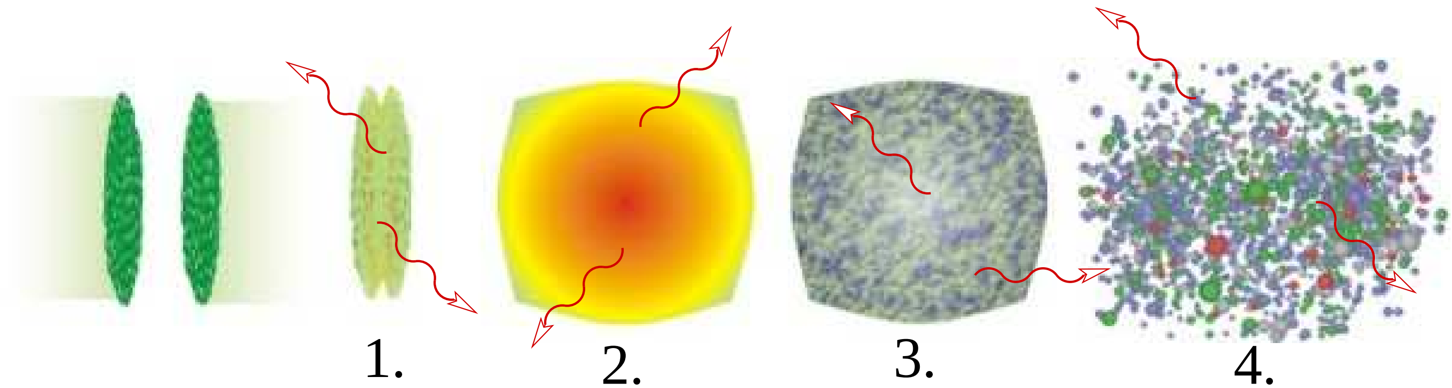
Photons and Transport at NLO

with Jacopo Ghiglieri, Juhee Hong, Aleksi Kurkela, Egang Lu, Derek Teaney

- Photons: motivation and basics
- Convergence of Perturbation Theory
- When soft physics is light-cone physics
- When light-cone physics is thermodynamics
- NLO photon production: results, prospects

Stages of a Heavy Ion Collision



1. Ions collide, making q, g, photons "primary"
2. q, g rescatter as QGP, make photons "thermal"?
3. Hadrons form, scatter, make photons "Hadronic"
4. Hadrons escape, some decay to photons "decay"

Photon re-interaction rare ($\alpha_{\rm EM} \ll 1$): direct info.

Thermal photons *may* act as a thermometer for QGP.

Production rate is interesting! Mostly for $E > 2$ GeV, several T.

Where do Photons Come From?

Since $\alpha_{\rm EM} \ll 1$, work to lowest order in $\alpha_{\rm EM}$:

- assume photon production *Poissonian* Find single-photon production

- neglect back-reaction on system cooling by γ emission insignificant...

Single-photon production at $\mathcal{O}(\alpha_{\rm EM})$

$$2k^0 \frac{d\mathrm{Prob}}{d\!\!\!^-{}^3 k} = \sum_X \mathrm{Tr}\, \rho\, U^\dagger(t) |X, \gamma(k)\rangle\langle X, \gamma(k)| U(t)$$

$U(t)$ time evolution operator, ρ density matrix.

Expand $U(t)$ in EM interaction picture:

$$U(t) = 1 - i \int^t dt' \int d^3x \, eA^\mu(x,t') J_\mu(x,t') + \mathcal{O}(e^2)$$

A^μ produces the photon. Get

$$\frac{d\mathrm{Prob}}{đ^3 k} = \frac{e^2}{2k^0} \int d^4Y d^4Z e^{-iK\cdot(Y-Z)} \sum_X \mathrm{Tr}\, \rho \, J^\mu(Y)|X\rangle\langle X|J_\mu(Z)$$

And $\sum_X |X\rangle\langle X| = \mathbf{1}$. *Assume* ρ slow-varying, near-equilibrium: $\int d^4Z \to Vt$: Get rate per 4-volume:

$$\frac{d\Gamma}{đ^3 k} = \frac{e^2}{2k^0} G^<(K) \,, \quad G^<(K) \equiv \int d^4Y e^{-iK\cdot Y} \left\langle J^\mu(Y) J_\mu(0) \right\rangle_\rho$$

Thermal Approx justified by Success of Hydro – but not true at early times ....

Calculational Approaches

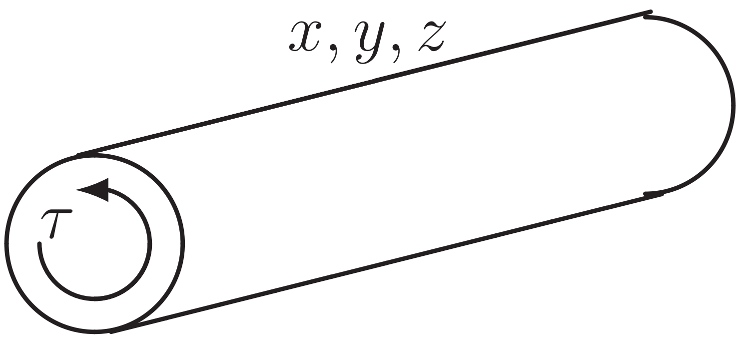
No first-principles, *nonperturbative* tool for $\langle J^\mu J_\mu \rangle(K)$. Only

- Lattice techniques (uncontrolled analytic continuation)
- Weak-coupling techniques (uncontrolled extrapolation from $\alpha_s < 0.1$)
- Strong-coupling $\mathcal{N}{=}4$ SYM (uncontrolled relation to QCD)

How bad is weak coupling?

- It fails at $T \sim$ few T_C ?
- It fails at $T \sim 10^6 T_C$?
- It fails at all temperatures? Truth: some of each!

