This is the solution to the Riddler problem for 15 September 2017. There are four questions:

1. Break a stick at two random points. What is the probability that the three pieces can form a triangle?
2. Pick three sticks of length randomly distributed from 0 to 1. What is the probability that they can form a triangle?
3. Repeat the first but requiring the triangle to be acute.
4. Repeat the second but for an acute triangle.

The condition that three pieces form a triangle is that the longest piece is shorter than the sum of the shorter pieces. For an acute triangle, it is that the sum of the squares of the shorter lengths exceeds the square of the longer length.

1. Call the points where the stick can be broken x and y. The choices for x, y represent the unit square. The three pieces have a total length 1, so a triangle is formed if none is longer than 1/2. The following triangles in the x, y plane are therefore excluded:

   - $x < y < 1/2$ third piece too long
   - $y < x < 1/2$ third piece too long
   - $x > y > 1/2$ first piece too long
   - $y > x > 1/2$ first piece too long
   - $x - y > 1/2$ middle piece too long
   - $y - x > 1/2$ middle piece too long

   each represent 1/8 of the square's area which is not allowed. That leaves $1 - 6/8 = 1/4$ of the square.

   So the answer is: 1/4

2. Call the longest length x, and the others y, z. There are 3 possibilities for which stick is longer. So the set of all possible stick lengths is represented by the integral

   $$3 \int_0^1 dx \int_0^x dy \int_0^x dz \,.$$

   Define $a = y/x$ and $b = z/x$, so this is rewritten as

   $$3 \int_0^1 x^2 dx \int_0^1 da \int_0^1 db \,.$$

   The condition that the sticks form a triangle is $y + z > x$, or equivalently, $a + b > 1$. Written in this way, the condition is independent of x, so we perform $3 \int_0^1 x^2 dx = 1$, and the problem becomes one of two variables in the unit square. The condition $a + b > 1$ restricts to the upper triangle, which is 1/2 the area of the square.

   so the answer is: 1/2

3. For side lengths $x > y > z$ we now need $x^2 < y^2 + z^2$ to make sure the angle is acute. (The case $x^2 = y^2 + z^2$ is a right angle by Pythagoras.) Therefore the triangle $y < x < 1/2$ is replaced by

$$y < x \qquad \text{and} \qquad (1-x)^2 < y^2 + (x-y)^2$$

which is the same as

$$x < \frac{1 - 2y^2}{2(1-y)}$$

The area of this shape (bounded by two straight lines and a curve) is

$$\int_0^{1/2} \frac{1-2y}{2-2y} dy = \frac{1 - \ln(2)}{2}$$

There are 6 shapes with this area cut out of the unit square, leaving an area of

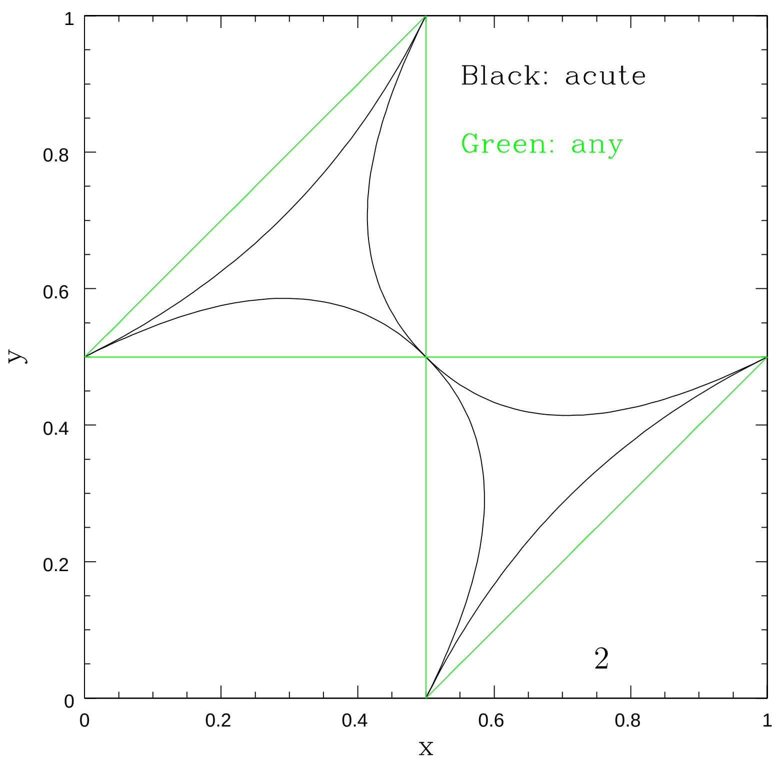
$$1 - 3(1 - \ln(2)) = 3\ln(2) - 2 = 0.07944154...$$

which is then the probability of an acute triangle.

4. This is much easier than 3. Now we need that part of the (a, b) unit square for which $a^2 + b^2 > 1$. This is the square with a quarter-circle excluded, which has area $1 - \pi/4$. so the answer is $1 - \pi/4 = 0.2146018366....$

Each problem came down to finding a fraction of the unit square where some criterion was true. Here are the pictures of the relevant fractions of the unit square:

For the case of breaking a stick in 2 places, the allowed areas in the x, y plane are those bounded by the black/green lines for the case of an acute/any triangle.



For the case of 3 sticks, in terms of the (a, b) plane (the fraction of the longest stick's length achieved by each shorter stick), the allowed regions are those above the black/green lines, again for an acute/any triangle.