Bose-Einstein Condensation:

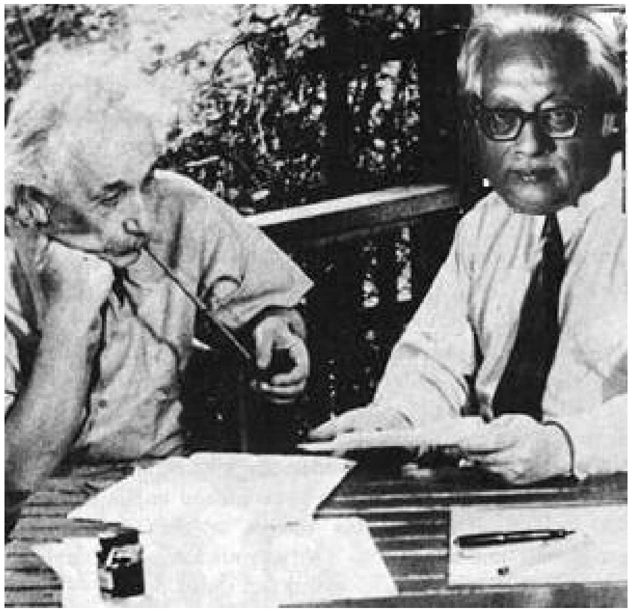
A New Kind of Matter

— or —

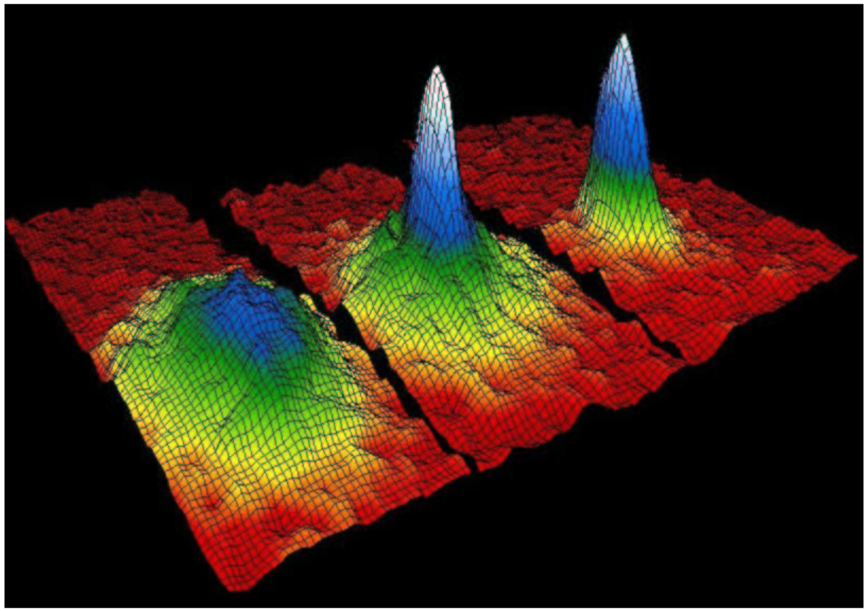
Fun with Ultracold Atomic Gases

R. Roth, GSI Theory Group

1924: There was a theory...



1995: ...and finally an experiment!



What is Bose-Einstein Condensation?

- History of BEC
- Remarks on Many-Body Quantum Mechanics
- BEC in an Ideal Bose-Gas
- BEC in Helium-4

The History of

Bose-Einstein Condensation

2. July 1924: A. Einstein translated a paper of S.N. Bose which contained a new derivation of Planck's radiation law based on a statistical treatment of light quanta

10. July 1924 / 8. Jan 1925: Einstein presented a similar treatment for an ideal gas of indistinguishable particles at the *preussische Akademie der Wissenschaften*. He predicted a new condensation phenomenon.

1938: Pyotr L. Kapitsa (Nobel Prize 1978) discovered the superfluidity of ^{4}He... the first experimental fingerprint of Bose-Einstein condensation in a dense system

5. June 1995: the advent of BEC in trapped ultracold dilute atomic gases...

^{87}Rb	5. June 1995	JILA (E. Cornell et al.)
^{7}Li	July 1995	Rice Univ. (R. Hulet et al.)
^{23}Na	Sept 1995	MIT (W. Ketterle et al.)
^{1}H	24. June 1998	MIT (D. Kleppner et al.)
^{4}He*	12. Feb 2001	ENS (A. Aspect et al.)

$\sim$ 150 groups world-wide are working on BEC in cold atomic gases...

Reminder I

Many-Body Quantum Mechanics

Many-Body Hilbert Space

- a N-body system is described by a state vector $|\Psi\rangle$ that is element of a N-body Hilbert space $\mathscr{H}$
- $\mathscr{H}$ can be decomposed according to symmetry under permutation of two particles

$$\mathscr{H} = \mathscr{H}_{+} \oplus \mathscr{H}_{-} \oplus \mathscr{H}_{\text{indef}}$$

- bosons: state is symmetric under perm.

$$|\Psi\rangle_{+} \in \mathscr{H}_{+} \quad \Leftrightarrow \quad \mathbf{P}_{ij}\,|\Psi\rangle_{+} = +\,|\Psi\rangle_{+}$$

- fermions: state is antisymmetric under perm.

$$|\Psi\rangle_{-} \in \mathscr{H}_{-} \quad \Leftrightarrow \quad \mathbf{P}_{ij}\,|\Psi\rangle_{-} = -\,|\Psi\rangle_{-}$$

- Spin-Statistic Theorem (QFT)

bosons	$\Leftrightarrow$	integer spin
fermions	$\Leftrightarrow$	half-integer spin

Fermions

- Pauli Principle: since the many-body state has to be antisymmetric two particles must not be in the same one-body state
- in the groundstate the particles fill the energetically lowest one-body states successively (non-interacting system)
  ➽ Fermi sphere

Bosons

- any occupation of the one-body states is allowed
- in the groundstate all particles occupy the energetically lowest one-body state (non-interacting system)
  ➽ Bose-Einstein Condensation

Quantum Statistical Mechanics

description of systems based on incomplete knowledge on their initial state

e.g. only expectation values of some observables (energy, particle number,...) are known

Conceptual: Density Operator

- state of the system is described by a density operator $\boldsymbol{\rho}$ rather than a state vector $|\Psi\rangle$

  pure state: $\boldsymbol{\rho}_{\text{pure}} \propto |\Psi\rangle\langle\Psi|$

  mixed state: $\boldsymbol{\rho}_{\text{mix}} \propto \sum_i w_i \, |\Psi_i\rangle\langle\Psi_i|$

- normalization and expectation values

$$1 = \text{Tr}(\boldsymbol{\rho}) \qquad \langle \mathbf{A} \rangle = \text{Tr}(\boldsymbol{\rho}\mathbf{A})$$

- under the constraint of our knowledge about the system (set of expectation values) the density operator has to maximize the entropy (measure of our ignorance)

$$S[\boldsymbol{\rho}] = -k_{\text{B}} \, \text{Tr}(\boldsymbol{\rho} \, \ln\boldsymbol{\rho}) \;\rightarrow\; \text{max.}$$

- standard ensembles result from that

Grand Canonical Ensemble

- assume the mean energy $E = \langle \mathbf{H} \rangle$ and the mean particle number $N = \langle \mathbf{N} \rangle$ are known