Story for static properties



Time is compact; large distances act like 3D theory

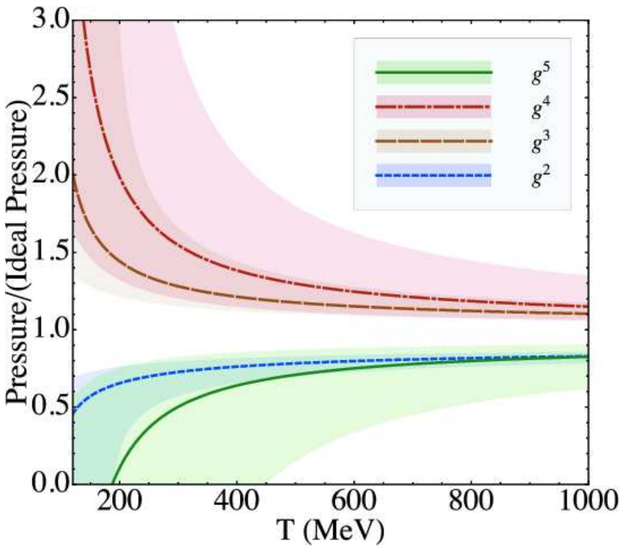
3D theory describes massive A_0, massless F_{ij}
Coupling $g^2_{3\mathrm{D}} = g^2_{4\mathrm{D}} T$ dimensionful
$g^2 T$ scale is where coupling becomes $\mathcal{O}(1)$

Reducing to 3D theory works for $T >$few T_c.
But must solve 3D theory nonperturbatively (or resum) unless T is *very* large.

Lessons from the Pressure

Divide degrees of freedom in 2 groups: *Hard and Soft*

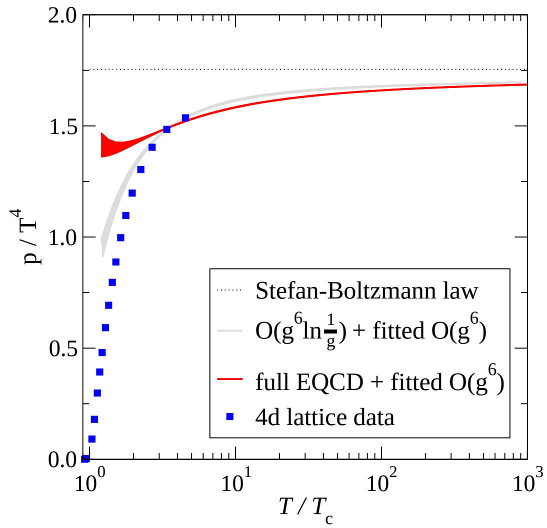
Naive order-by-order g-expansion



Converges if $T > 10^6 T_C$

Arnold-Zhai, Braaten-Nieto, *etc*

Integrate out *Hard*, solve *Soft* nonperturbatively: 3-D theory!



Works down to $T = 2T_C$.

Kajantie *et al*, *etc*

Hard physics is perturbative. There is hope!

Lattice

Lattice: find correlators at unequal *Euclidean* time τ.

How can I use that? Trade $G^{<}(K)$ for spectral function

$$\sigma(K) \equiv \int d^4Y e^{-iK\cdot Y} \left\langle \left[J^\mu(X)\,,\, J_\mu(0) \right] \right\rangle = \frac{1}{n_b(k^0)} G^{<}(K)$$

Related to Euclidean-frequency correlator via Kramers-Kronig

$$G_{\rm E}(\omega_{\rm E}, k) = \int \frac{dk^0}{2\pi} \, \frac{\sigma(k^0, k)}{k^0 - i\omega_{\rm E}}$$

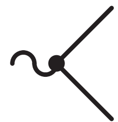
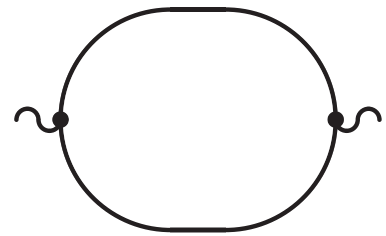
Transform to commonly measured $G_{\rm E}(\tau, k)$:

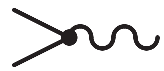
$$G_{\rm E}(\tau, k) = \int \frac{dk^0}{2\pi} \, \frac{\sigma(k^0)}{k^0} \; \times \; \frac{k^0 \cosh(k^0(\tau - 1/2T))}{\sinh(k^0/2T)}$$

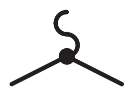
Problem: I want $\sigma(k^0)$, I know $G_E(\tau)$ with errors.

What do I expect for σ?

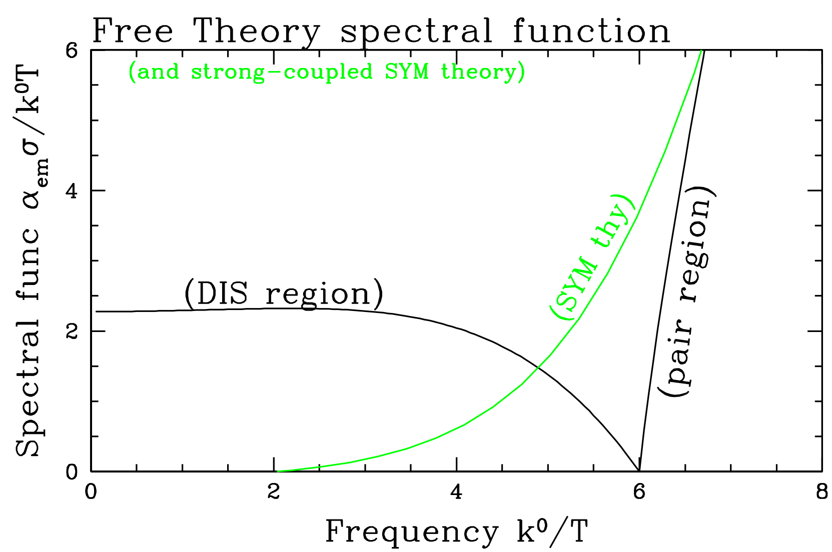
First consider lowest order perturbation theory:

$J^\mu = \sum_{q=uds} e_q \bar{q} \gamma^\mu q$:  so LO: $\langle JJ \rangle =$ 

Timelike K: pair annihilation  kinematically fine

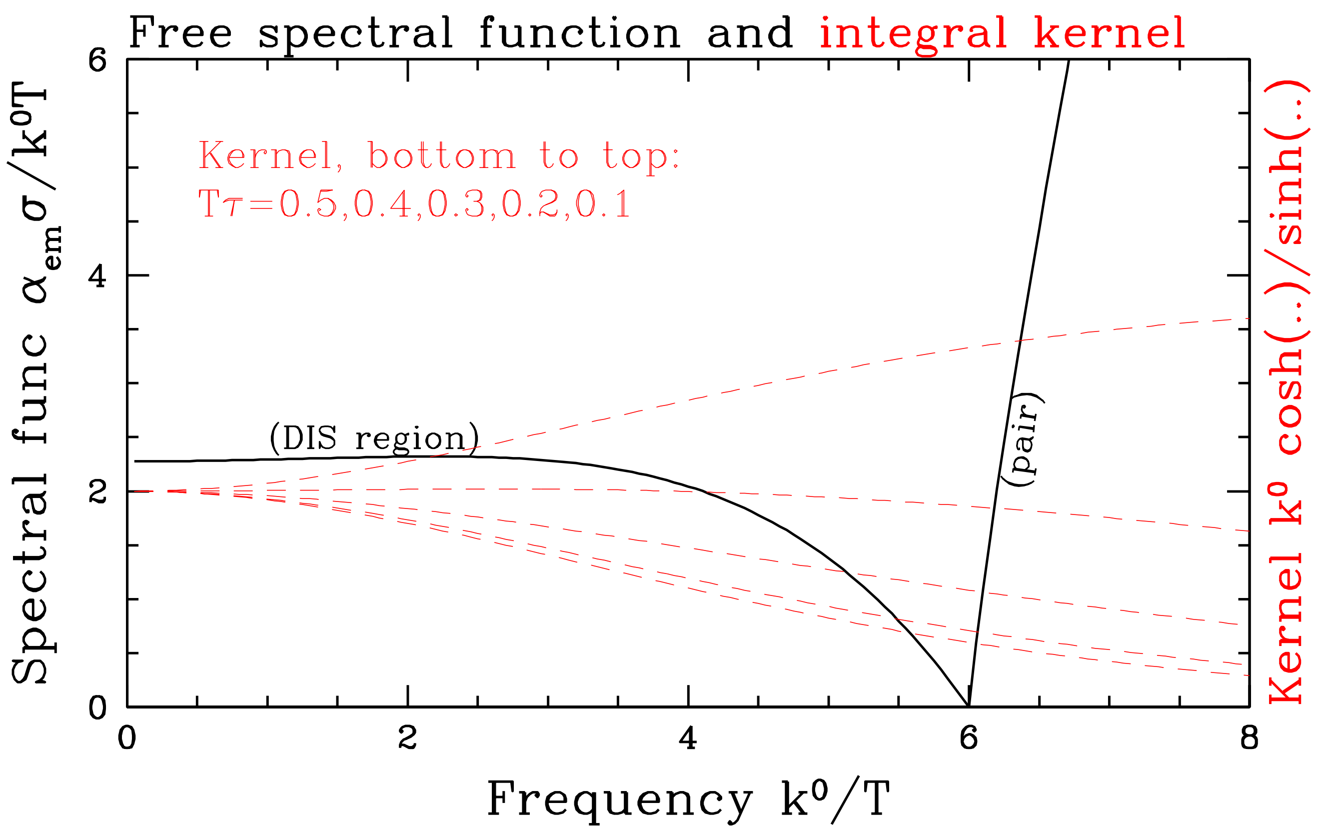
Spacelike K: DIS  also kinematically OK

Lightlike K: Neither allowed. Notch feature!



Lattice II

Can I capture sharp features with integral moments?

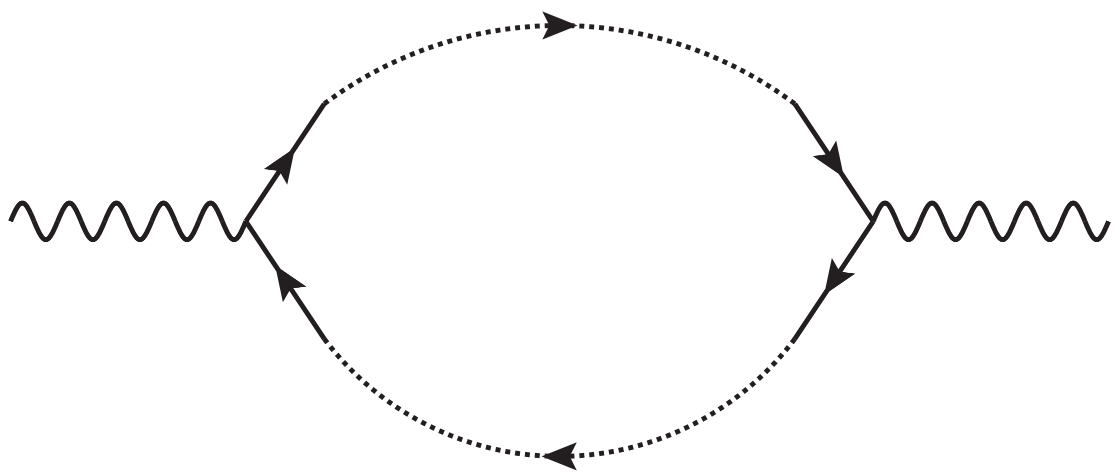


Not hopeful. Answer depends on input assumptions!

