Explosive phase transitions in the Early Universe?

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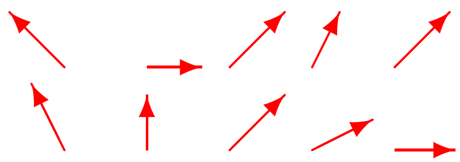
- Scalar fields in particle physics
- Scalar fields in symmetry-breaking phase transitions
- Electroweak phase transitions in the very early universe
- What limits the interface speed?
- Implications and Conclusions

(Quantum) Field Theory: Lightning review

There can be 3 sorts of fields:

- **Vector fields**: at each point in space, an "arrow"



Examples: $\vec{E}, \vec{B}$ fields, Vector Potential

- **Spinor fields**: at each point in space, two "anticommuting numbers"

- **Scalar fields**: at each point in space, a "number"

6 12 9 1 -7

10 14 12 8 3 Examples: scalar potential, pressure

Dynamics

Theory is described by an action $S = \int d^4x \mathcal{L}$

Equivalently, $U(t) = e^{-iHt}$, $H = \int d^3x \mathcal{H}$ so $U(t) = \exp(-i \int d^4x \mathcal{H})$

Lagrange density $\mathcal{L}$ describes all dynamics:

$$\frac{\vec{E}^2 - \vec{B}^2}{2} + \bar{\Psi}(m + \gamma^\mu \partial_\mu)\Psi + \left|\frac{\partial \Phi}{\partial t}\right|^2 - \left|\frac{\partial \Phi}{\partial \vec{x}}\right|^2$$

Kinetic terms: free particles propagate.

$$\Phi \bar{\Psi} \Psi + A_\mu J^\mu + |\Phi|^4$$

Interation terms, tell how particles interact.

Standard Model, Fermions and Masses

The mass term $m\bar{\Psi}\Psi$ for a fermion actually couples R-handed to L-handed parts of the field.

Experiment $\rightarrow$ Ψ_L *interacts differently* than Ψ_R!

Weak interactions *only* involve Ψ_L; Ψ_L has weak charge, Ψ_R does not.

(weak) charge conservation therefore forbids $m\bar{\Psi}_L\Psi_R$!

Fermion masses: coupling to a (weak)-charged scalar

$$\Phi\bar{\Psi}_L\Psi_R \quad \text{allowed!}$$

This "is" a mass provided that $\Phi \neq 0$

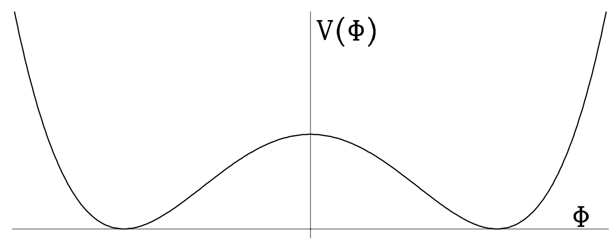
Higgs field

Fermion (and W, Z) masses in Standard Model demand scalar Φ

Scalar must have nonzero mean vacuum value $\langle\Phi\rangle = v$

Possible if potential energy (density) $V(\Phi)$ of form

$$V(\Phi) = \frac{\lambda}{4}(\Phi^2 - v^2)^2$$



Minimized when $\Phi = v$ away from zero.

"normal" vacuum state has $\langle\Phi\rangle = v \neq 0$.

Temperature and phase transitions

Universe is expanding and cooling, $T \sim t^{-1/2}$:

- 1 second: $T = 10^{10}\,\mathrm{K} = 8$MeV (nuclear binding)
- 10^{-6} sec.: $T = 6 \times 10^{12}\,\mathrm{K}$ =0.5GeV (hadron binding)
- 10^{-11} seconds: $T = 160$GeV (Higgs symmetry breaking)

Hot: Free energy, not energy, is relevant.

$\langle \Phi \rangle$ is dynamical, can change with T.

Does hi-Temp favor $\Phi \neq 0$ or $\Phi = 0$?