- grand canonical density operator follows from maximum entropy principle

$$\boldsymbol{\rho}_{\text{GC}} = \frac{1}{Z_{\text{GC}}} \exp[-\beta(\mathbf{H} - \mu\mathbf{N})]$$

$$Z_{\text{GC}} = \text{Tr}\{\exp[-\beta(\mathbf{H} - \mu\mathbf{N})]\}$$

  $\beta = 1/(k_B T)$ inverse temperature

  μ chemical potential

- bosonic/fermionic character enters in the evaluation of the partition sum Z_{GC}

- all relevant thermodynamic quantities can be determined from the derivatives of Z_{GC}

Bose-Einstein Condensation I

Particle Number

- non-interacting gas of bosons in a box of volume V with periodic boundary conditions
- logarithm of the partition sum of the grand canonical ensemble for identical bosons

$$\ln Z_{\text{GC}} = -\sum_k \ln\left[1 - z\exp(\beta\epsilon_k)\right]$$

$$z = \exp(\beta\mu) \qquad \text{fugacity}$$

$$\epsilon_k = \hbar^2 k^2/(2m) \qquad \text{single particle energies}$$

- expectation value of particle number N and occupation number n_k

$$N = \frac{1}{\beta}\frac{\partial}{\partial\mu}\ln Z_{\text{GC}}\Big|_{\beta,V} = \sum_k n_k$$

$$n_k = \frac{1}{z^{-1}\exp(\beta\epsilon_k) - 1}$$

- from $0 \leq n_k \leq N$ follows $0 \leq z \leq 1$

Integral Representation

- convert sum into an integral representation

$$\sum_k n_k \to \int \text{d}\epsilon\; g(\epsilon)\; n_k$$

$$g(\epsilon) = \frac{V(2m)^{3/2}}{(2\pi)^2\hbar^3}\sqrt{\epsilon}$$

- CAUTION: the sum contains a state with $k = 0$ that is not included in the integral representation since $g(0) = 0$!!!
- add number of particles in $k = 0$ explicitly

$$\begin{aligned} N &= N_0 + N_{\text{ex}} \\ &= N_0 + \int \text{d}\epsilon\; \frac{g(\epsilon)}{z^{-1}\exp(\beta\epsilon) - 1} \\ &= N_0 + V\,\lambda^{-3}\; f_{3/2}(z) \end{aligned}$$

$$\lambda = \sqrt{2\pi\hbar^2\beta/m} \qquad \text{thermal wave length}$$

$$f_{3/2}(z) \qquad \text{some polylog function}$$

Bose-Einstein Condensation II

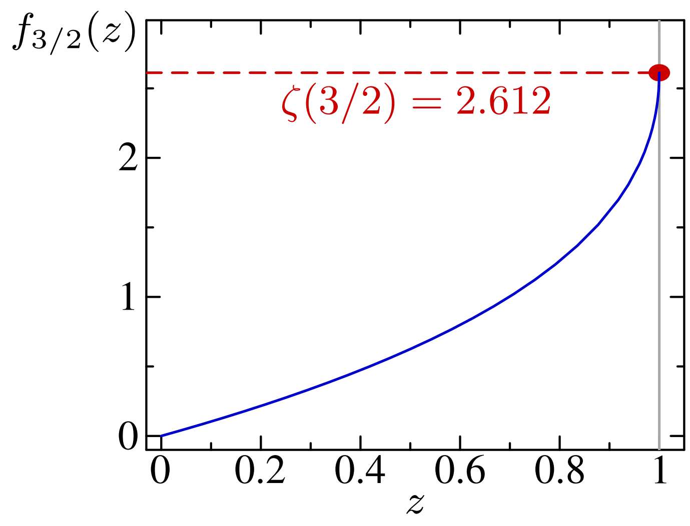
Occupation of $k > 0$ States

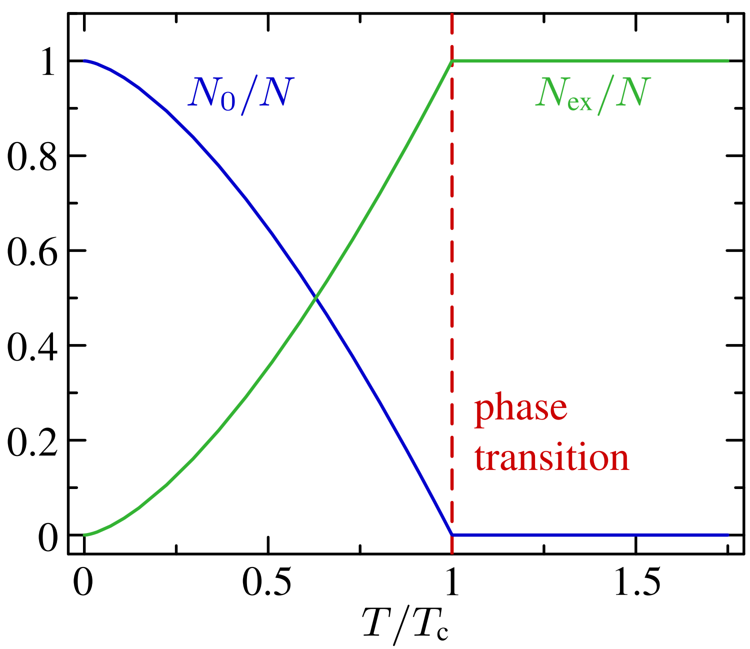
$$N_{\text{ex}} = V\,\lambda^{-3}\,f_{3/2}(z)$$

- because $f_{3/2}(z)$ has a maximum the particle number N_{ex} is limited for any given V and T

$$N_{\text{ex}} \leq N_{\text{ex}}^{\max} = 2.612\,V\,\lambda^{-3}$$

- BUT: where do the particles go if there are more than $N_{\text{ex}}^{\max}$ in the system ?...





Bose-Einstein Condensation

- "macroscopic" occupation of the ground state if

$$N > N_{\text{ex}}^{\max} \quad \Rightarrow \quad N_0 = N - N_{\text{ex}}^{\max}$$

- critical temperature for BEC from $N = N_{\text{ex}}^{\max}$

$$k_B T_c = \frac{2\pi\hbar^2}{(2.612)^{2/3}\,m}\left(\frac{N}{V}\right)^{2/3}$$

- ground state occupation as function of temperature

$$\frac{N_0}{N} = \begin{cases} 0 & ;T \geq T_c \\ 1 - (T/T_c)^{3/2} & ;T < T_c \end{cases}$$

Characteristics of BEC

BEC

a macroscopic fraction of the particles is in the same quantum mechanical state

Phase Transition

condensation of particles in momentum space

Coherence

long-range spatial order appears in the condensate

Superfluidity

flow without friction; energy-gap for elementary excitations

Complex Order Parameter

$$\Phi(\vec{x}) = \sqrt{n_0(\vec{x})}\, \mathrm{e}^{\mathrm{i}S(\vec{x})}$$

Irrotational Flow

rotation of the flow velocity ($\vec{v} \propto \vec{\nabla} S(\vec{x})$) vanishes; except...

