Science 285 (1999) 1703, 10 Sept. 1999

Onset of Fermi Degeneracy in a Trapped Atomic Gas

B. DeMarco and D. S. Jin*†

An evaporative cooling strategy that uses a two-component Fermi gas was employed to cool a magnetically trapped gas of 7×10^5 ^{40}K atoms to 0.5 of the Fermi temperature T_F. In this temperature regime, where the state occupation at the lowest energies has increased from essentially zero at high temperatures to nearly 60 percent, quantum degeneracy was observed as a barrier to evaporative cooling and as a modification of the thermodynamics. Measurements of the momentum distribution and the total energy of the confined Fermi gas directly revealed the quantum statistics.

Fermions, such as electrons, protons, and neutrons, compose all of the matter around us, and phenomena derived from the qu… degeneracy of Fermions are ubiquit… nature. Fermi-Dirac (FD) statistics g… the structure and behavior of such … systems as atoms, nuclei, electrons in … and white dwarf and neutron stars. Ho… Fermi systems are generally dens… strongly interacting. The only realizati… low-density Fermi sys… …resent has been a dil… …dis-

… temperature (11). Another prospect in thi… system is the possibility of a phase tran- … evaporative cooling … measuring the scaled temper… …on of the number of a…

cooling a gas of ^{40}K atoms

$$N \approx 10^5 - 10^6$$

$$T \approx 0.3\,\mu\mathrm{K} = 0.5\,T_F$$

properties of ^{40}K atoms

- fractional total spin ➽ fermion

$$F = \underset{I}{4} \pm \underset{S}{1/2} = \frac{9}{2}, \frac{7}{2}$$

- $(2F+1)$ magnetic sub-states: $-F \leq m_F \leq F$
- p-wave interaction among equal m_F atoms
  s-wave interaction only among non-equal m_F atoms

Trapping Mechanisms

magnetic trapping

- external magnetic field couples to the dipole-moment of the atoms
- inhomogeneity of the magnetic field causes a force that can be formulated as an anisotropic HO-potential

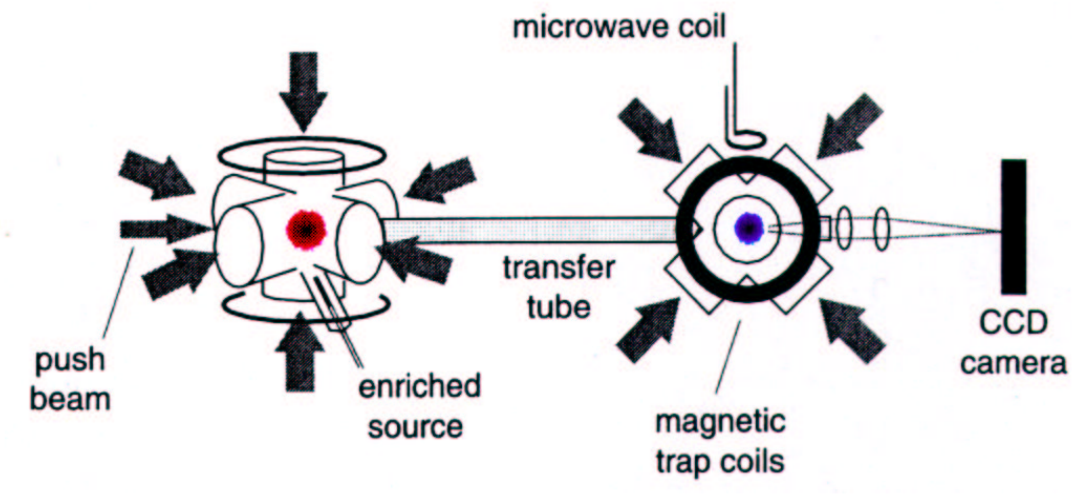
$$\ell_r = 0.8\,\mu\text{m} \qquad \ell_z = 3.6\,\mu\text{m} \qquad B_0 = 5\,\text{G}$$

- collisions with residual room temperature atoms limit the lifetime

$$\tau_{\text{trap}} \approx 300\,\text{s}$$

- the "condensate" is extremely dilute

$$n_{\text{trap}} \;\approx\; 10^{-2} n_{\text{air}} \;\approx\; 10^{-5} n_{\text{He-lq}}$$



...and a zoo of alternatives

- tightly confining quadrupole traps, wire traps
- optical dipole traps using off-resonance laser light

Cooling Strategy

forced evaporative cooling

- selective removal of "hot" atoms from the trap (by microwave induced transitions to an untrapped hf state)
- re-thermalization due to elastic binary collisions, cooling rate is given by thermalization rate

✔ effective cooling needs a mixture of two m_F states in order to allow s-wave scattering

✘ scatterings in a single component gas have p-wave character ➻ inefficient without further tricks...