Thermal bath: excitations (particles) present with

$$f(k) = \frac{1}{\exp(E_k/T) \pm 1} \qquad + \text{ Fermion, } - \text{ Boson}$$

If Φ changes, F changes according to $dE/d\Phi$ of particles:

$$\frac{dF}{d\Phi} = \sum_{\text{species}} \int \frac{d^3k}{(2\pi)^3} f \frac{dE}{d\Phi} = \sum_{\text{species}} \frac{dm^2}{d\Phi} \int \frac{d^3k}{(2\pi)^3} \frac{f}{2E}$$

Particles get heavier at larger Φ: favors small Φ.

Temperature favors state where particles are light!

First crude approximation

Take particle occupancies to be *free, massless* values:

$$\int \frac{d^3k}{2\pi}^3 \frac{f}{2E} = \frac{T^2}{24} \text{ bosons}, \quad \frac{T^2}{48} \text{ fermions}$$

Therefore effective Higgs mass increased:

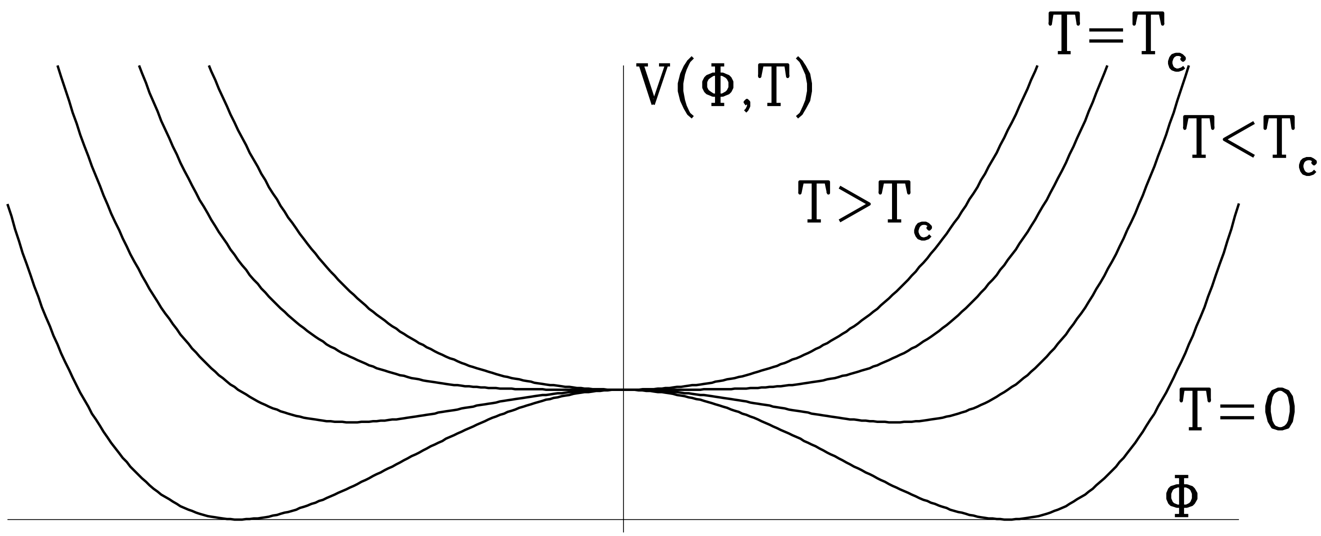
$$\frac{\lambda}{4}(\Phi^2 - v^2)^2 = \frac{\lambda}{4}\Phi^4 + \frac{m_\Phi^2}{2}\Phi^2 + \frac{\lambda v^4}{4}, \quad m_\Phi^2 = -\lambda v^2$$

shifted by

$$m_\Phi^2 = -\lambda v^2 + \sum_{\text{species}} \nu \frac{m^2}{v^2} \times \frac{T^2}{12 \text{ or } 24}$$

ν=multiplicity

Potential as Function of T (crude approx.)



