The Similarity Renormalization Group

- Renormalization Group methods in nuclear physics

TECHNISCHE UNIVERSITÄT DARMSTADT

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Topics

- Introduction

- Flow equation

- Decoupling

- NN-interaction-evolution & Universality

- 3N-interaction-evolution

- Many-body-interaction-evolution & In-Medium SRG

Introduction

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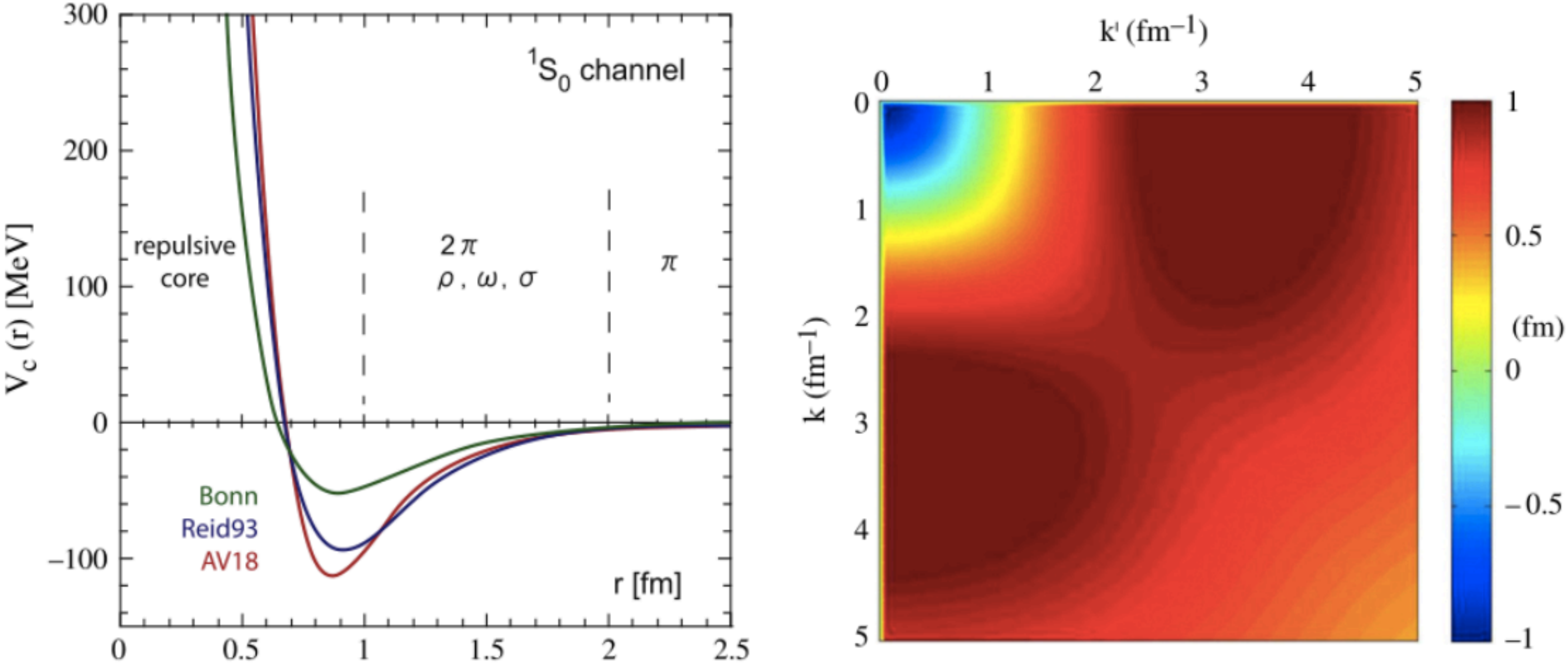


Left panel: three phenomenological potentials as function of interparticle distance that accurately describe proton-neutron scattering up to laboratory energies of 300 MeV. Right panel: alternative momentum space representation of the Av18 potential in the $^1S_0$ channel [2].