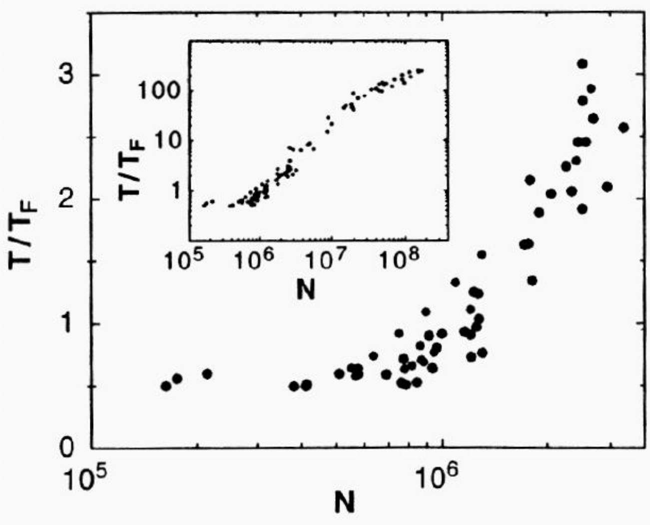


Fig. 3. Evaporation trajectory. A plot T/T_F versus N shows the result of evaporation; the inset displays the entire trajectory, starting at $T/T_F \approx 240$ and $N = 10^8$ atoms, and the main figure shows the low-temperature region. The bulk of the evaporation is very efficient, as seen in the large slope of the T/T_F versus N curve. However, the cooling process becomes limited at $T/T_F \approx 0.5$, where effects of FD statistics are observed in the momentum distribution of the gas.

[Science 285 (1999) 1703]

some other approaches

- sympathetic cooling of a boson/fermion mixture
- tuning of the p-wave collision rate by an external electric field (inducing a zero-energy resonance)

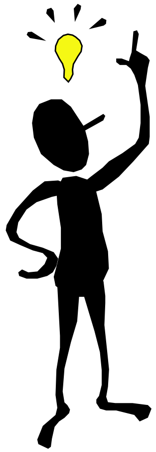
What Makes Its Attraction...

a macroscopic system which exhibits quantum properties

all relevant quantities are observable & tunable

size, density, particle number, mass, statistic, composition, temperature, distributions, interaction strength & type...

large composite bosons/fermions



meta stable many-body state

realization of a dilute Fermi gas

mean-field is appropriate

BE condensation $T_{\mathrm{BEC}} \sim \mu\mathrm{K}$

BCS transition $T_{\mathrm{BCS}} \sim \mathrm{nK}$

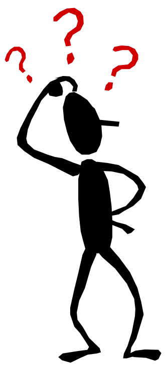
Some Questions to be Asked

static properties

- structure of the ground- & excited states, energy spectrum
- effects of quantum statistics & the atom-atom interaction
- mechanical stability of the condensate (separation & collapse)

dynamic properties

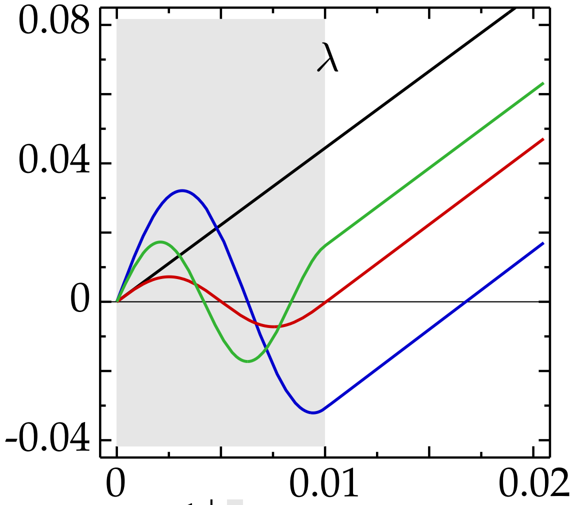
- dynamics of cooling
- ultracold collisions & trap loss processes
- collective excitations, vortex formation & evolution
- matter waves & atom lasers



What is a proper effective interaction to attack these questions?

