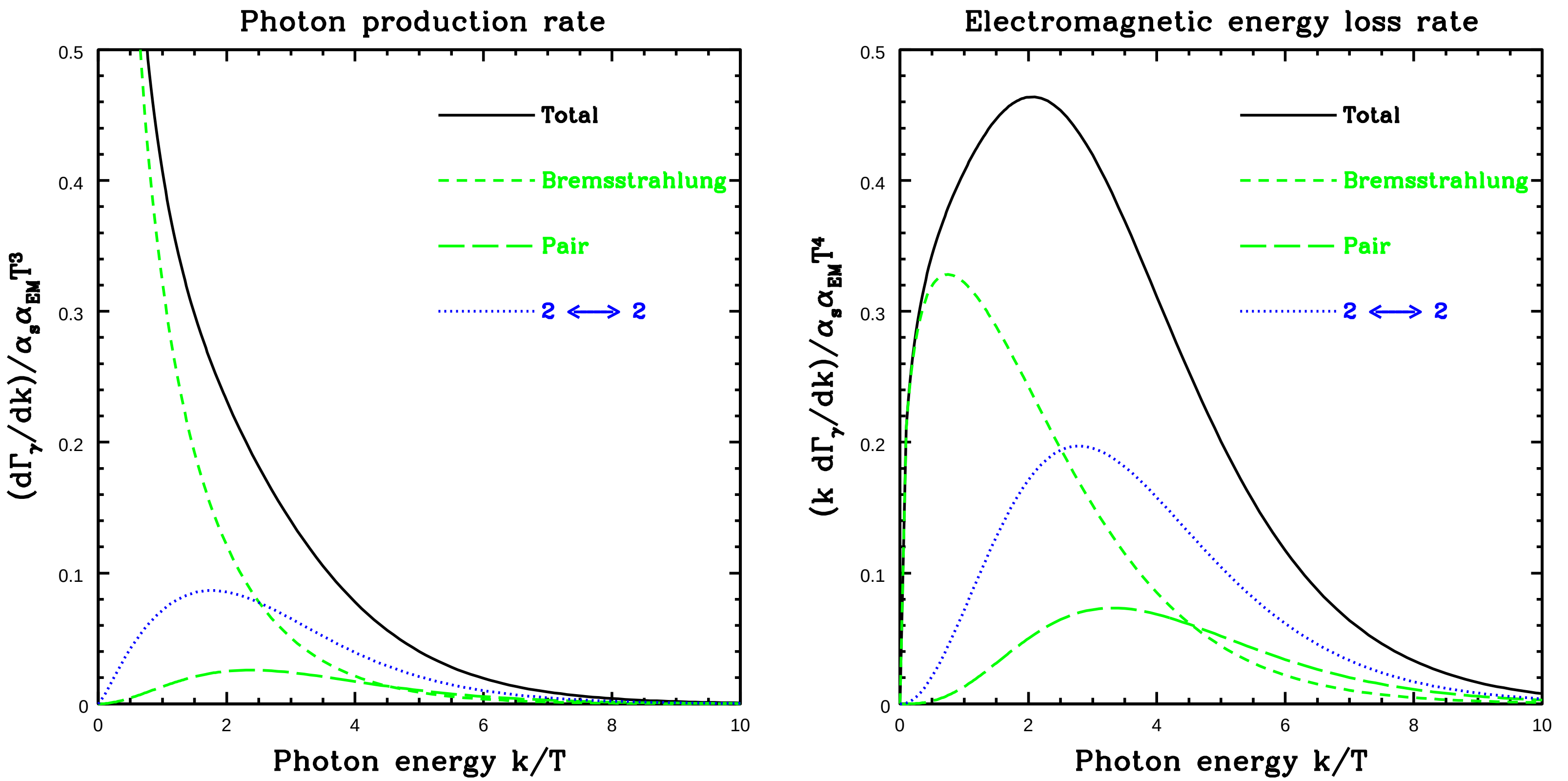
The physical picture of multiple scattering suppressed Bremsstrahlung worked out by Migdal in 1955

Case of photons in QCD treated by Baier Dokshitzer Mueller Peigne Schiff in 1996 and by B.G. Zakharov in 1996.