Perturbative treatment

We want $G^{<}(K) \equiv \int d^4Y e^{-iK\cdot Y} \left\langle J^\mu(Y) J_\mu(0) \right\rangle_\rho$

$J^\mu = \bar{\psi}\gamma^\mu\psi$. Correlator of two quarks. Something like

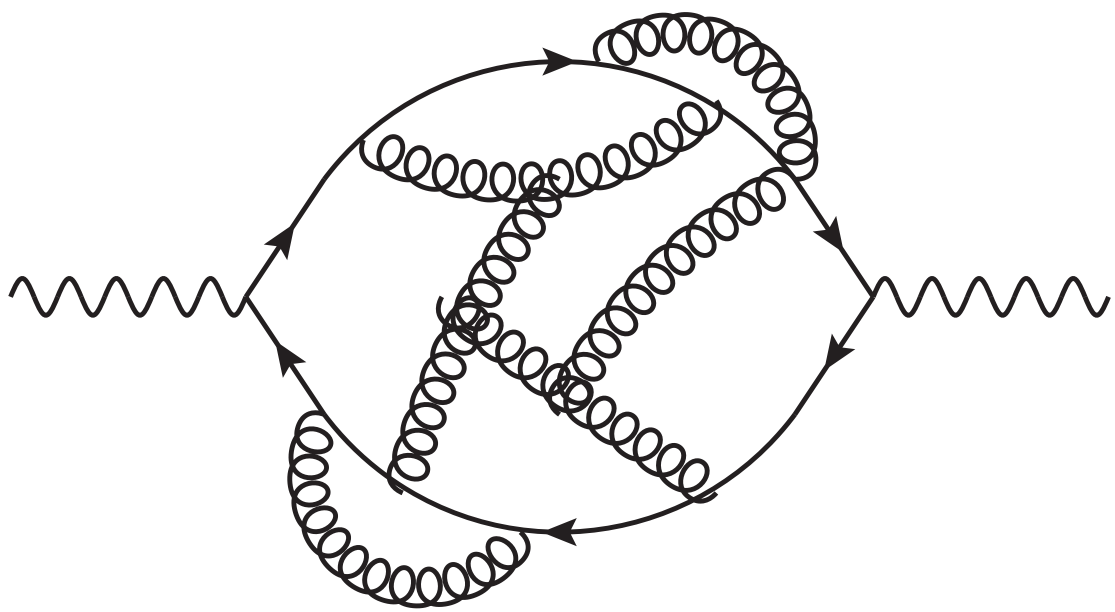


Really, need to put in *all* ways the 4 quark operators can connect with each other and with environment.

Perturbative treatment

We want $$G^{<}(K) \equiv \int d^4Y e^{-iK\cdot Y} \left\langle J^\mu(Y) J_\mu(0) \right\rangle_\rho$$

$J^\mu = \bar\psi \gamma^\mu \psi$. Correlator of two quarks. In general,

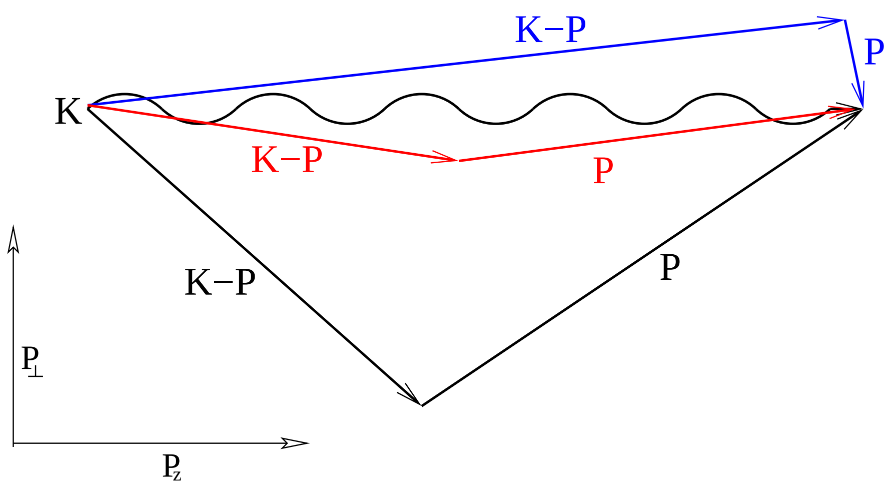


Worse: dynamics *complex*, no nice effective 3D theory!

Start with Kinematics

$$\gamma \text{ produc: } \sum_{\psi_f} \overbrace{\langle \psi_i | A^\mu \bar{\psi} \gamma_\mu \psi | \psi_f \rangle}^{\overline{\mathcal{M}}} \overbrace{\langle \psi_f | A^\nu \bar{\psi} \gamma_\nu \psi | \psi_i \rangle}^{\mathcal{M}}$$

In $\mathcal{M}$, $\psi, \bar{\psi}$ momenta $p, k-p$ must add to k of photon:



Black: way off-shell, but big phase space

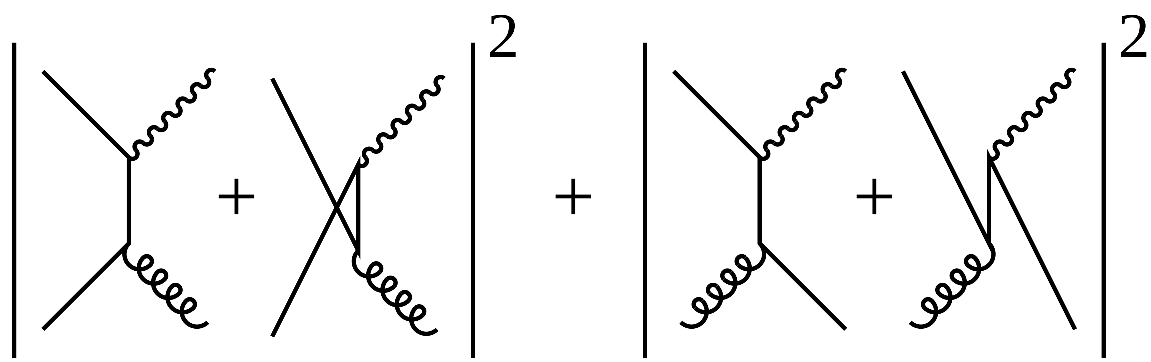
Blue: less phase sp, but soft enhancement

Red: both can be almost on-shell.

Call these regions Hard, Soft, and Collinear.

Hard case

If all momentum components (transverse and longitudinal) are large, physics is simple: short distance-and-time correlators, PQCD works. Loop corrections are $\mathcal{O}(g^2)$ and should get large around $T \sim 2T_C$.