Introduction

- coupling of low & high momenta (repulsive core)

- slowly converging system (harmonic oscillator basis)

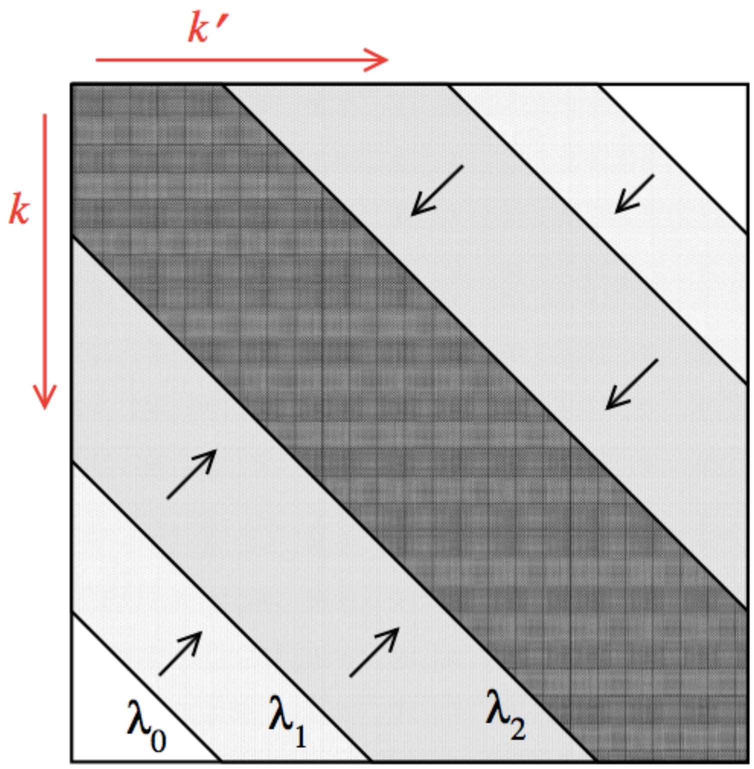
- expansive numerical calculations of energies

→ decoupling of low & high momenta

→ faster converging hamiltonian

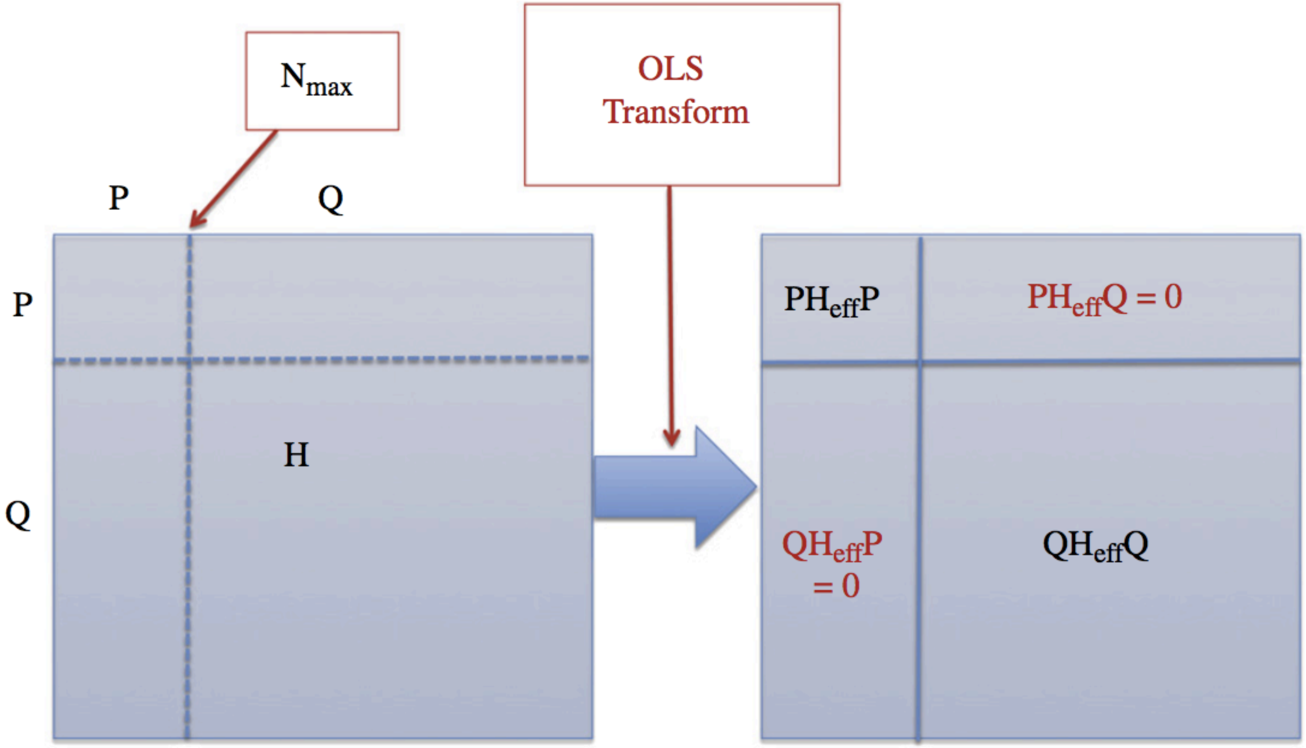
Introduction

- *Similarity Renormalization Group* (SRG) methode:



Introduction

- *Okubo-Lee-Suzuki similarity transformation* (OLS) methode:



Introduction

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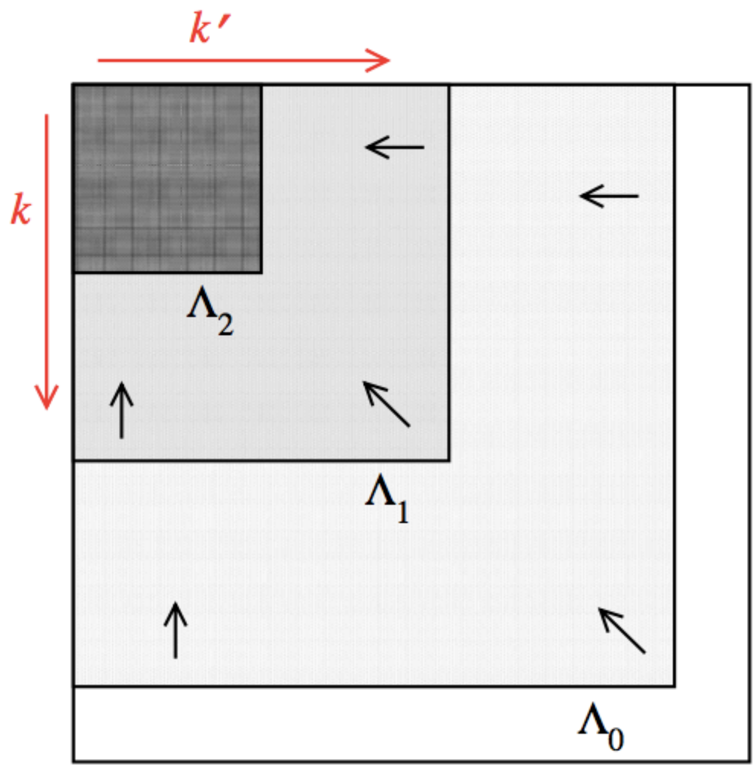
- $V_{\mathrm{low,k}}$ methode:



Flow equation

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$$H_s = U_s H U_s^\dagger = T + V_s$$

with $T = \frac{kk'}{2\mu} \cdot \delta^3(k' - k) = \kappa^2$

$$\frac{dH_s}{ds} = [\eta_s, H_s] = [[G_s, H_s], H_s]$$

with the anti-hermitian generator $\eta_s = \frac{dU_s}{ds} U_s^\dagger$

Decoupling

$$\frac{dH_s}{ds} = G_s H_s H_s - 2 H_s G_s H_s + H_s H_s G_s$$

$$\frac{dV_s}{ds} = 2(T V_s T - V_s T V_s) - T^2 V_s - V_s T^2 + T V_s^2 + V_s^2 T$$

inserting an $1 = \frac{2}{\pi} \int dq\ q^2$ to all V_s^2-terms

$$\frac{dV_s}{ds}(\kappa, \kappa') = -\left(\kappa^2 - \kappa'^2\right)^2\ V_s(\kappa, \kappa') + \frac{2}{\pi} \int dq\ q^2 \left(\kappa^2 + \kappa'^2 - 2q^2\right) V_s(\kappa, q) V_s(q, \kappa')$$

Decoupling

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$$\frac{dV_s}{ds}(\kappa,\kappa') = -\left(\kappa^2 - \kappa'^2\right)^2 \; V_s(\kappa,\kappa') + \sigma\left(V_s^2(\kappa,\kappa')\right)$$