What we did was

- give a rigorous diagrammatic derivation
- work consistently to leading order
  (dynamic medium, medium dispersion effects)
- Fourier transform to frequency domain, where problem easier to solve without further approximation

Result: Thermal Medium

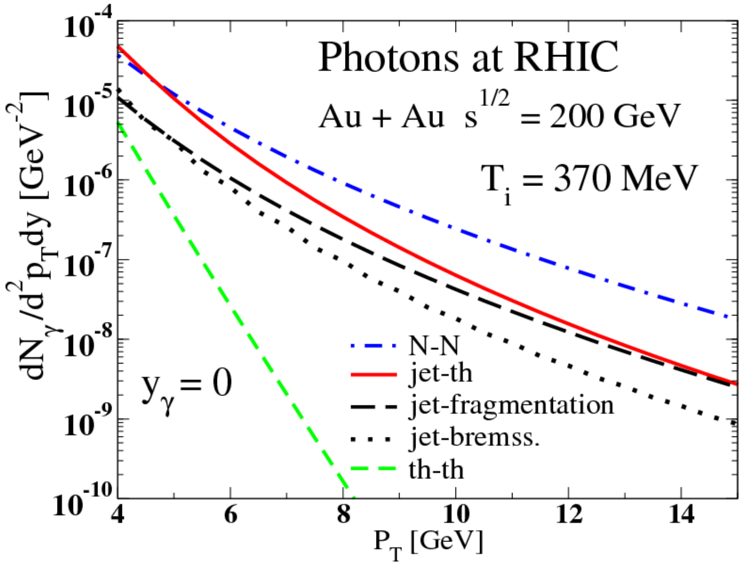


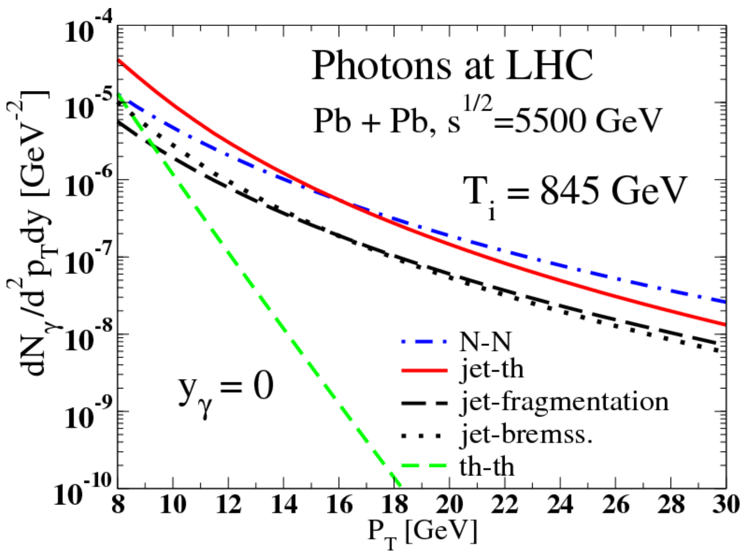
Brem/pair, $2 \leftrightarrow 2$ rates comparable. Used $\alpha_s = 0.2$, 3 flavors

Medium-Jet Interactions

Formalism does not require equilibrium.

Just stick in distribution function $n(p)$ incl. jets:





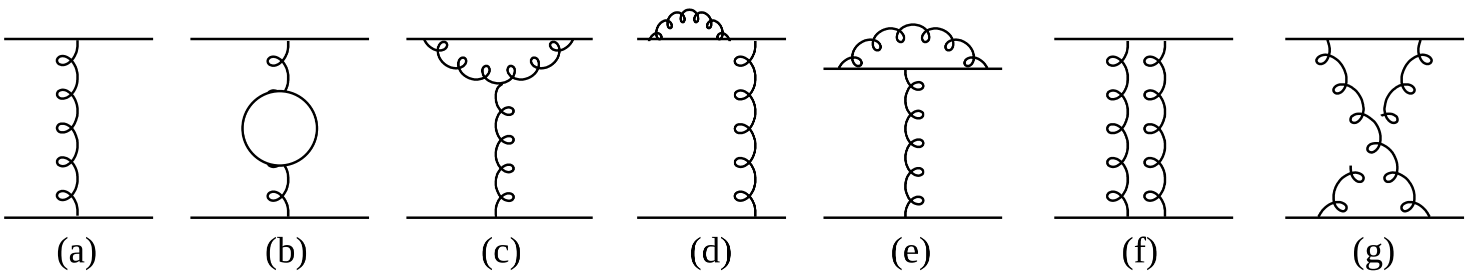
Jet-medium photons may exceed thermal.