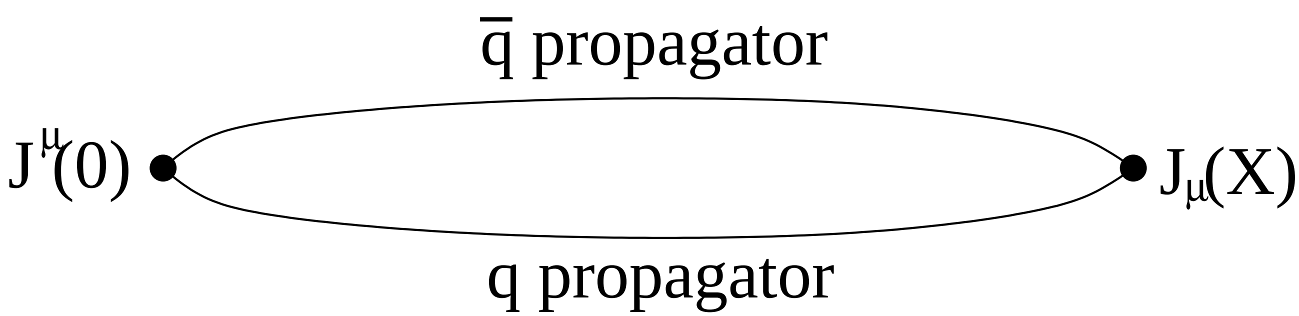
The challenge is the other two regions, where Pert. Thy. need not work as well.

Momentum-space vs Coordinate space

Momentum K lightlike $\not\longrightarrow$ lightlike X-separation:

$$f(K) = \int d^4 X e^{-iK\cdot X} f(X) \quad \text{involves } \textbf{all } X$$

But $q, \bar{q}$ start and end at same place:

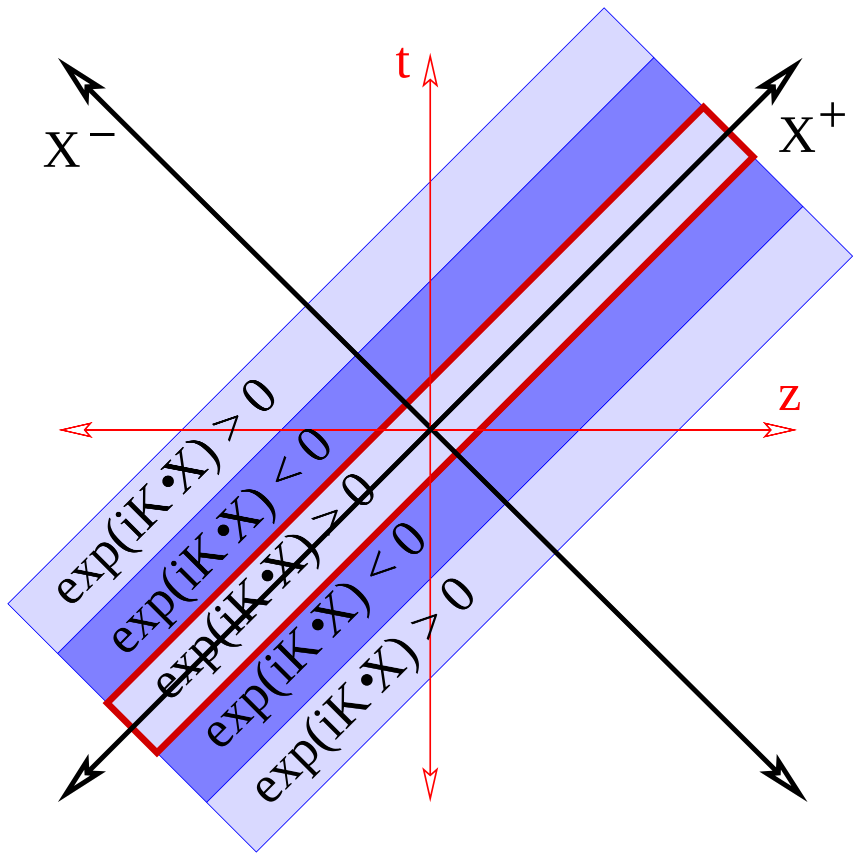


X determined by Fourier properties of P *and* $K - P$.

P small (or $p_\perp$ small): X (or $x_\perp$) large.

K big, but X in $\int d^4X \exp(-iK \cdot X)$ also big. How?

Need phase $\exp(-iK \cdot X)$ small. Occurs in narrow region. Write t, z as $X^- = (t - z)$, $X^+ = (t + z)/2$.



Since $-K \cdot X = K^+X^- + K^-X^+$, K^+ big, contribution is from region $X^- \simeq 0$ (Light Cone)

Lightcone correlators are Simple!

$x^- = 0$ $(x = t)$ is "Lightcone" of photon

Separation lightlike if $x_\perp = 0$, spacelike if $x_\perp \neq 0$.

Causality $\rightarrow$ only *pre-existing* correlators.

Unequal times usually means *Complicated Dynamics*.

Now ~~Complicated dynamics~~ Simple Thermodynamics!

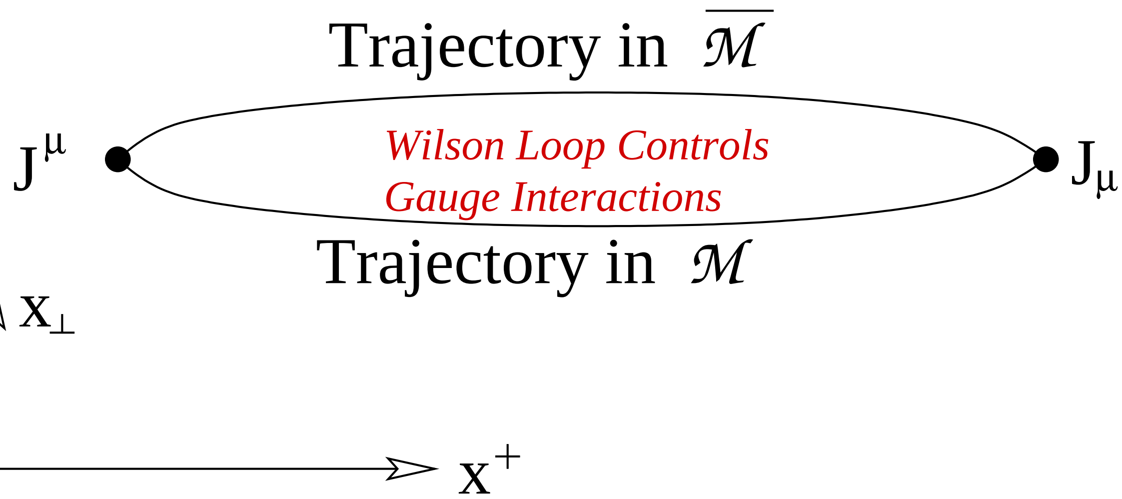
- Energy-dependent: Just Thermal Masses!
- Energy-independent: Classical (3-D theory) correlators!