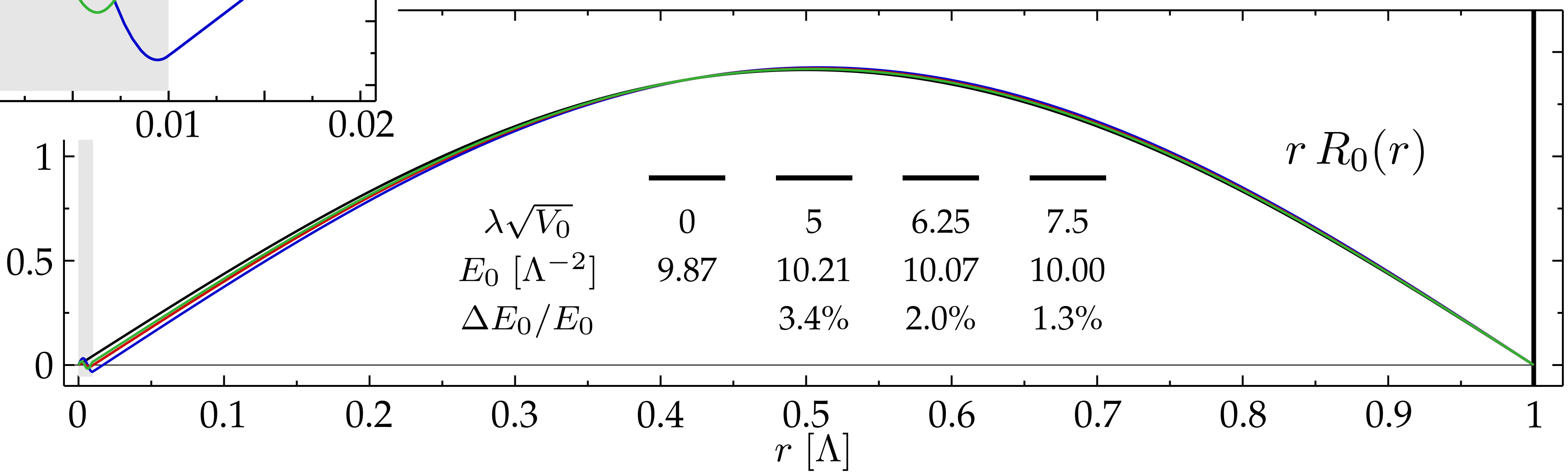
Example: Square-Well Potential

- two particles ($m = 1$) interacting via an attractive square-well potential of radius $\lambda = 0.01\Lambda$ and depth $V_0 > 0$
- boundary condition $R_l(\Lambda) = 0$ for radial wave function



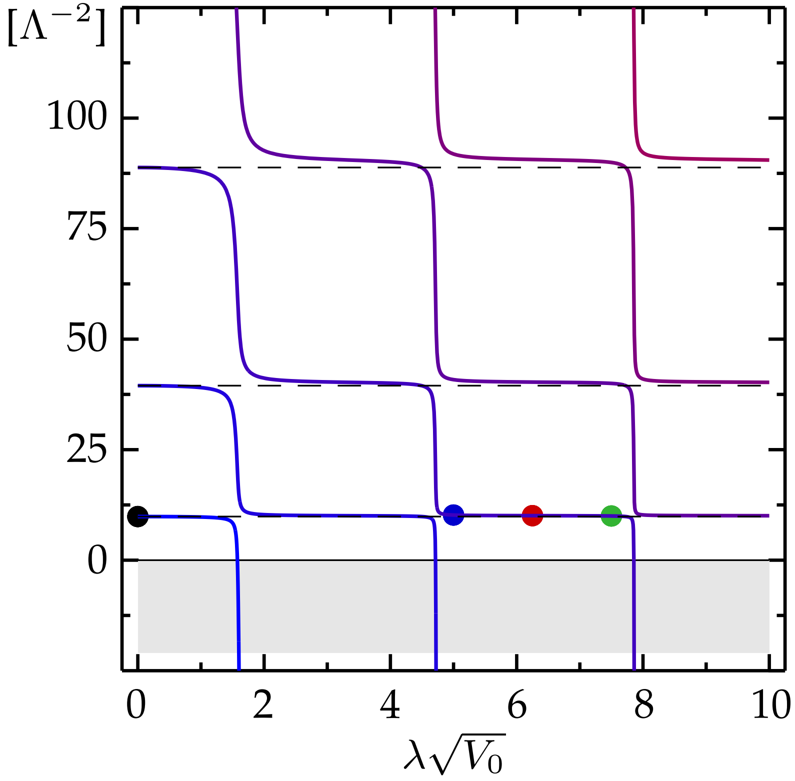
✘ structure of the *positive energy* eigenstates

$$\left[-\frac{1}{r}\frac{\partial^2}{\partial r^2} r + \frac{l(l+1)}{r^2} \right] R_l(r) = \begin{cases} (E_l + V_0)\, R_l(r) & ; r < \lambda \\ E_l \quad R_l(r) & ; r \geq \lambda \end{cases}$$