How reliable are LO Calculations?

Bad news 1: first corrections are $\mathcal{O}(g)$, not $\mathcal{O}(\alpha_s)$

Soft gluons involved! Loop gives α_s *and* Bose factor $\sim T/gT \sim 1/g$

And there are a lot of $\mathcal{O}(g)$ corrections!

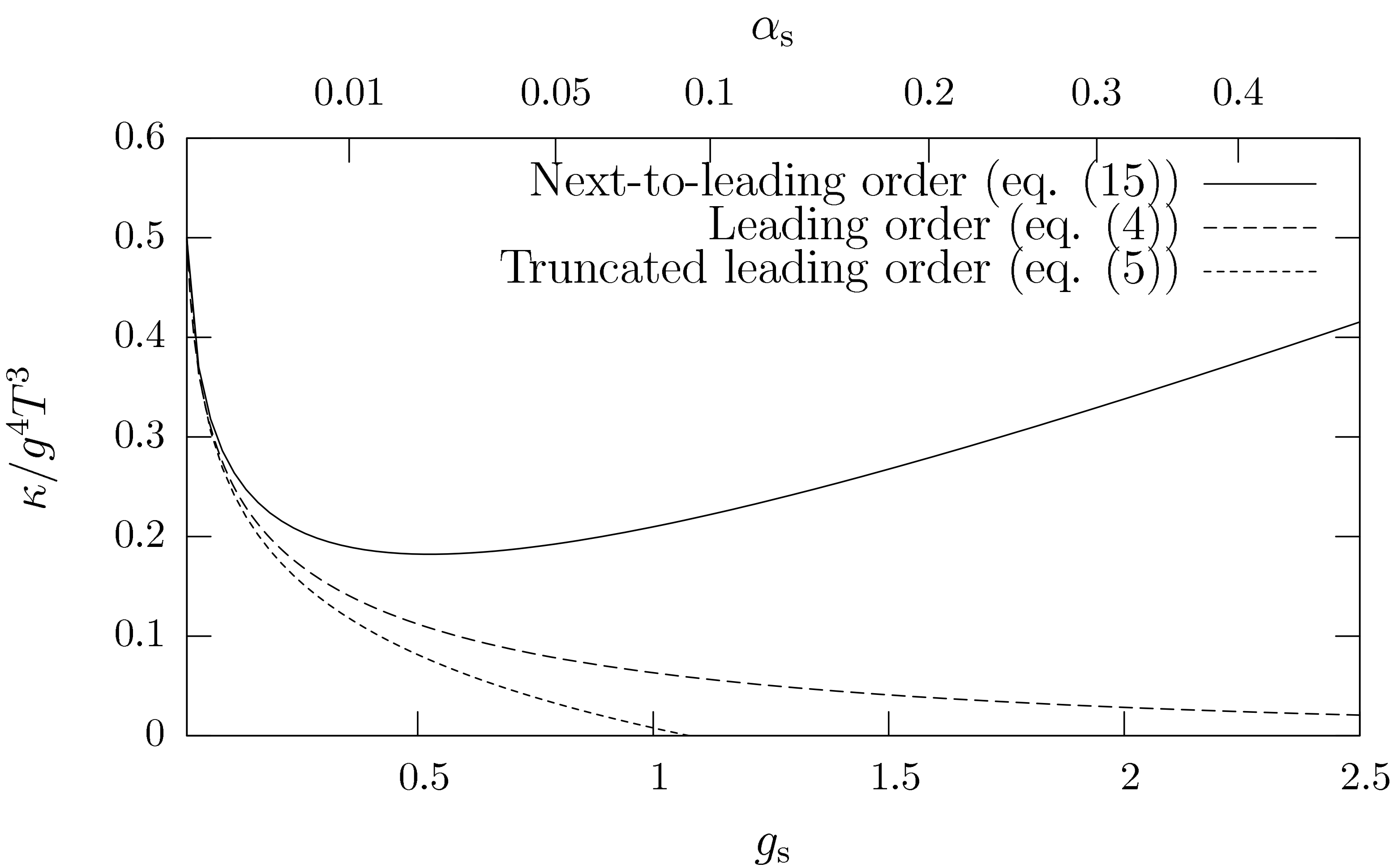


LO requires using (a) as rung. NLO requires all!

Bad news 2: $\mathcal{O}(g)$ coefficient likely to be large!

NLO Not Computed! But similar computation for heavy quarks indicate large $\mathcal{O}(g)$ NLO corrections. Similar to pressure at $\mathcal{O}(g^2)$, $\mathcal{O}(g^3)$, possibly for similar reasons

Similar Problem: Heavy Quark Diffusion



NLO calculation has been done. Correction is HUGE!

Why shouldn't perturbation theory work?

If $\alpha_{\rm s} \sim 1/20$, most of DOF are weakly coupled quasipart.
Quarks with 5–10T energy weakly coupled.

Scattering and HTL's sample particles as

$$N_f \int \frac{d^3p}{p} n_f(p) + N_c \int \frac{d^3p}{p} n_b(p)$$

Gluon n_b dominates; *big* contribution from $p \lesssim T$.

But these gluons get *large* corrections from HTL's.