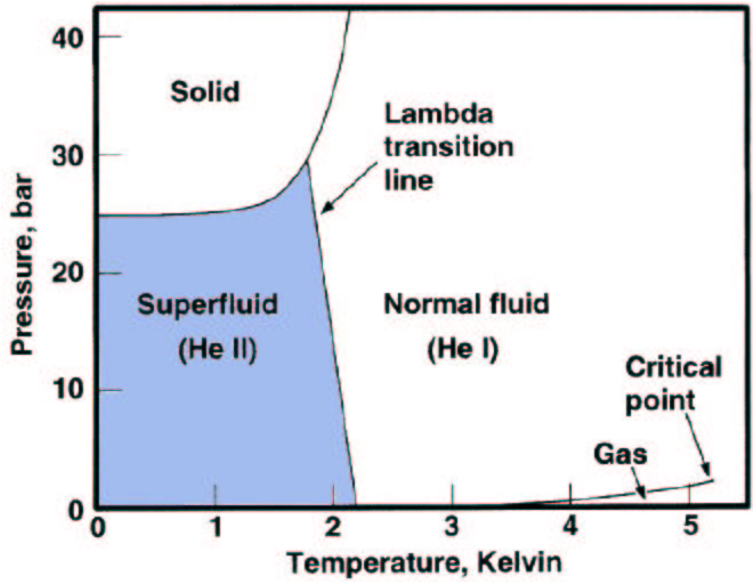
Quantized Vortices

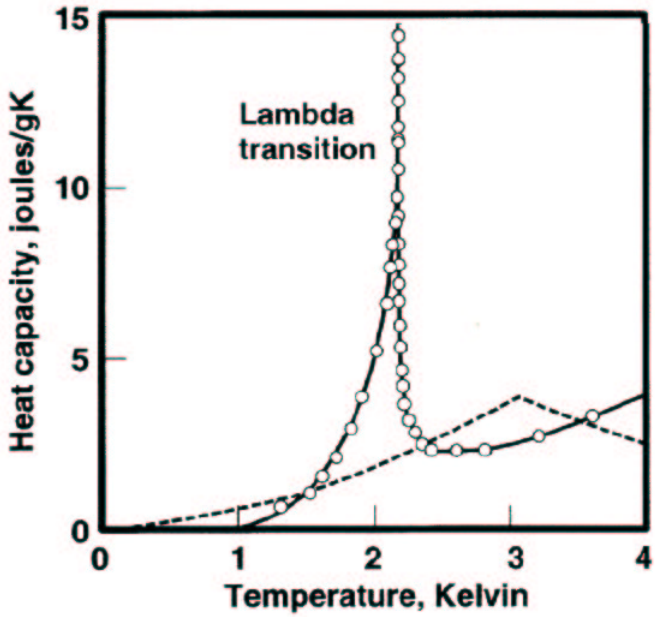
phase may change by multiples of 2π along a closed path

The Liquid Helium-4 Era

Superfluidity in Helium-4

- Helium is the only element that remains liquid for $T \to 0$K (at atmospheric pressure)
- ^{4}He is the only "conventional" candidate to observe BEC/superfluidity in a cold atomic system
- 1938: Pyotr L. Kapitsa (Nobel Prize 1978) discovered the superfluidity of liquid Helium II





Problems with Helium-4

- Helium liquids are dense and the atoms feel a strong two-body interaction (Lennard-Jones potential)
- strong interactions change the low-temperature properties dramatically compared to the ideal system
- due to the interactions only $\sim 10\%$ of the atoms are in the $k = 0$ state even at $T = 0$K
- effects of quantum statistics and interaction mix up in a complicated way... theory difficult... experiment too.

1995: The Beginning of a New Era

Ultracold Atomic Gases

Imagine you could...

...gather millions of atoms in a trap made of light and magnetic fields

...use different bosonic species and even mixtures

...choose the density and temperature you wish

...decide whether the interaction should be attractive, repulsive or vanishing

...tune the interaction strength and see how the BEC responds

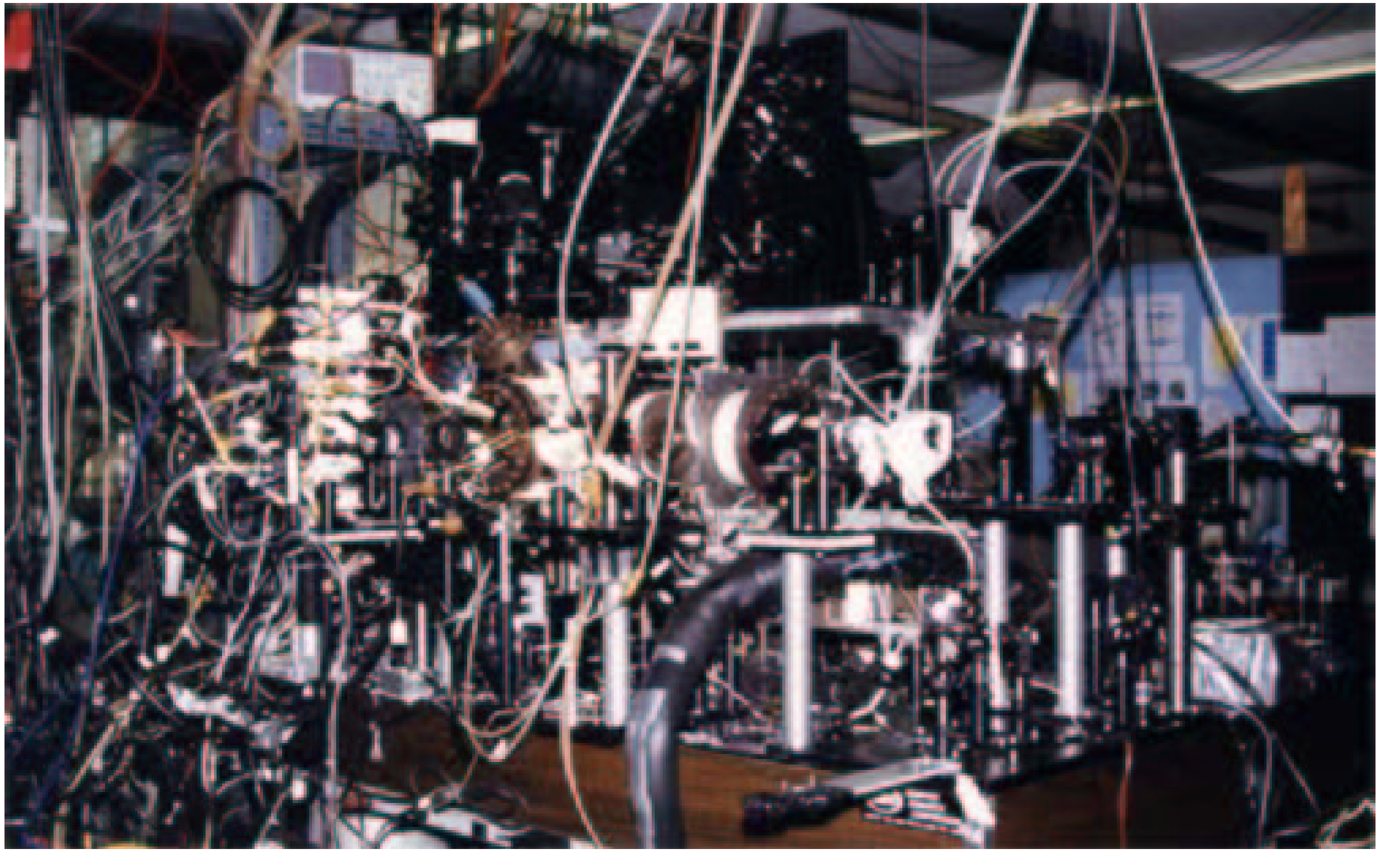
...manipulate the BEC with knifes and tweezers of light and radio waves