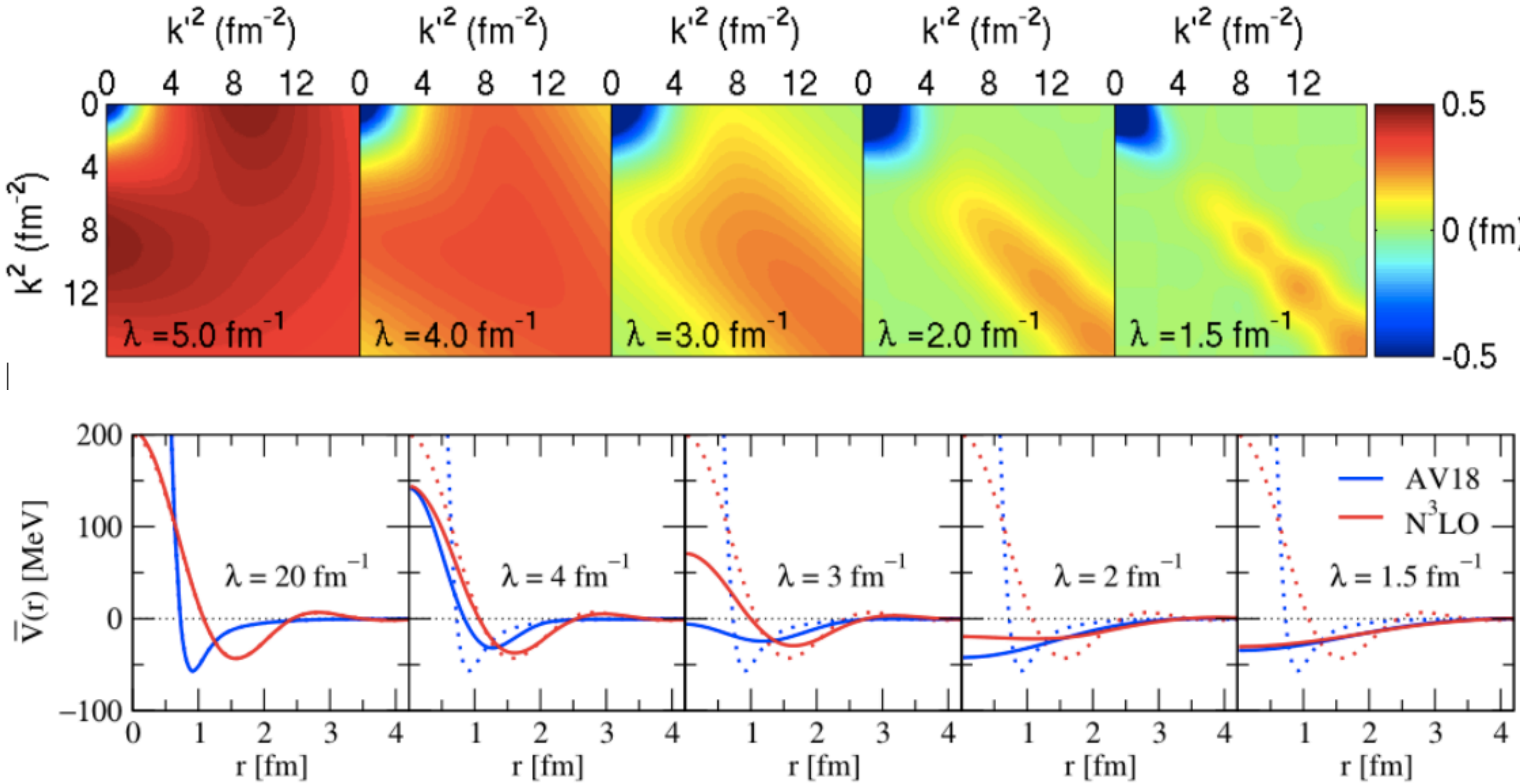
$$\Rightarrow \quad V_s(\kappa,\kappa') \approx V_{s=0}(\kappa,\kappa') \cdot \exp\left(-s\left(\kappa^2 - \kappa'^2\right)^2\right)$$

$$(s : 0 \to \infty)$$

$$\Rightarrow \quad V_\lambda(\kappa,\kappa') \approx V_{\lambda=\infty}(\kappa,\kappa') \cdot \exp\left(-\frac{1}{\lambda^4}\left(\kappa^2 - \kappa'^2\right)^2\right)$$