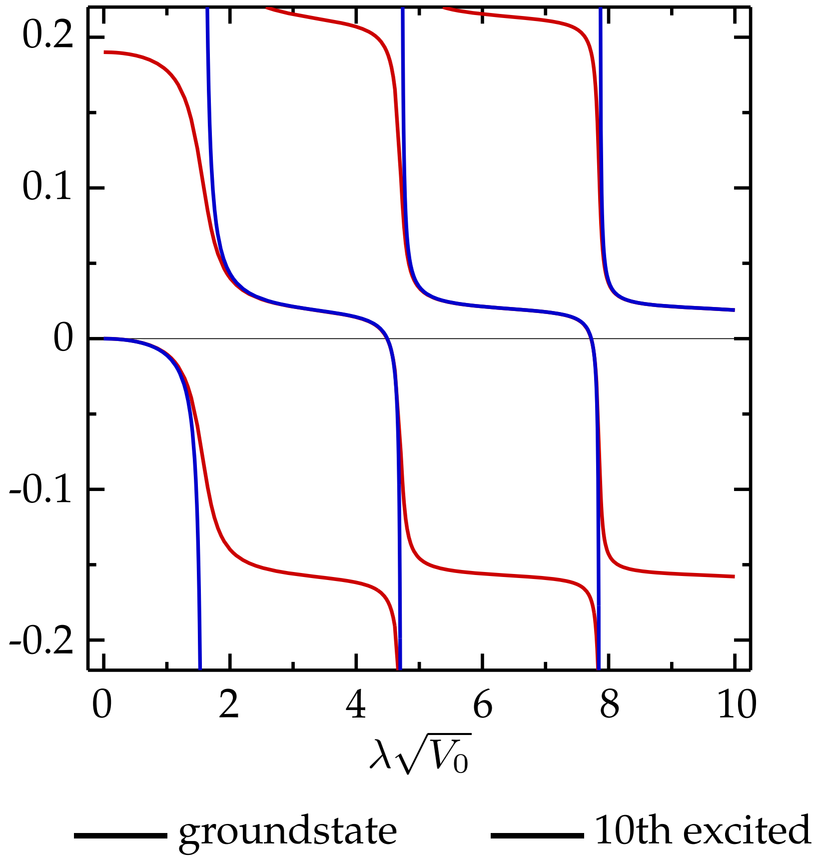
Example: Energy Spectrum & Energy Shift

E_0 versus $\lambda\sqrt{V_0}$

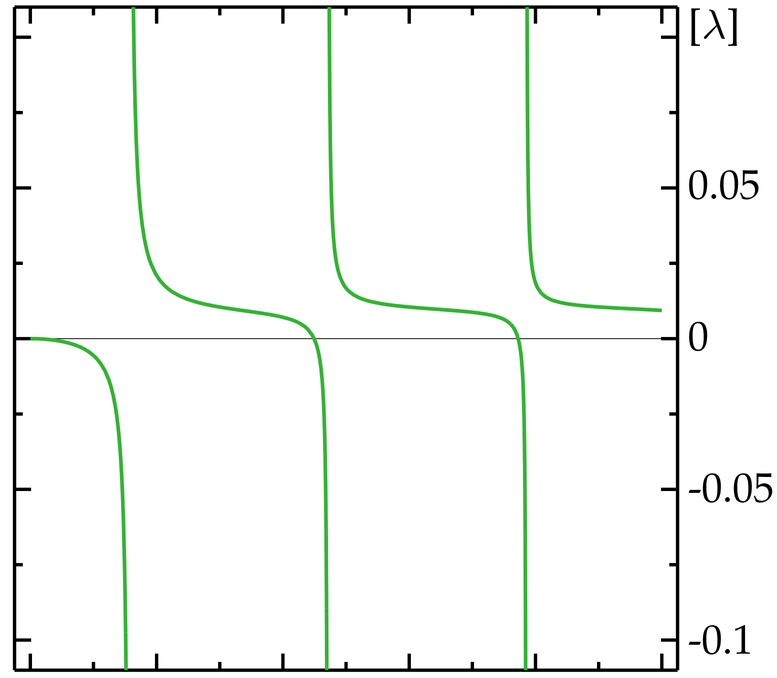


- extended plateaus near energy levels of the free system
- steep falls whenever the potential gains another bound state