...watch the time evolution of collective exactions and vortices

...see quantum mechanics at work with the naked eye

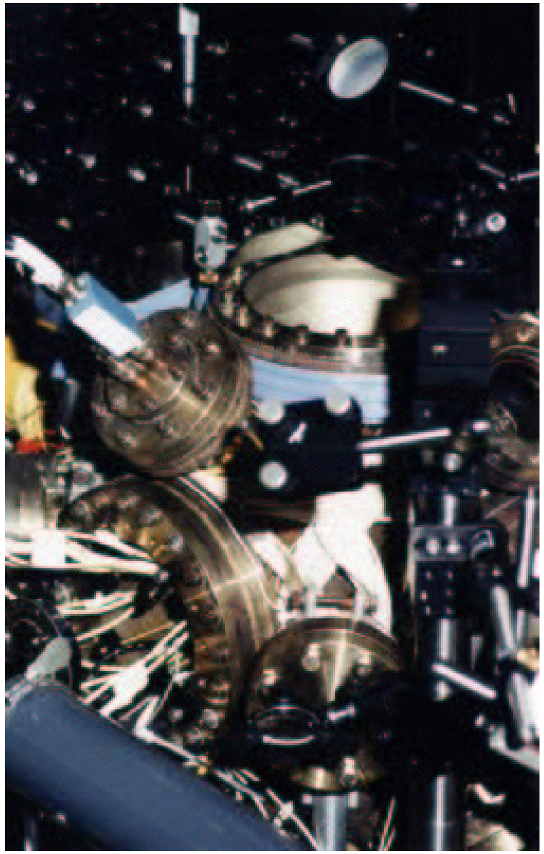
That's How It Looks in the End...

- *Full Setup*



^{23}Na setup @ MIT; *W. Ketterle, et al.*

- *Cold MOT*



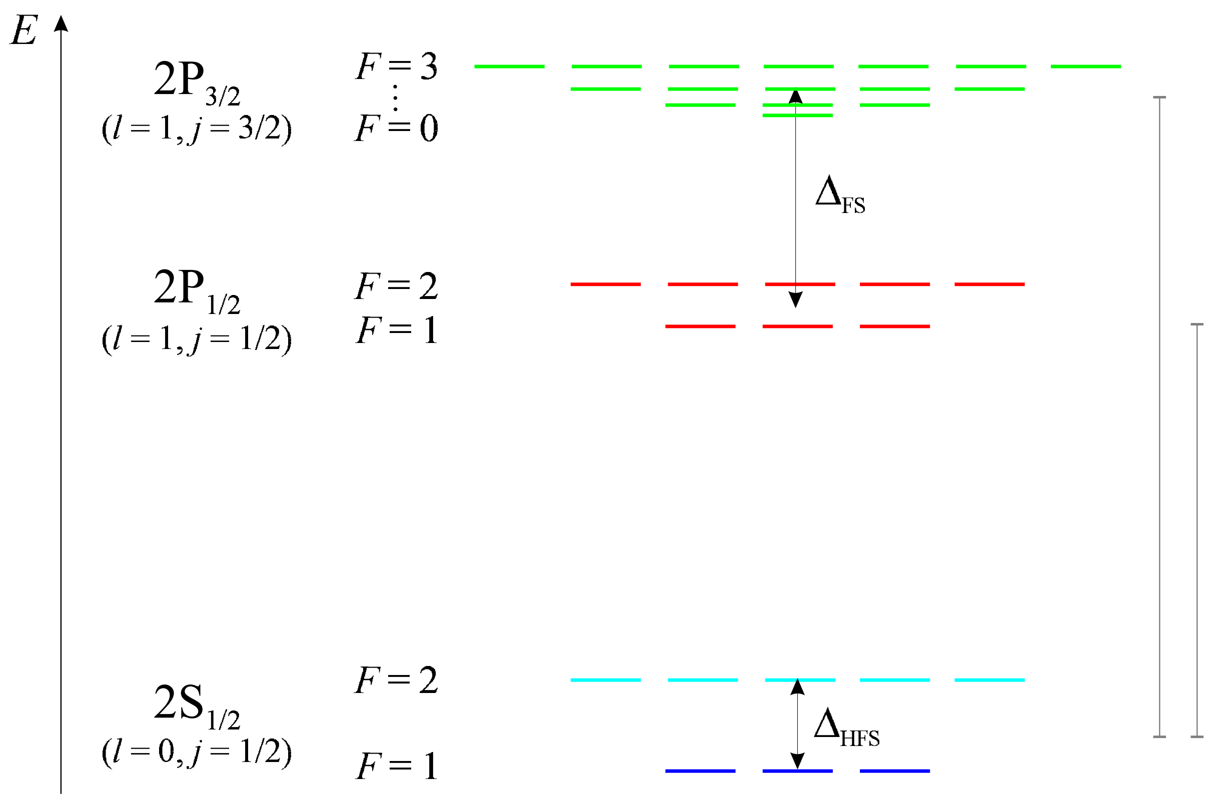
Elements of a BEC Apparatus...
...A Construction Kit

- Excitation Spectrum & Zeeman Shift
- Magneto-Optical Trap
- Magnetic Trap
- Evaporative Cooling
- Imaging

Excitation Spectrum of Alkali Atoms

Example: ^{87}Rb

nuclear spin	$I = 3/2$	$I = 3/2$	$F = 1, 2\ (1, 2; 0, ..., 3)$
electron spin	$s = 1/2$	$j = 1/2\ (1/2, 3/2)$	
orbital-ang. mom.	$l = 0\ (1)$		