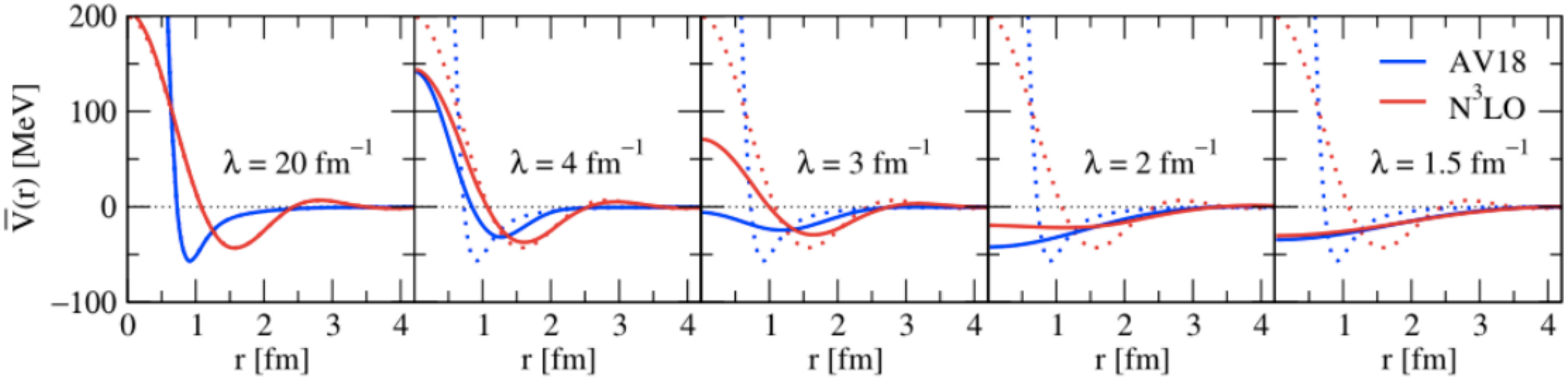
$$\left(\lambda = s^{-\frac{1}{4}} : \infty \to 0\right)$$

Decoupling & SRG-evolution



Top: The Av18 potential in the $^3S_1$ channel is evolved with SRG. Bottom: The local projections of Av18 and N^3LO (500 MeV) potential in $^3S_1$ channel at different resolutions [2].

NN-interaction-evolution & Universality



The local projections of Av18 and N^3LO (500 MeV) potential in $^3S_1$ channel at different resolutions [2].

- dissolution of the repulsive core to more attractive form
- shift of the long-range tail to a more attractive form

- NN-potentials converge to less repulsive forms
- evolved potentials vary hardly

3N-interaction-evolution

- Types of 3N-interaction SRG-treatments:

(i) full evolution of NN- & 3N-forces

(ii) evolution of NN-interaction & additional N2LO 3N-forces

(iii) evolution of 3N-forces in a continuous plane wave basis

3N-interaction-evolution

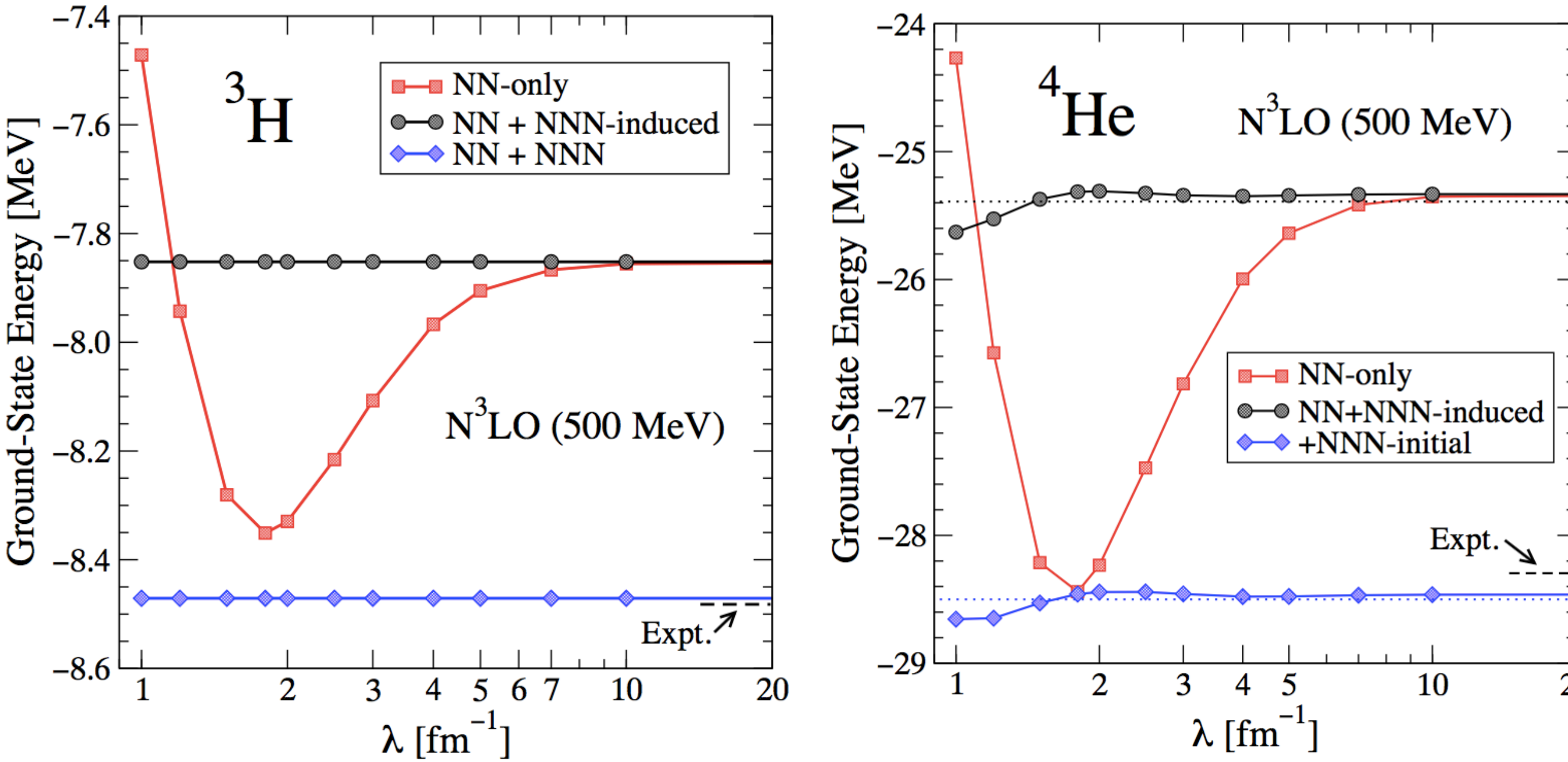
$$H_\lambda = T + \sum_{i,j=1}^{3} V_{ij} + V_{123} = T + V_\lambda$$

$$\Rightarrow \quad \mathrm{d}_\lambda V_\lambda = \mathrm{d}_\lambda V_{12} + \mathrm{d}_\lambda V_{23} + \mathrm{d}_\lambda V_{13} + \mathrm{d}_\lambda V_{123}$$
$$= \left[\left[T, V_\lambda\right], H_\lambda\right]$$

$$\Rightarrow \quad \mathrm{d}_\lambda V_{123} = [[T_{12}, V_{12}], V_{13} + V_{23} + V_{123}]$$
$$+ [[T_{13}, V_{13}], V_{12} + V_{23} + V_{123}]$$
$$+ [[T_{23}, V_{23}], V_{12} + V_{13} + V_{123}]$$
$$+ [[T, V_{123}], H_\lambda]$$

$$\Rightarrow \quad \mathrm{d}_\lambda V_{123}\,|_{V_{123}=0} \neq 0$$

3N-interaction-evolution



Ground-state energy of ^{3}H (left) and ^{4}He (right) as a function of the SRG evolution parameter λ for three different SRG flows [4].

Many-body-interaction-evolution & In medium-SRG

$$H_\lambda = \langle T\rangle a^\dagger a + \langle V_\lambda^{(2)}\rangle a^\dagger a^\dagger a a + \langle V_\lambda^{(3)}\rangle a^\dagger a^\dagger a^\dagger a a a + \sigma(V_\lambda^{(4)})$$

- SRG-evolution will induce A-body-forces, if applied in an A-body subsystem

- reference to a state of finite density instead of the vacuum

- 3- to A-body-forces by using 2-body machineries

References

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[3] S. K. Bogner, R. J. Furnstahl, A. Schwenk - *From low-momentum interactions to nuclear structure*, Progress in Particle and Nuclear Physics 65 (2010) 94-147

[4] E. D. Jurgenson, P. Navrátil, R. J. Furnstahl - *Evolution of Nuclear Many-Body Forces with Similarity Renormalization Group*, Physical Review Letter (2009), LLNL-JRNL-412933

[5] B. R. Barrett, P. Navrátil, J. P. Vary - Ab initio no core shell model, Progress in Particle and Nuclear Physics 69 (2013) 131-181