Collinear case

Collinear $\Rightarrow$ almost on-shell $\Rightarrow$ large x separation

$x^- \ll x_\perp \ll x^+$ $(1/T \ll 1/gT \ll 1/g^2T)$

Consider *spacetime trajectory* of $q, \bar{q}$:



Need $x_\perp$-separated Wilson loop.

Spacetime picture pioneered by B. Zakharov, hep-ph/9607440,9807540

Nontrivial analysis B. Zakharov, BDMPS, AMY

γ-rate, quark stat-functions, DGLAP split-kernel

$$\frac{dN_\gamma}{d^3\mathbf{k}d^4x} = \frac{\alpha_{\rm EM}}{\pi^2 k}\int_{-k/2}^{\infty}\frac{dp^+}{2\pi} n_f(k{+}p)\,[1{-}n_f(p)]\,\frac{p^2+(p{+}k)^2}{2[p\,(p{+}k)]^2} \times \lim_{\mathbf{x}_\perp\to 0} 2\,{\rm Re}\,\partial_{\mathbf{x}_\perp}\mathbf{f}(x_\perp)$$

$$2\nabla_\perp\delta^2(x_\perp) = \left[\mathcal{C}(x_\perp) + \frac{ik}{2p^+(k{+}p^+)}(m_\infty^2+\nabla_{x_\perp}^2)\right]\mathbf{f}(x_\perp)$$

J-operator, Wilson-line, $x_\perp$-diffusion

$\mathbf{f}(x_\perp)$: density matrix $|\psi_{P+K}\rangle\langle\gamma_K\psi_P|$ or $|\psi_P\bar\psi_{K-P}\rangle\langle\gamma_K|$

Eikonal evolution (Evolution in x^+) – $x_\perp$ diffusion, *AND*
Wilson-loop interaction with medium $\mathcal{C}(x_\perp)$.

$\mathcal{C}(x_\perp)$ is Euclidean!

$\mathcal{C}(x_\perp)$: Wilson loop with space-separated lightlike lines. All points at spacelike or lightlike separation.

Soft contribution is Euclidean!! S. Caron-Huot, 0811.1603

Calculate it with *simple* perturbation theory (EQCD)

Calculate it on the lattice?!

NLO corrections to $\mathcal{C}(x_\perp)$ computed. NNLO would be nonperturbative; possible via lattice Panero Rummukainen

For us: stick $C_{NLO}(x_\perp)$ into Eq $\Rightarrow$ Get NLO answer.

How Things Get Euclidean

S. Caron-Huot

Consider correlator $G^<(x^0,\mathbf{x})$ with $x^z > |x^0|$. Fourier representation

$$G^<(x^0,\mathbf{x}) = \int d\omega \int dp_z d^2p_\perp e^{i(x^z p^z + \mathbf{x}_\perp\cdot\mathbf{p}_\perp - \omega x^0)} G^<(\omega, p_z, p_\perp)$$

Use $G^<(\omega,\mathbf{p}) = n_b(\omega)(G_R(\omega,\mathbf{p}) - G_A(\omega,\mathbf{p}))$ and define $\tilde{p}^z = p^z - (t/x^z)\omega$:

$$G^< = \int d\omega \int d\tilde{p}^z d^2p_\perp e^{i(x^z\tilde{p}^z + \mathbf{x}_\perp\cdot\mathbf{p}_\perp)} n_b(\omega)\left(G_R(\omega,\tilde{p}^z + \omega\frac{x^0}{x^z},\mathbf{p}_\perp) - G_A\right)$$

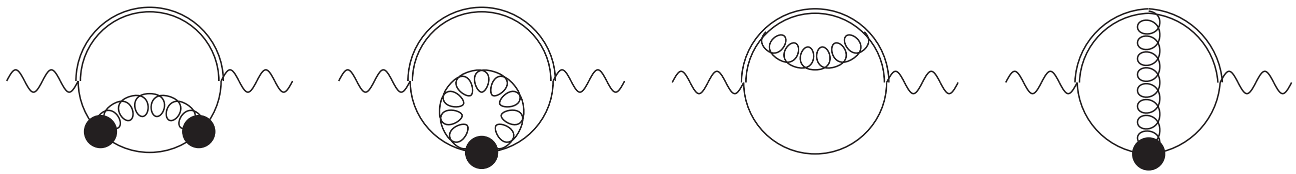
Perform ω integral: upper half-plane for G_R, lower for G_A, pick up poles from n_b:

$$G^<(x^0,\mathbf{x}) = T\sum_{\omega_n=2\pi nT}\int dp^z d^2p_\perp e^{i\mathbf{p}\cdot\mathbf{x}} G_E(\omega_n, p_z + i\omega_n(x^0/x^z), p_\perp)$$

Large separations: $n \neq 0$ exponentially small. $n=0$ contrib. is x^0 independent!

Soft momenta

Start with brute force: do the diagrams



Cut hard line: $p^- \simeq 0$, hard-line approx. p^+ independent.