"visible" transitions

$\lambda_{D1} = 780.0\text{nm}$

$\lambda_{D2} = 794.8\text{nm}$

Hyperfine Structure & Zeeman Shift

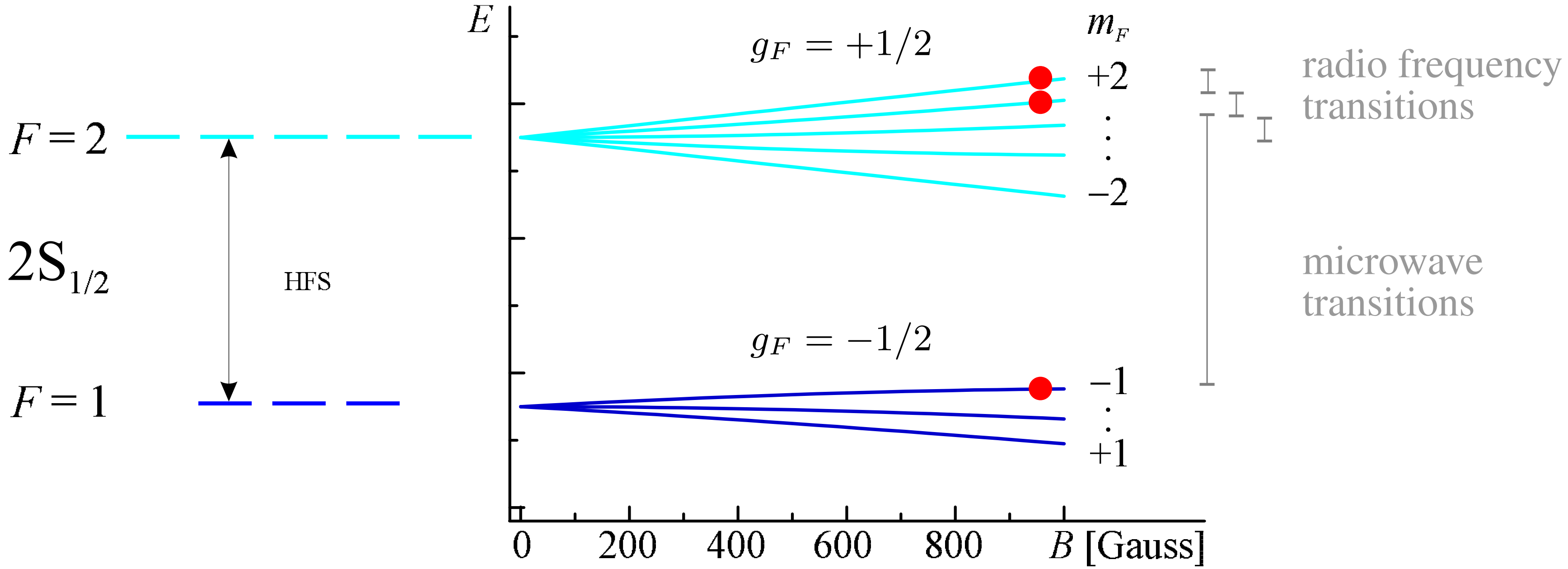
- Zeeman shift of the $2S_{1/2}$ energy levels in an external magnetic field B

$$\Delta E_{\text{Zeeman}}(B) = -\vec{\mu}\cdot\vec{e}_z B = \mu_{\text{B}}\, g_F m_F\, B$$

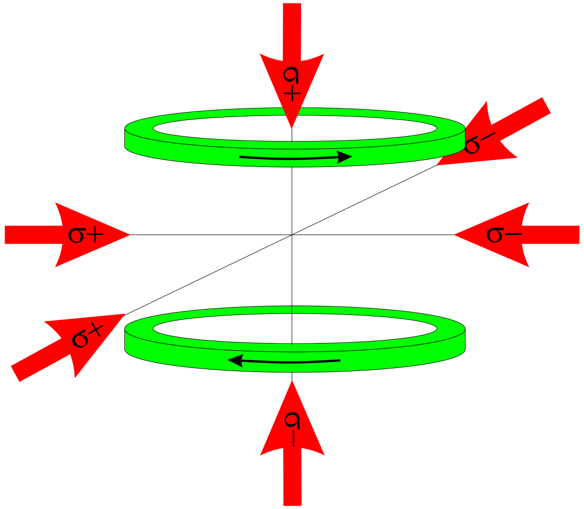
$$g_F \approx g_j \frac{F(F+1) - I(I+1) + j(j+1)}{2F(F+1)}$$

$$g_j = 1 + \frac{j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$$

- weak-field seekers: energy decreases with decreasing field strength for states with $g_F m_F > 0$



Magneto-Optical Trap (MOT)

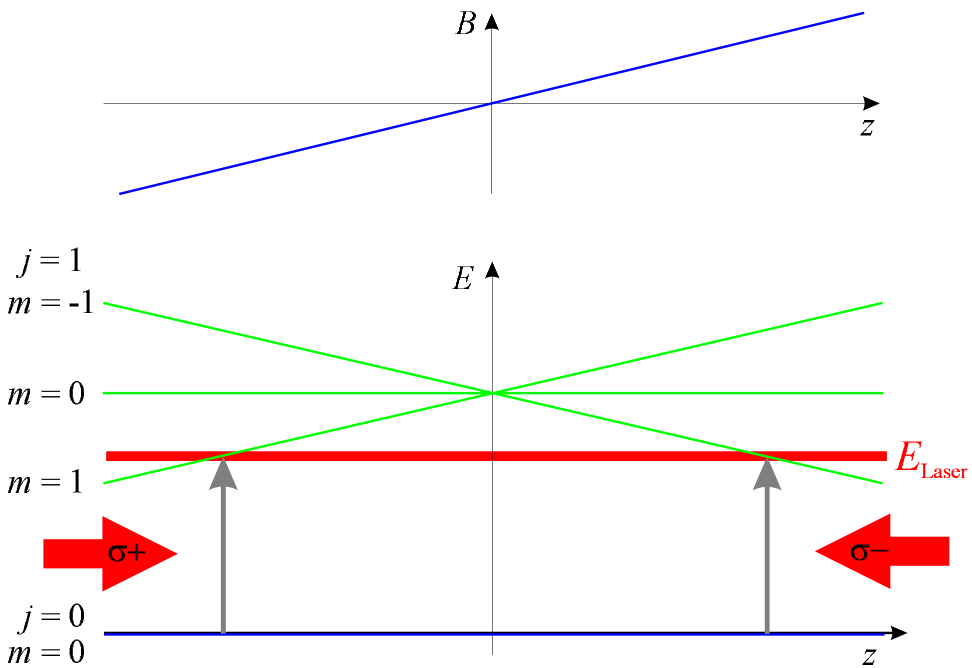


Setup

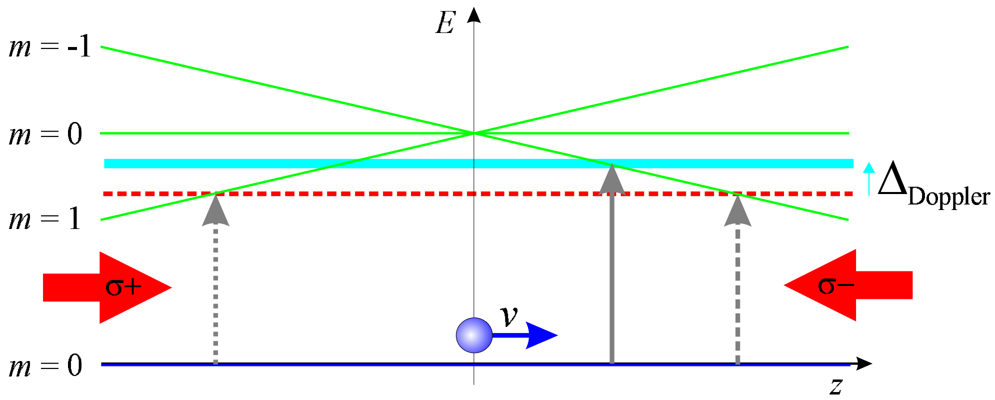
- anti-Helmholtz coils produce a quadrupole field; field strength proportional to distance from center
- 3 orthogonal pairs of counter-propagating circular polarized lasers; red-detuned with respect to D_1 or D_2-line

Trapping Mechanism

- neglect Doppler effect for a moment
- Zeeman shift brings one of the m_F-levels into resonance with the laser in the outer regions
- scattering force: absorption of a counter-propagating photon and spontaneous reemission results in a net force towards the center ➽ trapping



Magneto-Optical Trap (MOT)



Enhanced Trapping Mechanism

- Doppler effect shifts the counter-propagating laser towards higher energies and closer to resonance
- position and velocity dependent restoring force active in the whole trap volume
- typical trap depth $\sim 0.5\text{K}$

Doppler Cooling

- atom absorbs a laser photon of energy $E_{\text{laser}} = E_{\text{reso}} - \Delta_{\text{Doppler}}$ and re-emits an one-resonance photon E_{reso}
- the energy difference Δ_{Doppler} is payed with kinetic energy ➻ cooling
- recoil of re-emission increases momentum fluctuations and thus limits the achievable coolness ➻ Doppler limit
- minimal temperature $T_{\text{Doppler}} \sim 100\mu\text{K}$

workhorse for atomic physics at low temperatures

BUT: need an additional step to realize BEC!