Second order Phase Transition to $\Phi = 0$ at hi T

More careful treatment: First order if λ small,

No phase transition if λ large ($m_H > m_W$)

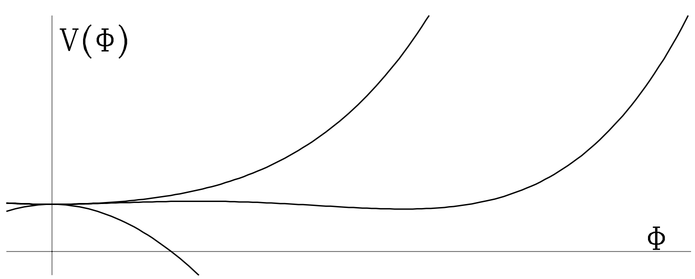
Fluctuation induced phase transition

Our "crude approximation" above misses a subtlety:

as Φ grows, particle number empties out, effect reduces.

$$\int \frac{d^3k}{(2\pi)^3} \frac{1}{2E(\exp(E/T)-1)} < \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k(\exp(k/T)-1)}$$

Result: at larger Φ potential turns up less



two local minima:

1'st order transition

Heavy Higgs ($m_H > 80$GeV$=m_W$)

Previous arguments predict *very weak* phase transition

Φ induced masses $<$ inter-particle $E_{\rm interaction}$

Calc. unreliable. $\rightarrow$ nonperturbative (latt) treatment
which shows there is *no* phase transition!

Phase transition can re-appear in extensions of SM:

- MSSM: new scalars which get large masses from Φ.
  1'st order if they are light (Light Right Scalar Top)

- Extended scalar sectors:
  strong 1'st order transition is possible

Simplest scalar extension

Add 1 scalar field χ:

$$V(\chi, \Phi) = \frac{\lambda}{4}(\Phi^2 - v^2) + \frac{\lambda'}{4}(\chi^2 - u^2) + \tilde{\lambda}\Phi^2\chi^2 \;\; (+\chi\Phi^2, \chi^3)$$

If v^2, u^2 positive, potential can have two minima.

If $\tilde{\lambda}^2 > 4\lambda\lambda'$, cannot have both $\Phi \neq 0$ and $\chi \neq 0$

If m_Φ^2 more temperature sensitive, $\Phi \neq 0$ favored at low T, $\chi \neq 0$ at medium T, both 0 at high T.

Strong 1'st order phase transition

Why is Electroweak Phase Transition interesting?

Gravity Wave Background: Low-density bubbles form in high-density medium. Time-changing density contrast $\rightarrow$ grav. waves. $\sim 10^{-3}$ Hz today. Observable (LISA) if transition is very strong [latent heat $\sim$ energy density]

Electroweak Baryogenesis: $\Phi = 0$ phase allows rapid violation of baryon number ($\not B$ condition). 1'st order Phase transition: Out of Equilibrium. Matter abundance of Universe may have formed if $\not C$, $C\!\!\!/P$. [requires latent heat $\sim 10^{-2}$ energy density and new CP viol]

Nature of phase transition?

Two important kinds of questions:

- Thermodynamics: first order/second order/crossover? Phase structure, strength of transition, degree of supercooling.
  Analytic + lattice gauge theory techniques

- Dynamics: structure of nonequilibrium phase interface "Bubble Wall," propagation velocity of "Bubble Wall," Physics near "Bubble Wall"
  nonequilibrium quantum field theory techniques

Bubble wall velocity in similar theories?

Nonrelativistic phase transitions: phase interfaces either propagate as *deflagrations* ($v < v_{\rm sound}$) or *detonations* ($v > v_{\rm sound}$) but always with terminal velocity.

Vacuum Quantum Field Theory: moving wall is Lorentz transform of wall at rest. Wall accelerates with constant proper acceleration towards $v = c$.