The $p \sim T$ gluons are NOT weakly coupled, and *they matter* Related to poor pert. behavior of EQCD, problems with QCD pressure

What do you do?

- Do the NLO calculation to see if things really are bad. In progress (see Aleksi Kurkela talk)
- Try any resummation you can think of
- Include big theory error bars in rate when doing phenomenology Strong T dependence – may still be a thermometer?
- Ask what happens in *some* theory at strong coupling

$\mathcal{N}$=4 SYM theory

Consider QCD with a Majorana-Weyl fermion in 10D

$A^M = \{A^\mu, A^i\}$: $i = 4\ldots9$; ψ is 4 Majoranas ψ_a

Now Dim. Reduce the i dimensions. $A^i \equiv \Phi^i$ scalars!

$$F^{MN}F_{MN} = F^{\mu\nu}F_{\mu\nu} + 2F^{\mu i}F_{\mu i} + F^{ij}F_{ij} = F^2 + |D\Phi|^2 + \Phi^4$$
$$\bar{\psi}_a \Gamma^M_{ab} D_M \psi_a = \bar{\psi}_a \not{D} \psi_a + g\Gamma^i_{ab}\Phi_i \bar{\psi}_a \psi_b$$

Gauge fields, 6 scalars 4 Majorana fermions

scalar quartics and Yukawa interactions all fixed by g

And an SO(6) symmetry, rotation in "missing" 6-dimensions

Consider this theory as QCD-like. A^μ like gluons (only SU(N) with $N \gg 1$) and ψ, Φ are "like" quarks.

Gauge a U(1) subgroup of SO(6) and call it E&M.