Magnetic Traps

Concept

- magnetic moment of the atoms couples to an inhomogeneous magnetic field

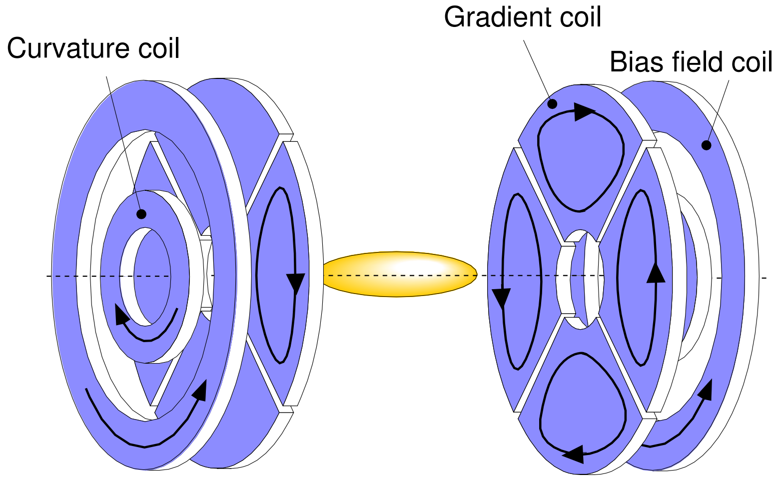
$$U(\vec{x}) = -\vec{\mu} \cdot \vec{B}(\vec{x}) = \mu_{\text{B}} g_F m_F \, B_z(\vec{x})$$

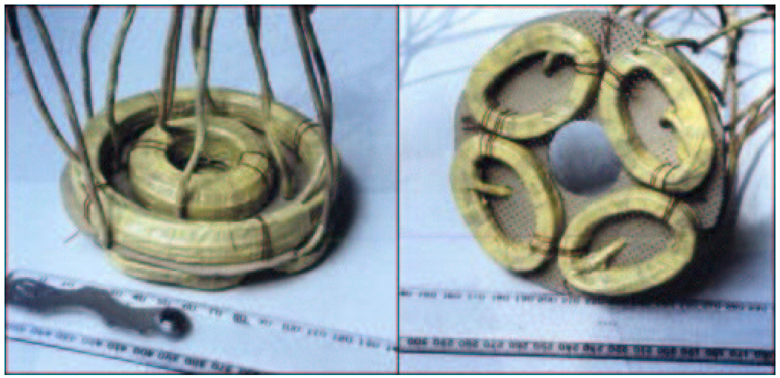
- weak-field seekers ($g_F m_F > 0$) can be trapped in a local field minimum
- strong-field seekers cannot be trapped magnetically (no field maximum possible in free space)

Realization

- anti-Helmholtz configuration produces a local field minimum; but anisotropic, non-central and weak restoring force
- more complex configurations necessary to generate "harmonic oscillator" potential, e.g. Pritchard-Ioffe trap and modifications
- typical net field strength ~ 100G
- typical trap depth ~ 1mK

■ *Cloverleaf Trap*





MIT; *W. Ketterle, et al.*

Construction Kit: Atom Cooling

Forced Evaporative Cooling

Concept

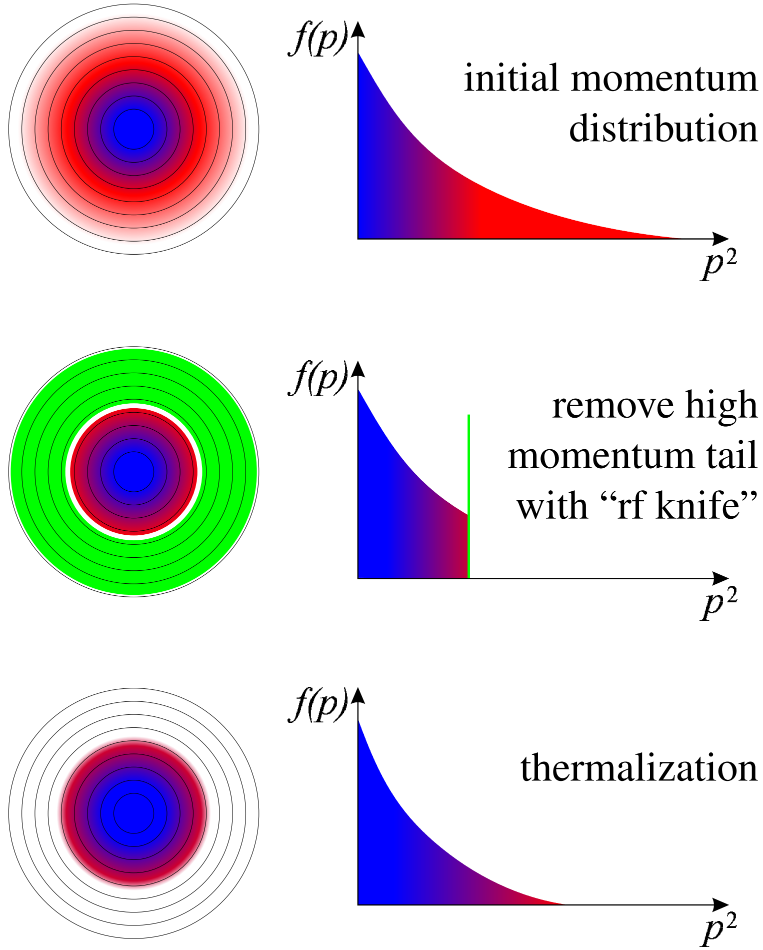
- selective removal of high-momentum atoms and re-thermalization of the remnant will decrease the temperature

Realization

- momentum of the trapped atom is connected to it's position, i.e., atoms that contribute to the peripheral density have high momenta
- due to Zeeman shift the transition frequencies between different m_F states depend on the position in the magnetic trap
- induce a transition of the peripheral atoms to an untrapped state ($F = 2, m_F = 1 \rightarrow m_F = 0$) by an rf-sweep with decreasing frequency
- after re-thermalization the gas contains less particles at a lower temperature