$\Delta E_0/E_0$ versus $\lambda\sqrt{V_0}$



- $\Delta E_0/E_0$ is independent of the absolute energy within the plateaus

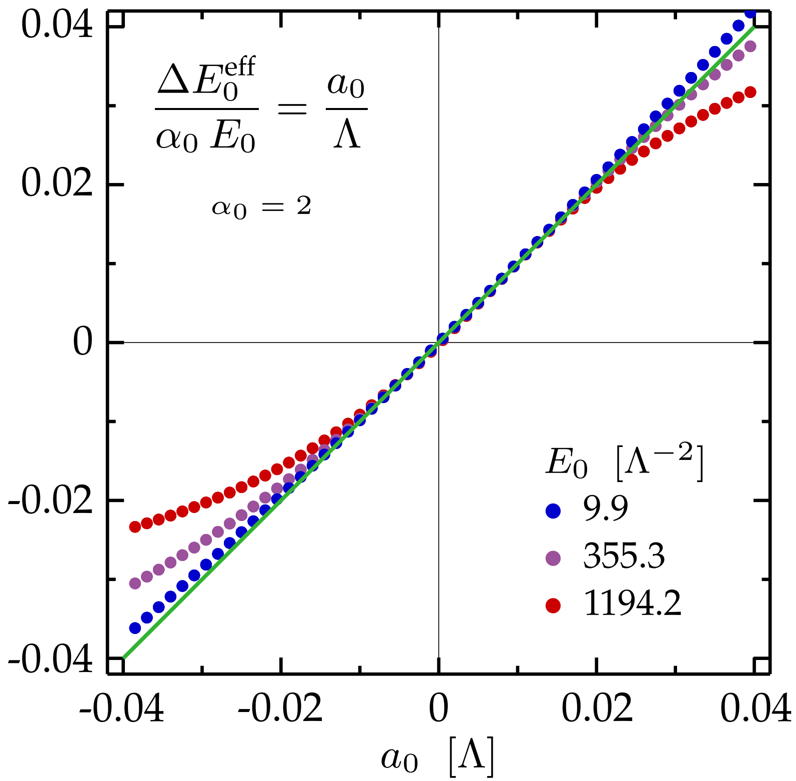


s-wave scattering length a_0

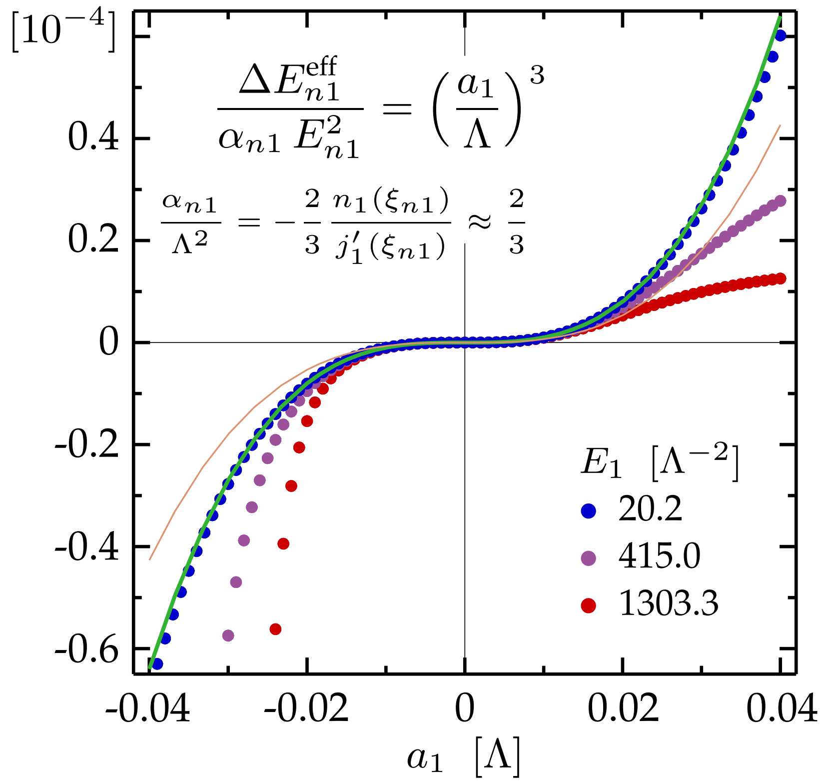
- $\Delta E_0/E_0 \propto a_0$

Example: Comparison with Exact Energy Shifts

✘ *s-wave energy shift*



✘ *p-wave energy shift*



• • • exact groundstate   • • • exact 5th excited   • • • exact 10th excited   —— effective interaction

- effective interaction gives very good description for low energies/momenta *or* for small scattering length
- 1% accuracy goal $\Rightarrow$ $|k\, a_0| < 0.1$ resp. $|k\, a_1| < 0.3$

A Proper Effective Interaction!?

formal requirements

- symmetries: translation, rotation, Galilean, parity, time reversal
- hermiticity

system properties

- low density

  $\rho^{-1/3} \gg$ “range of interaction”

- low temperature

  $k^{-1} \gg$ “range of interaction”

model space

- mean-field type model with a restricted model space that does not contain two-body correlations

relevant observables

- mean-field quantities like energy spectra, densities & related

effective interaction

- zero-range potential (for each partial wave)
- expectation value in two-body model-states equals the energy shift induced by the full interaction

Energy Shift in General

- relative asymptotic two-body wave function ($r > \lambda$)

$$R_{nl}(r) = A_{nl}\, j_l(k_{nl} r)$$

$$\bar{R}_{nl}(r) = \bar{A}_{nl}\left[j_l(\bar{k}_{nl} r) - \tan\eta_l(\bar{k}_{nl})\, n_l(\bar{k}_{nl} r)\right]$$

- boundary condition $R(\Lambda) = 0$

$$j_l(k_{nl}\Lambda) = 0 \quad \Rightarrow \quad k_{nl} = \xi_{nl}/\Lambda$$

$$j_l(\bar{k}_{nl} r) = \tan\eta_l(\bar{k}_{nl})\, n_l(\bar{k}_{nl} r)$$

- expansion in $\bar{k}_{nl}$ around the free momentum k_{nl} with $\Delta k_{nl} = \bar{k}_{nl} - k_{nl}$

$$\frac{\Delta k_{nl}}{k_{nl}} = \tan\eta_l(k_{nl})\, \frac{n_l(\xi_{nl})}{\xi_{nl}\, j_l'(\xi_{nl})} + \mathcal{O}[(\tfrac{\Delta k_{nl}}{k_{nl}})^2] + \mathcal{O}[\tfrac{\Delta k_{nl}}{k_{nl}}\ \tan\eta_l(k_{nl})]$$

- typical relative wave length is large compared to the interaction range

$$|k_{nl} a_l| \ll 1 \quad \rightarrow \quad |\tan\eta_l(k_{nl})| \ll 1 \quad \& \quad |\tfrac{\Delta k_{nl}}{k_{nl}}| \ll 1$$

➽ energy shift as function of the phase shifts for $|k_{nl} a_l| \ll 1$

$$\Delta E_{nl} = E_{nl}\ \tan\eta_l(k_{nl})\, \frac{2\, n_l(\xi_{nl})}{\xi_{nl}\, j_l'(\xi_{nl})} = -\frac{1}{2\mu}\, \frac{\tan\eta_l(k_{nl})}{k_{nl}}\, A_{nl}^2$$

Effective Contact Interaction

- Ansatz for the l-th partial wave

$$\mathbf{v}_l^{\text{eff}} = \int \mathrm{d}^3 r \, |\vec{r}\rangle \frac{\overleftrightarrow{\partial}^l}{\partial r^l} \; g_l \, \delta^{(3)}(\vec{r}) \; \frac{\overrightarrow{\partial}^l}{\partial r^l} \langle \vec{r}|$$