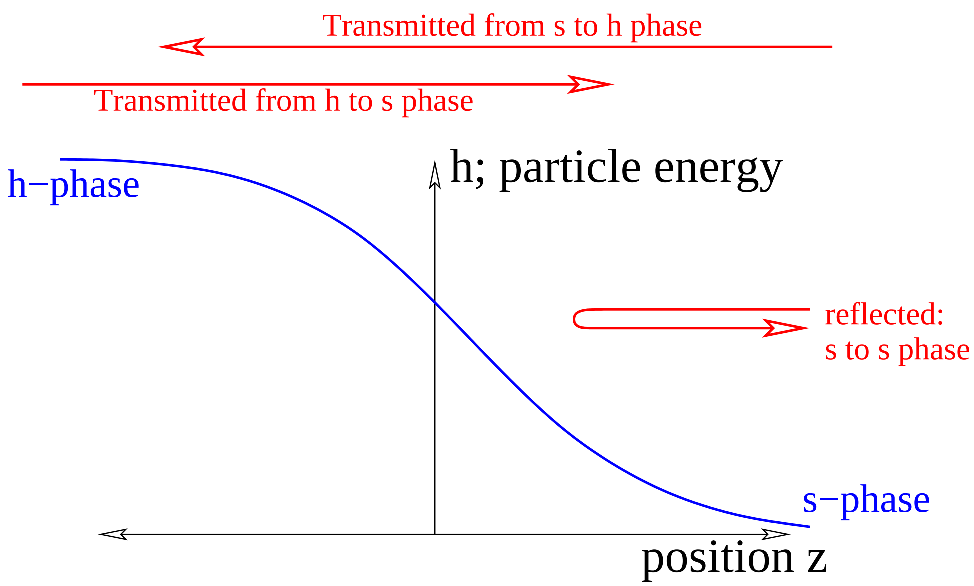
Which behavior applies for Electroweak case?

Bubble wall velocity: force balance

Vacuum favors large Φ pushing wall forward:

$$\frac{dE}{dz} = \frac{dE}{d\Phi}\frac{d\Phi}{dz} \Rightarrow \frac{dF}{d\,\mathrm{Vol}} = -\frac{dV_{\mathrm{vac}}}{d\Phi} d\Phi dz$$

Particles push wall backward.



Force from particles, *in equilibrium*:

Wall rest-frame: particle moves in static background

$$F_{\rm particle} = \frac{dp_z}{dt}. \quad \frac{dE^2}{dt} = 0 \to \frac{2p_z dp_z}{dt} = -\frac{dm^2}{dt} = -\frac{dm^2}{d\Phi}\frac{d\Phi}{dz}\frac{dz}{dt}$$

$$\frac{dF}{d\,{\rm Vol}} = -\frac{d\Phi}{dz}\sum\int\frac{d^3k}{(2\pi)^3}\frac{f}{2E}\frac{dm^2}{d\Phi} = -\frac{d\Phi}{dz}\frac{dV_{\rm therm}}{d\Phi}$$

Reproduces thermal contribution of full potential.

Vacuum effect + particle effect describe effective potential difference between phases.

Moving wall

The same formula applies (in wall rest frame):

$$\frac{dF}{d\,\mathrm{Vol}} = -\frac{d\Phi}{dz}\left(\frac{dV_{\mathrm{vac}}}{d\Phi} + \sum \frac{dm^2}{d\Phi}\int \frac{d^3k}{(2\pi)^3}\frac{f_{\mathrm{noneq}}(\vec{k},\vec{x})}{2E_k}\right)$$

Problem is determining $f_{\mathrm{noneq}}(\vec{k},\vec{x})$ – out of equilibrium!

Evolution in changing backgrnd+scatt. $\rightarrow$ Boltzmann eq's

Standard Model and MSSM: results give $v_{\mathrm{wall}} \sim 0.1c$.

What about two-scalar case, which can provide strongest transition?

Simpler question

Does bubble wall reach a finite terminal velocity? Or keeps accelerating so γv grows linearly with distance?