before evaporation: $N \sim 10^9,\ T \sim 100\,\mu\text{K}$

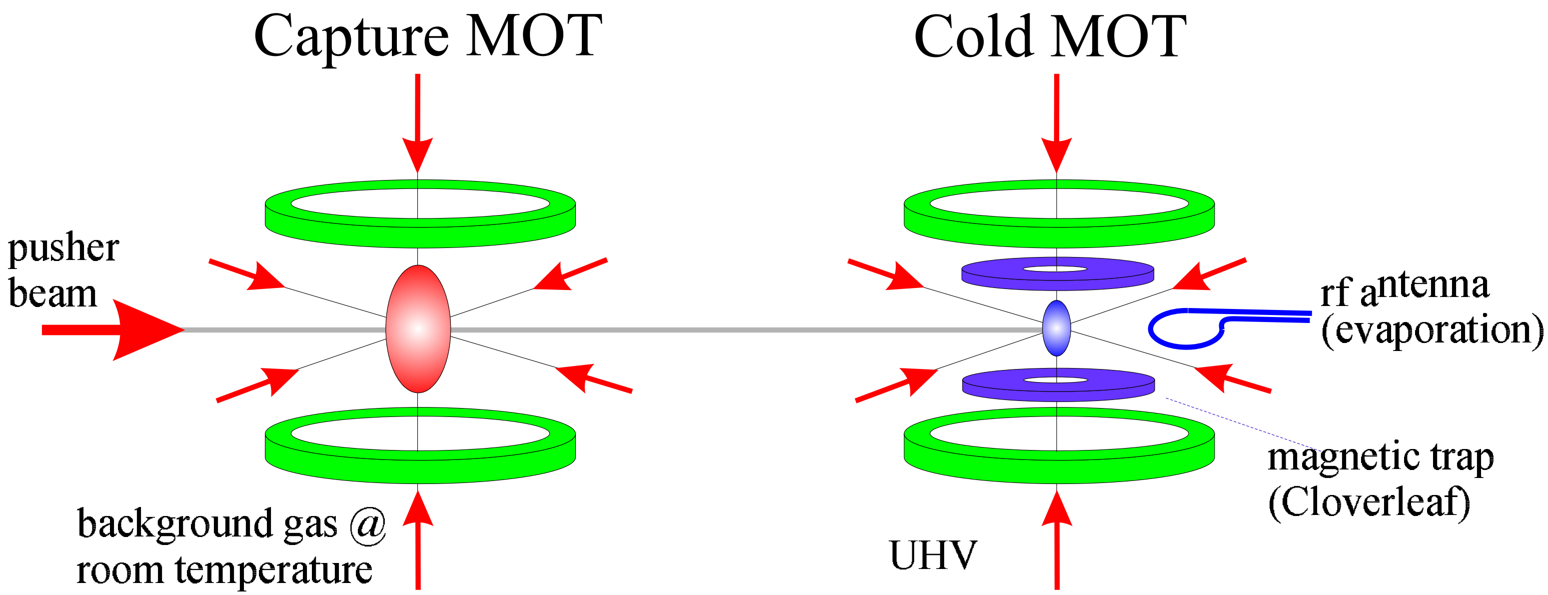
after evaporation: $N \sim 10^6,\ T \sim 100\,n\text{K}$



R. Roth - 02/2001

Double-MOT System

A Prototypical BEC Apparatus



① capture atoms from room-temperature background gas and precool ($\tau \sim 100\,\mathrm{s}$, $N \sim 10^{10}$)

② transfer precooled cloud to cold MOT (pusher beam or gravity)

③ cool with MOT to Doppler limit ($\tau \sim 1\,\mathrm{s}$, $N \sim 10^9$, $T \sim 100\,\mu\mathrm{K}$)

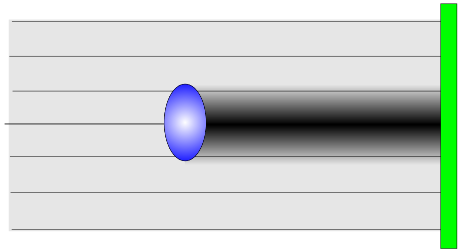
④ switch off MOT and switch on magnetic trap

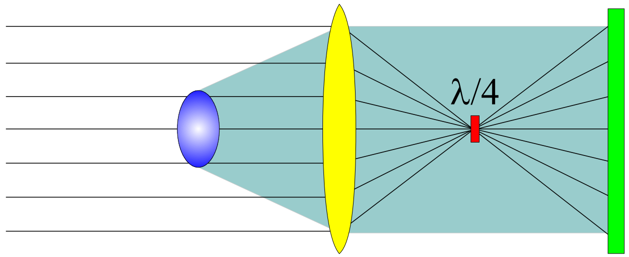
⑤ start evaporative cooling

- sweep radio-frequency down to a fixed final frequency ($\tau \sim 50\,\mathrm{s}$, $N \sim 10^6$)
- wait for thermalization ($\tau \sim 2\,\mathrm{s}$, $T \sim 100\,\mathrm{nK}$)

⑥ shoot a picture of your BEC...

Optical Imaging





Absorption Imaging

- shine a high-intensity resonant laser beam onto the trapped cloud and image the shadow
- imaginary part of index of refraction: absorption is proportional to the column density of atoms
- destructive: atoms get excited, recoil momenta, heating,...

Phase Contrast Imaging

- shine a far off-resonance laser beam onto the cloud and observe interference of scattered and unscattered light
- real part of index of refraction: phase shift of scattered light is proportional to density
- quasi non-destructive: only shakes the spin a little bit

Ballistic Expansion

- imaging after switching off the trap and expansion maps momentum distribution

In-Situ Imaging

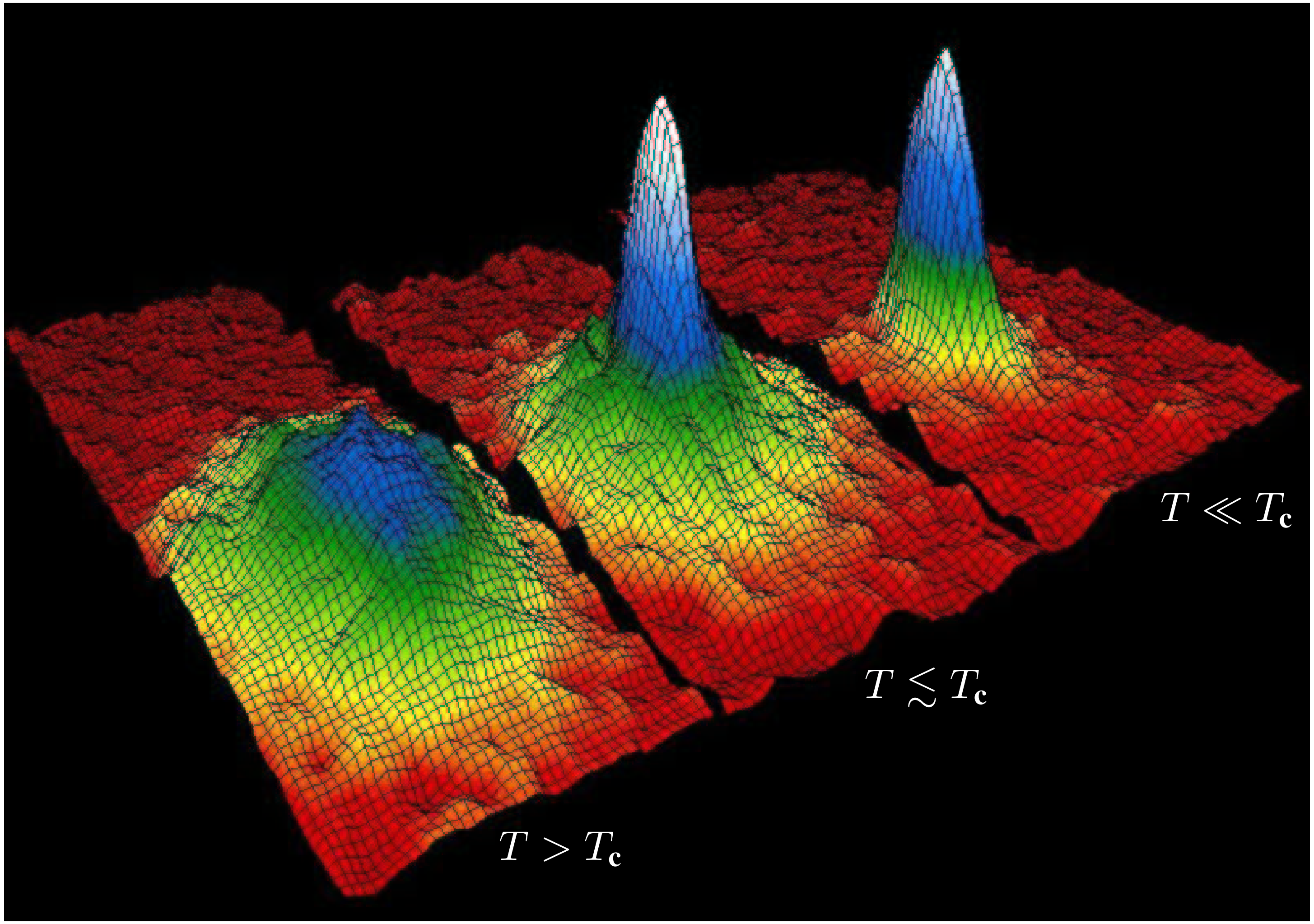
- in-situ imaging of the atoms kept in the trap maps the spatial density distribution