Remaining integrals (using KMS) (P, Q are resp. soft quark, gluon momenta)

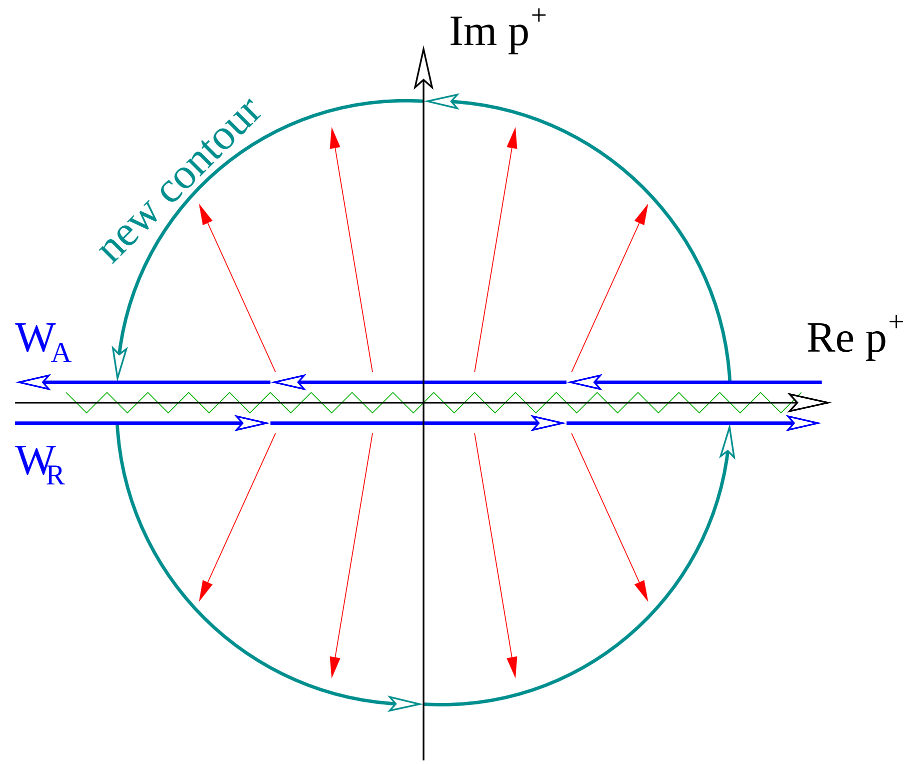
$$\int_{\sim gT} d^2p_\perp dp^+ \int_{\sim gT} d^4Q n_b(k^0)(G_{\rm R} - G_{\rm A})$$

$G_{\rm R}$: retarded function of sum of all 4 diagrams' guts.

Momentum p^+ is **null**. Any R/A function is analytic in upper/lower half plane for time-like **or null** p-variable.

Analytically continue in p^+!!

Deform p^+ contour into complex plane



Now $p^+ \gg p_\perp, Q$. (On mass-shell) Expand in $p^+ \gg p_\perp, Q$

$$G_{\mathrm{R}}[\text{4 diagrams}] = C_0(p^+)^0 + C_1(p^+)^{-1} + \ldots$$

C_0 is on-shell width, gives linear in p^+ divergence.

C_1 is on-shell dispersion correction, dp^+/p^+ gives const.

Huh? Continuation possible because J^μ light-cone separated. And light-cone correlators are simple!

- C_0 **term:** Exactly the limit of collinear calculation when one quark momentum gets small. Already included.

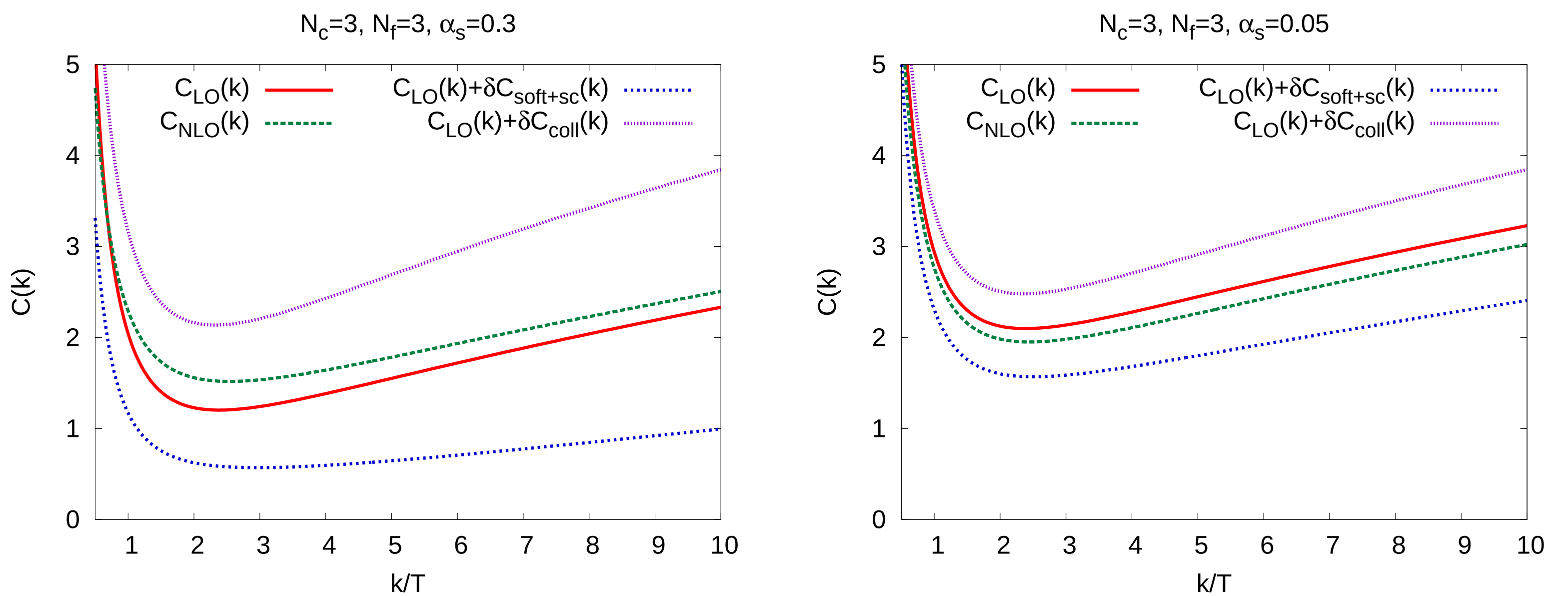
- C_1 **term:** real dispersion-correction. Really simple:

$$\gamma\text{-rate} \propto \int \frac{d^2 p_\perp}{(2\pi)^2} \frac{m_\infty^2}{p_\perp^2 + m_\infty^2}$$

  where m_∞^2 is dispersion correction. Has leading-order piece (hard modes) and subleading piece (dispersion correction of soft modes). *both are known.*

Remaining region–similar story. Null-separation physics, all condensates.