- free angular momentum eigenstates

$$\langle \vec{r}|nlm\rangle = R_{nl}(r) Y_{lm}(\Omega) \qquad R_{nl}(r) \overset{k_{nl} r \ll 1}{=} A_{nl} \frac{(k_{nl} r)^l}{(2l+1)!!}$$

- expectation value shall give the energy shift

$$\langle nlm | \, \mathbf{v}_l^{\text{eff}} \, | nlm \rangle = \frac{g_l}{4\pi} \left[\frac{l!}{(2l+1)!!} \right]^2 A_{nl}^2 \, k_{nl}^{2l} \overset{!}{=} \Delta E_{nl}$$

➽ interaction strength as function of phase shifts

$$g_l = -\frac{4\pi}{2\mu} \left[\frac{(2l+1)!!}{l!} \right]^2 \frac{\tan \eta_l(k_{nl})}{k_{nl}^{2l+1}}$$

- parameterization of the phase shifts for $k a_l \ll 1$

$$\frac{\tan \eta_l(k_{nl})}{k_{nl}^{2l+1}} \overset{k a_l \ll 1}{=} -\frac{(2l+1)}{[(2l+1)!!]^2} \, a_l^{2l+1}$$

➽ interaction strength as function of scattering length

$$g_l = \frac{4\pi}{2\mu} \frac{(2l+1)}{(l!)^2} \, a_l^{2l+1}$$

Summary: The Effective Contact Interaction

conception

- effective hermitian contact interaction that reproduces the exact (low-energy) spectrum within a restricted model space that does not contain two-body correlations
- assuming low density & temperature: $|k\, a_l| \ll 1$

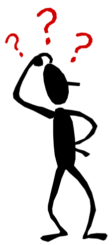
realization

$$\langle \vec{r}|\, \mathbf{v}_l^{\text{eff}}\, |\vec{r}\rangle = \frac{\overleftrightarrow{\partial}^{\,l}}{\partial r^l}\; g_l\, \delta^{(3)}(\vec{r})\; \frac{\overleftrightarrow{\partial}^{\,l}}{\partial r^l}$$

$$g_l = -\frac{4\pi}{m}\left[\frac{(2l+1)!!}{l!}\right]^2 \frac{\tan\eta_l(k_{nl})}{k_{nl}^{2l+1}} \approx \frac{4\pi}{m}\,\frac{(2l+1)}{(l!)^2}\; a_l^{2l+1}$$

application

- energy-density functional for bosonic & fermionic matter including p-wave interactions
- local density approximation for trapped ultracold Bose & Fermi gases, exploration of static properties
- dynamical description of cooling, expansion, vortex formation...



what's about
the *conventional*
pseudo-potential
of Huang et al. !??

Pseudo-Potential of Huang

[K. Huang, Statistical Mechanics, J. Wiley & Sons, 1963]

concept of the pseudo-potential

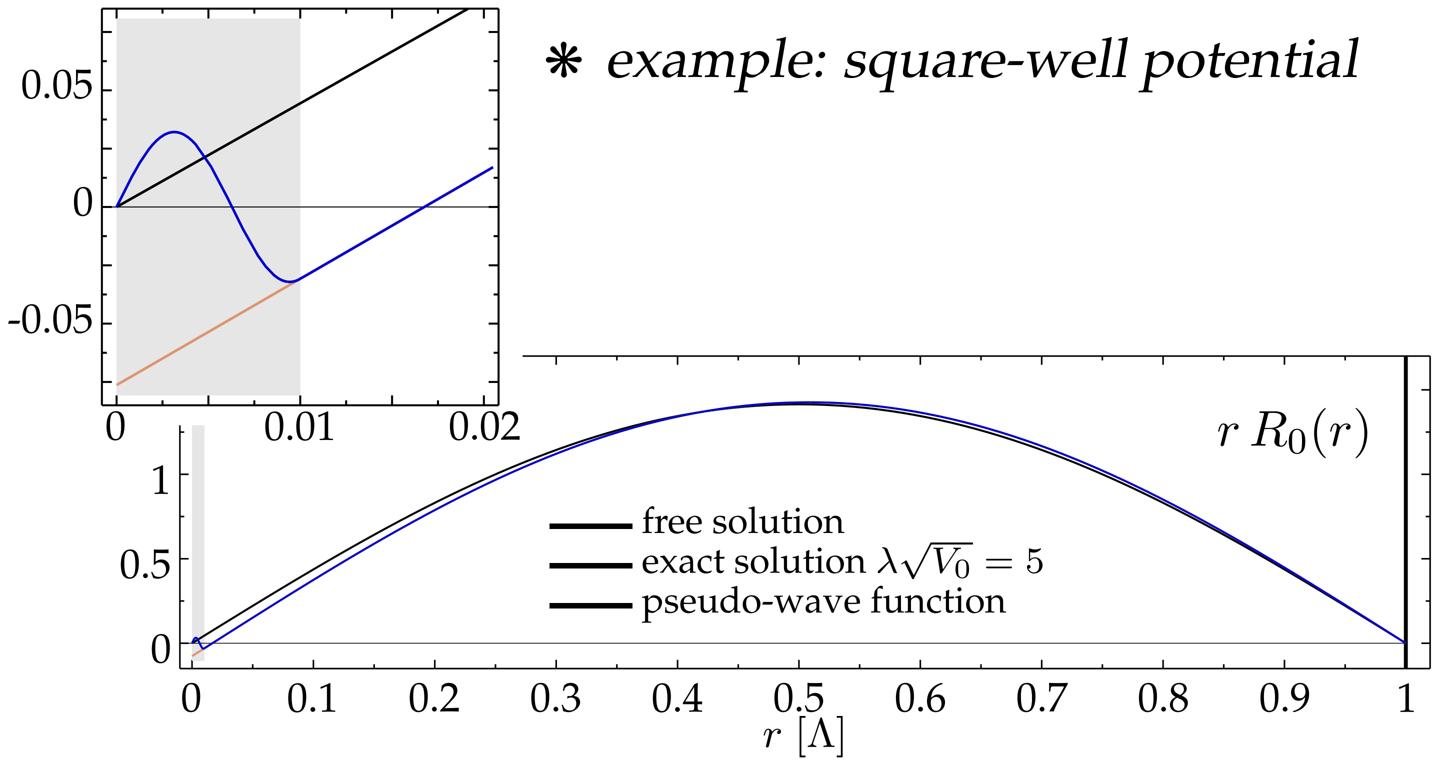
- reformulate the effect of the interaction as *boundary condition* for the relative wave function at $r = 0$
- such that the pseudo-wave function matches the exact one outside the potential range ➻ *phase-shift equivalent*
- inside it's the continuation of the asymptotic wave function, with distribution character near the origin ➻ *unphysical*