Still tells us something interesting:

- If wall accelerates without limit, much more interesting gravity wave signature
- If wall accelerates without limit, Baryogenesis becomes very difficult ($\not{B}$ in front of wall, noneq. only behind wall)

$\gamma \gg 1$ Simplifications

Wall frame: particles rush at wall ultra-relativistically:

- All particles approach wall from one side.
- All particles start out in equilibrium, $\Phi = 0$ state
- All particles have $E \sim \gamma T \gg m$: no reflection
- Particles have no time to scatter while traversing wall

$\gamma \gg 1$ Calculation

Very high particle density $\sim \gamma$, but
Very small p_z transfer per particle, $\sim \gamma^{-1}$.

$$-\frac{dF}{d\,\mathrm{Vol}} = \frac{d\Phi}{dz} \sum \frac{dm^2}{d\Phi} \int \frac{d^3k}{(2\pi)^3 2E} f(\vec{k}, z)$$

Small p_z transfer: occupancy changes are $\mathcal{O}(\gamma^{-1})$,
$f(\vec{k}, z) = f(\vec{k}, z = \infty)$ (to $\mathcal{O}(1/\gamma)$)

$$-\frac{dF}{d\,\mathrm{Vol}} = \frac{d\Phi}{dz} \sum \frac{dm^2}{d\Phi} \int \frac{d^3k}{(2\pi)^3 2E} f_{\mathrm{eq}}(\gamma E + \gamma v p_z)$$

Integral is easy!

$$\int \frac{d^3k}{(2\pi)^3 2E} \quad \text{Lorentz invariant}$$

Perform integral in plasma rest-frame, $f = f_{\rm eq}$ in hi-temp phase.

$$-\frac{dF}{d\,{\rm Vol}} = \frac{d\Phi}{dz} \sum \frac{dm^2}{d\Phi} \int \frac{d^3k}{(2\pi)^3 2E} f_{\rm eq}(k)$$

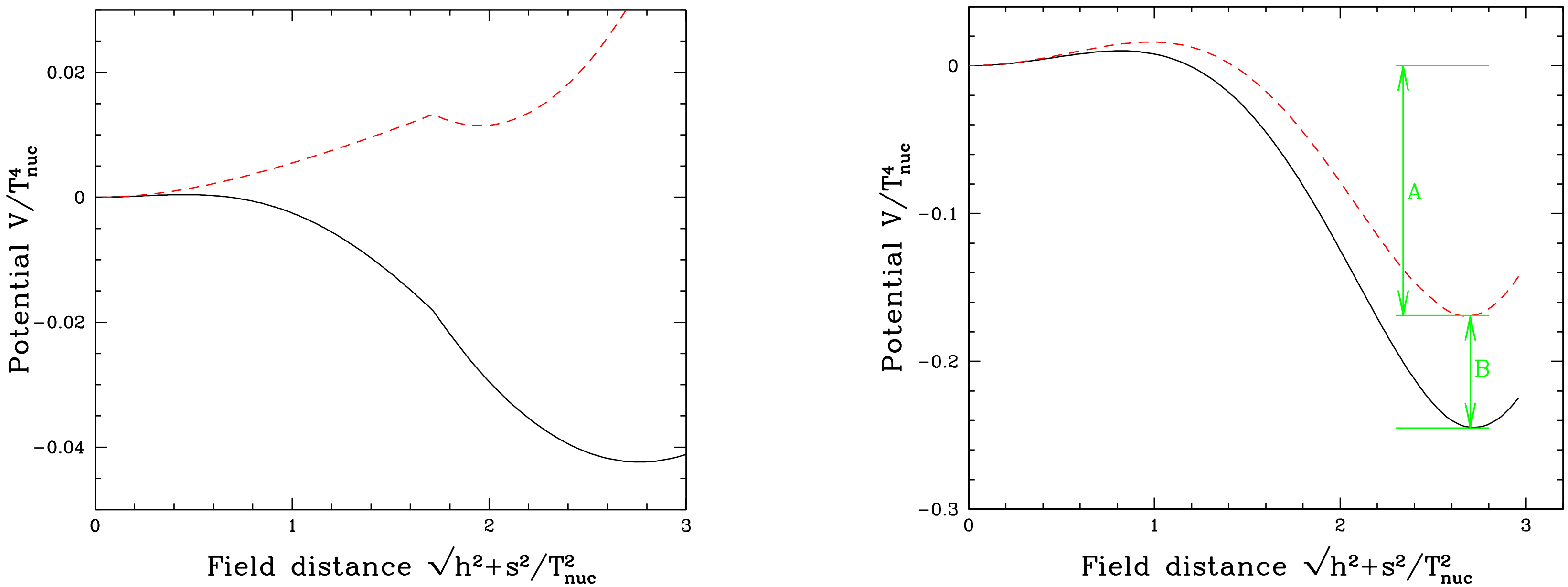
Exactly our "Crude Approx." from before!