Summing it up: two corrections



Upward correction: more scattering at NLO.

Downward correction: fewer soft gluons, less dispersion corr.

Numerical conspiracy: effects nearly cancel [Accidental!]

Main lesson

All the sticky IR physics shows up in a few condensates.

Some are dispersion corrections – physically simple.

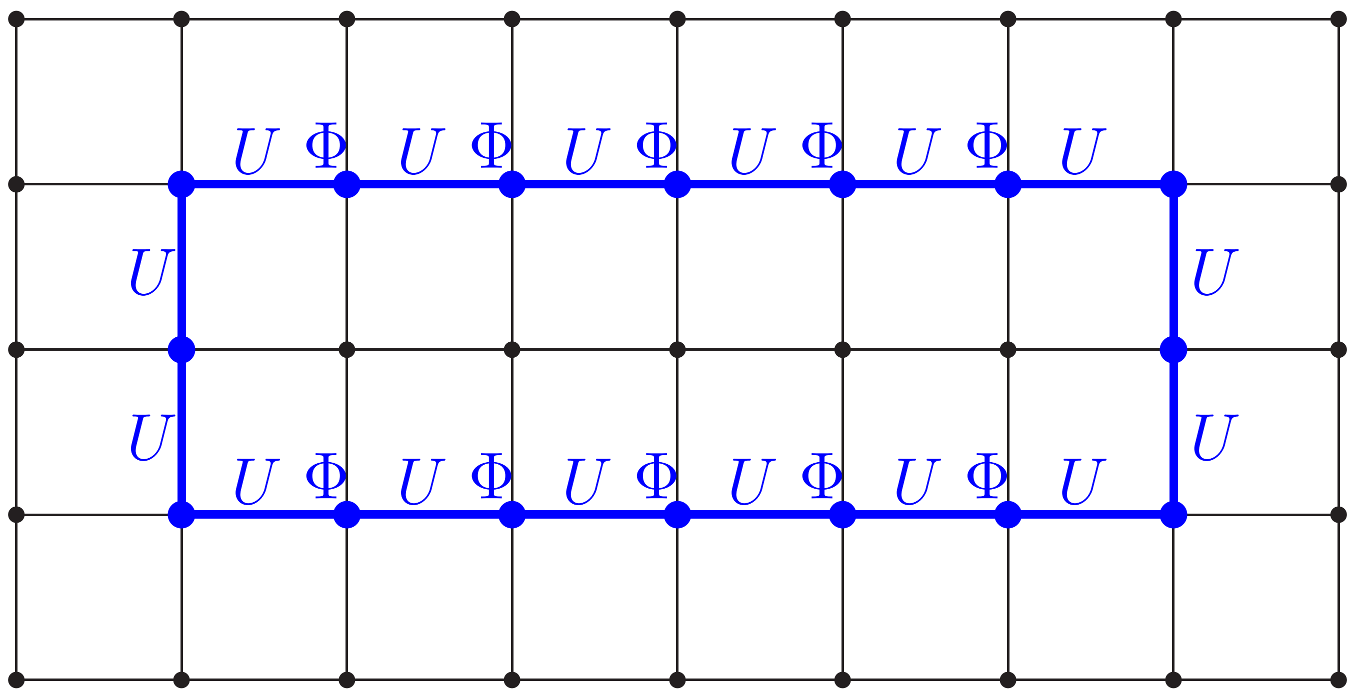
Some are Euclidean – get directly on the lattice.

Bad news: $\mathcal{O}(g)$ corrections big even for $\alpha_s = 0.1$ or $1000\, T_c$. Sort of expected that.

Good news: A few condensates. Determine them nonperturbatively, maybe get down to few T_c?

Get them on the lattice?

$\mathcal{C}(x_\perp)$ on the lattice



Short side: $x_\perp$ Wilson line $\exp \int iA_\perp \cdot x_\perp \Rightarrow U_\perp U_\perp \ldots$

Long side: x^+ Wilson line $\exp \int i(A^z + A^0) dz \Rightarrow U_z e^{a\Phi} U_z e^{a\Phi} U_z \ldots$

The latter is a new beast. Lattice renormalization properties?

In Preparation.

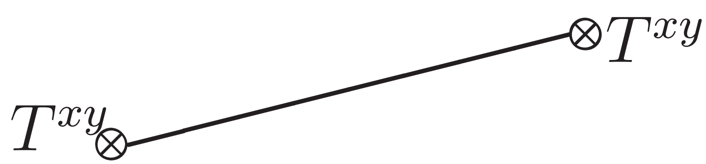
Other transport coefficients?

We want Baryon Diffusion D and (especially) shear η!

Both controlled by high-energy $E =$ several T particles

Lightlike correlators should again dominate:

T^{xy} disturbs a particle

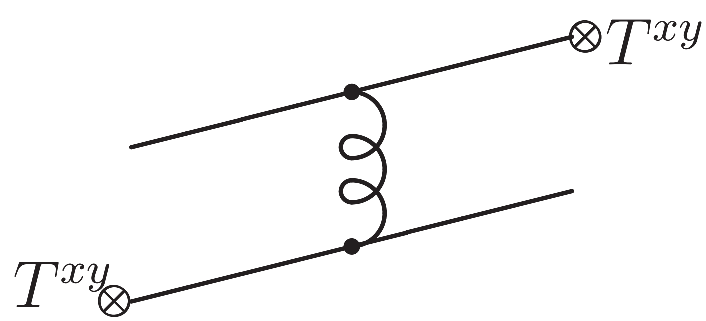


It induces a T^{xy}

NLO effects arise along particle's lightlike trajectory.

Problem: transfer of stress to someone else

Lines are timelike separated



There are $\mathcal{O}(g)$ corrections. Dang!

Conclusions

- Photon production is worth computing
- "Enhanced" Pert. calculation – few T_C??
- NLO corrections to transport are *large but simple*
- Need a few correlators at lightlike-separated points
- Most can be extracted from the lattice
- Shear and diffusion will be harder. Stay tuned