➻ "potential" described by a *non-hermitian* operator

$$\mathbf{v}_l^{\text{ps}} = \int \mathrm{d}^3 r \, |\vec{r}\rangle \, \frac{1}{r^l} \; \gamma_l \, \delta^{(3)}(\vec{r}) \; \frac{\overrightarrow{\partial}^{2l+1}}{\partial r^{2l+1}} \, r^{l+1} \, \langle \vec{r}|$$

$$\gamma_l = -\frac{4\pi}{2\mu} \, \frac{(l+1)\,[(2l+1)!!]^2}{(2l+1)\,(2l+1)!} \, \frac{\tan \eta_l(k_{nl})}{k_{nl}^{2l+1}}$$

✻ *example: square-well potential*



Energy Shift by Pseudo-Potential

- look at a two-body system described in a model space of free states with boundary condition $R_{nl}(\Lambda) = 0$

$$\langle \vec{r} | nlm \rangle = R_{nl}(r) Y_{lm}(\Omega) \qquad R_{nl}(r) \overset{k_{nl} r \ll 1}{=} A_{nl} \, \frac{(k_{nl} r)^l}{(2l+1)!!}$$

- the exact — *model independent* — energy shift for $|k\, a_l| \ll 1$ is given by

$$\Delta E_{nl} = -\frac{1}{2\mu} \, \frac{\tan \eta_l(k_{nl})}{k_{nl}} \, A_{nl}^2 \overset{!}{=} \Delta E_{nl}^{\text{eff}}$$

- energy shift induced by the pseudo-potential

$$\begin{aligned} \Delta E_{nl}^{\text{ps}} &= \langle nlm | \, \mathbf{v}_l^{\text{ps}} \, | nlm \rangle \\ &= \frac{\gamma_l}{4\pi} \, \frac{(2l+1)!}{[(2l+1)!!]^2} \, A_{nl}^2 \, k_{nl}^{2l} \\ &= -\frac{1}{2\mu} \, \frac{(l+1)}{(2l+1)} \, \frac{\tan \eta_l(k_{nl})}{k_{nl}} \, A_{nl}^2 = \frac{(2l+1)}{(l+1)} \Delta E_{nl} \end{aligned}$$

the pseudo-potential is
not a proper effective interaction
to describe energy related quantities!

LDA for Trapped Fermions

- take the energy-density for infinite matter & assume local homogeneity

$$\rho \to \rho(\vec{x}) \ , \quad \mathcal{E}(\rho) \to \mathcal{E}[\rho(\vec{x})]$$