Force on wall given by $V_{\rm therm,eff} = \int d\Phi({\rm above})$

Same as 2-order Taylor expansion about high-T phase.

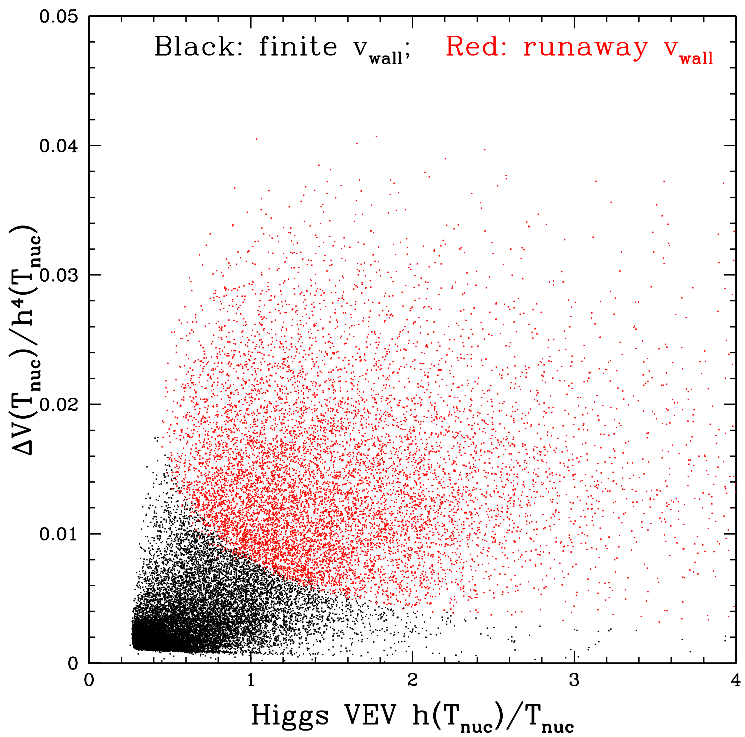
Recipe: replace V_{therm} with $V_{\mathrm{therm,eff}}$.

Example for two Φ, χ theory cases:



Finite terminal v wall runs away

Wall runs away when transition is sufficiently strong:



Finite v for small Φ/T or small supercooling.

Cases which give strong transition in sense of large supercooling lead to runaway bubble walls.

Can electroweak bubble walls run away?

- Fluctuation induced phase transition (SM, MSSM):
  No. 2'nd min. appears because $V_{\rm therm}(\Phi)$ nontrivial.
  Quad. approximation $\rightarrow$ low temp minimum disappears
  $\rightarrow$ restraining force on $\gamma \gg 1$ wall is stronger than forward force. γv always small.

- First order in crude approximation:
  Generically the bubble wall *does* "run away".

Conclusions

- Electroweak phase transition: physically interesting
  - ∗ Gravitational wave background?
  - ∗ Electroweak Baryogenesis?
- Transition not generic: requires extended scalar sector
- Bubble wall velocity can be finite, can run away!
  - ∗ Fluctuation induced transition (MSSM): finite
  - ∗ “naive” first order transition (NMSSM, χ extension): bubble wall runs away in cases with strong phase transition.