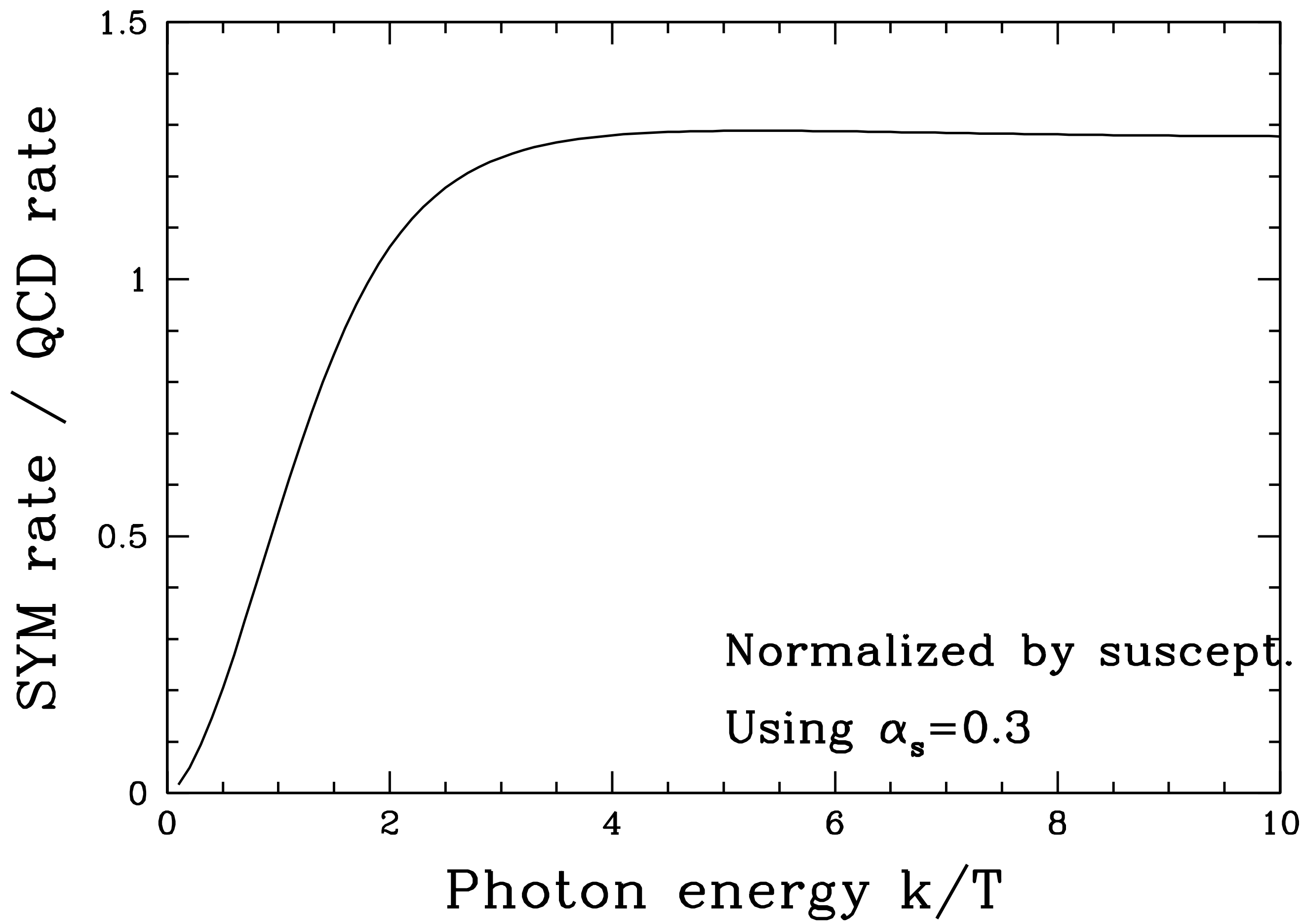
Ask about its $\langle JJ \rangle$ correlators

Can do this at weak coupling but *also* at strong coupling using the gravity-dual tricks. Result proportional to

- how many charge carriers you have
- how much charge they carry (coupling to E&M)
- how much they get kicked around

Number and size of charges: charge susceptibility χ

Interesting quantity: Photon production / χ



SYM at infinite coupling has *marginally* larger photon production rate than Leading-Order QCD calc. at $\alpha_{\mathrm{s}} = 0.3$

Suggests that full rate may *not* exceed LO rate by much

Weak-coupling rate actually *larger* at small k understood

Strong-coupled QCD: Guesses, anyone?

- Large positive corrections in other quantities
- Infinite-coupling limit in SYM is not much larger

Conclusions

- Photon rate is physically interesting

  though observing THERMAL photons may be tough!

- Computation: $\langle JJ \rangle$ at lightlike momentum

- Phase space, perturbation theory subtle! Resummations

- LO calculation done, but may have big corrections

- SYM case suggests strong-coupled rate not *much* larger than perturbative estimates

Someone should do the NLO calculation!