- functional variation defines ground-state density $\rho(\vec{x})$

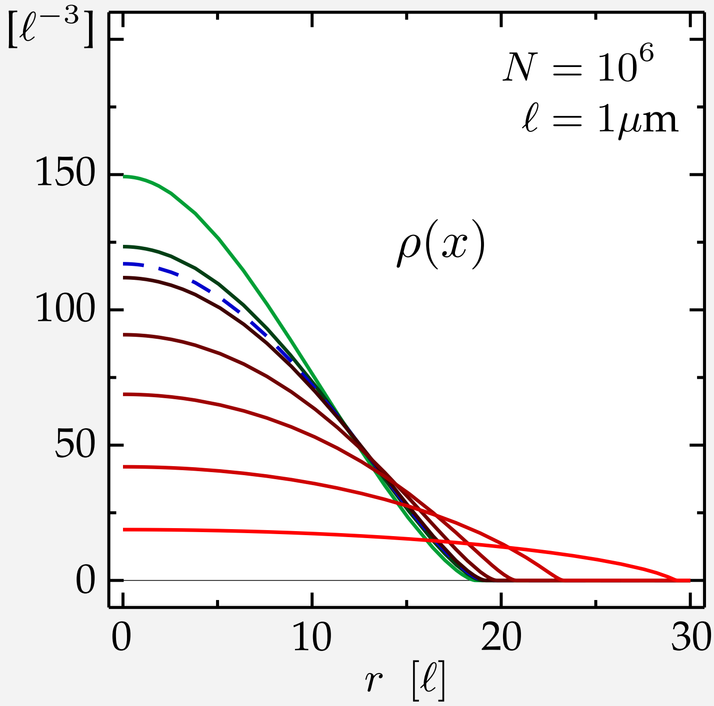
$$E = \int \mathrm{d}^3x \ \mathcal{E}[\rho(\vec{x})] \to \min$$

$\mathbf{H} =$	*1 component* $\mathcal{E}[\rho(\vec{x})] =$	*2 components* $\mathcal{E}[\rho_1(\vec{x}), \rho_2(\vec{x})] =$
$\mathbf{T}$	$\alpha_t \ \rho^{5/3}(\vec{x})$	$\alpha_t \ [\rho_1{}^{5/3}(\vec{x}) + \rho_2{}^{5/3}(\vec{x})]$
$+ \mathbf{V}_0^{\text{eff}}$		$+ \alpha_{v0} \ \rho_1(\vec{x}) \ \rho_2(\vec{x})$
$+ \mathbf{V}_1^{\text{eff}}$	$+ \alpha_{v1} \ \rho^{8/3}(\vec{x})$	$+ \alpha_{v1} \ [\rho_1{}^{8/3}(\vec{x}) + \rho_2{}^{8/3}(\vec{x})$ $+\frac{1}{2} \ \rho_1(\vec{x}) \ \rho_2{}^{5/3}(\vec{x}) + \frac{1}{2} \ \rho_1{}^{5/3}(\vec{x}) \ \rho_2(\vec{x})]$
$+ U(\vec{\mathbf{x}})$	$+ U(\vec{x}) \ \rho(\vec{x})$	$+ U(\vec{x}) \ [\rho_1(\vec{x}) + \rho_2(\vec{x})]$

$$\alpha_t = \frac{6^{5/3}\pi^{4/3}}{20m} \ , \qquad \alpha_{v0} = \frac{2\pi}{m} \ a_0 \ , \qquad \alpha_{v1} = \frac{6^{5/3}\pi^{7/3}}{5m} \ a_1^3 \ , \qquad U(\vec{x}) = \frac{1}{2m \, \ell^4} \ x^2$$

Some LDA Results

1 component gas: p-wave interaction



$a_1 \;\; [a_B]$

- - - 0

— 500, 1000, 1500, 2500, 5000

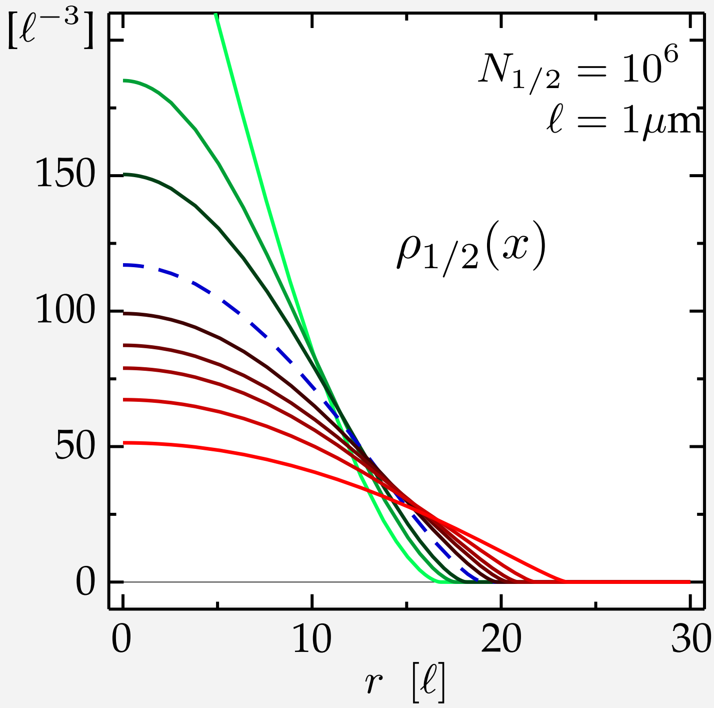
— -500, -750

- condensate stability condition for $a_1 < 0$

$$\sqrt[6]{N}\frac{|a_1|}{\ell} < 0.445$$

➽ here: $a_1 > -840a_\mathrm{B}$

2 component gas: s-wave interaction



$a_0 \;\; [a_B]$

- - - 0

— 500, 1000, 1500, 2500, 5000

— -500, -750, -1000

- condensate stability condition for $a_0 < 0$

$$\sqrt[6]{N}\frac{|a_0|}{\ell} < 0.546$$

➽ here: $a_0 > -1032a_\mathrm{B}$