Experimental Results...
...Pictures of an Unique System

- BEC with Rubidium and Sodium
- Tuning of the Interaction
- Vortices
- Atom Lasers

Boulder / Colorado — June 5th, 1995 — 10:54 am

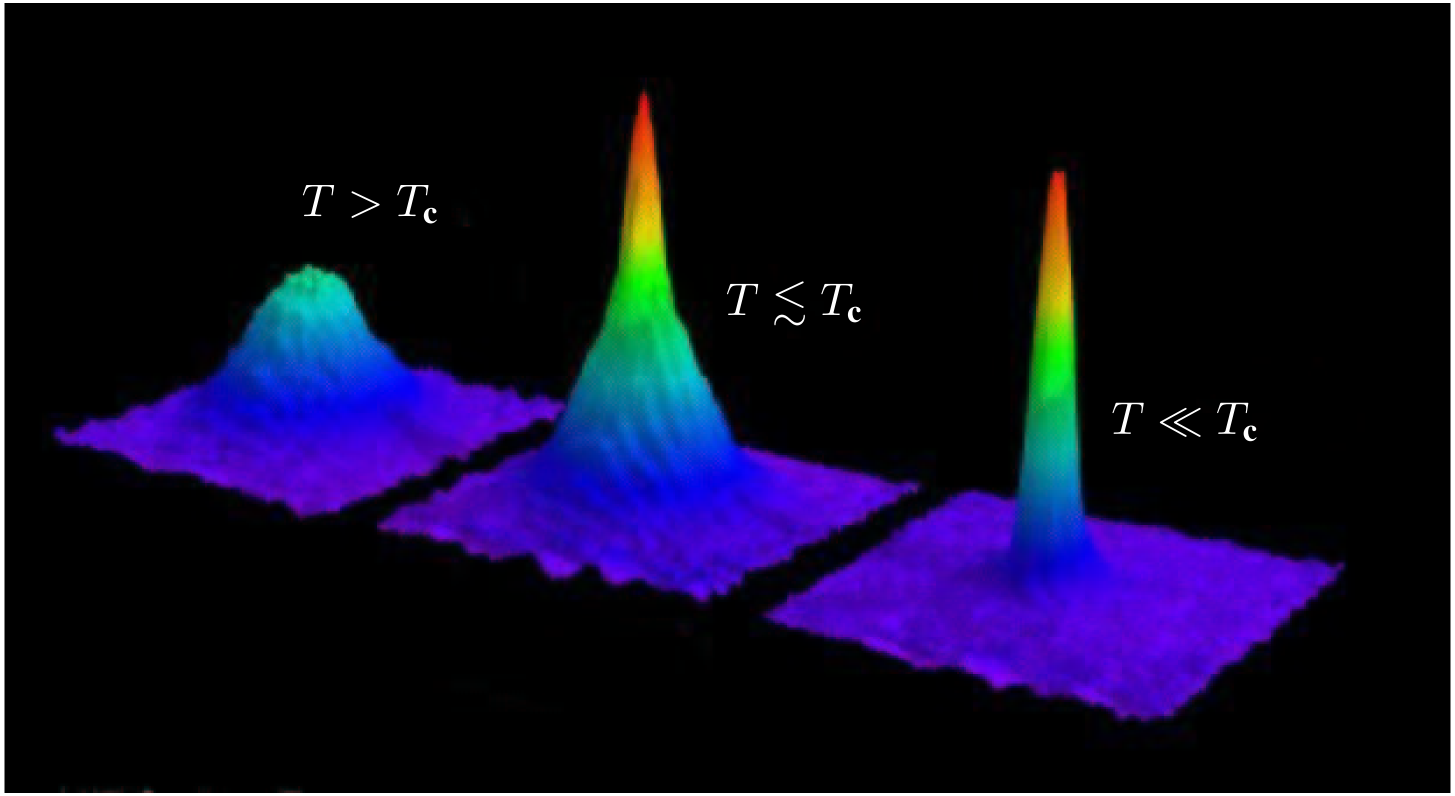
First Rubidium BEC



JILA, NIST, U of Colorado; *E. Cornell, C. Wieman, et al.*

- ^{87}Rb $(F=2, m_F=2)$
- $N_{\text{initial}} \approx 10^6$
- $N_{\text{BEC}} \approx 2000$
- $T_c \approx 170$nK
- absorption image after 60 ms expansion
- 0.2mm × 0.27mm

First Sodium BEC



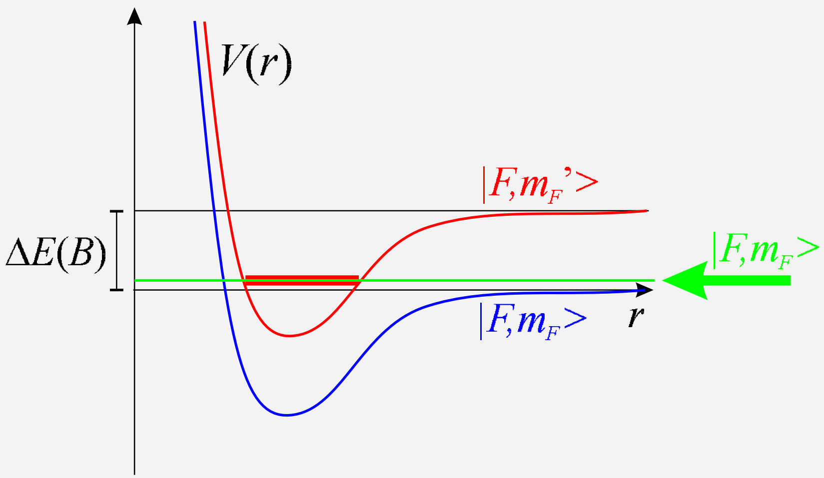
MIT; *W. Ketterle, et al.*

- ^{23}Na $(F=1, m_F=-1)$
- $N_{\text{initial}} \approx 10^9$
- $N_{\text{BEC}} \approx 5 \times 10^5$
- $T_c \approx 2\,\mu$K
- absorption image after 60 ms expansion
- 1mm $\times$ 1mm

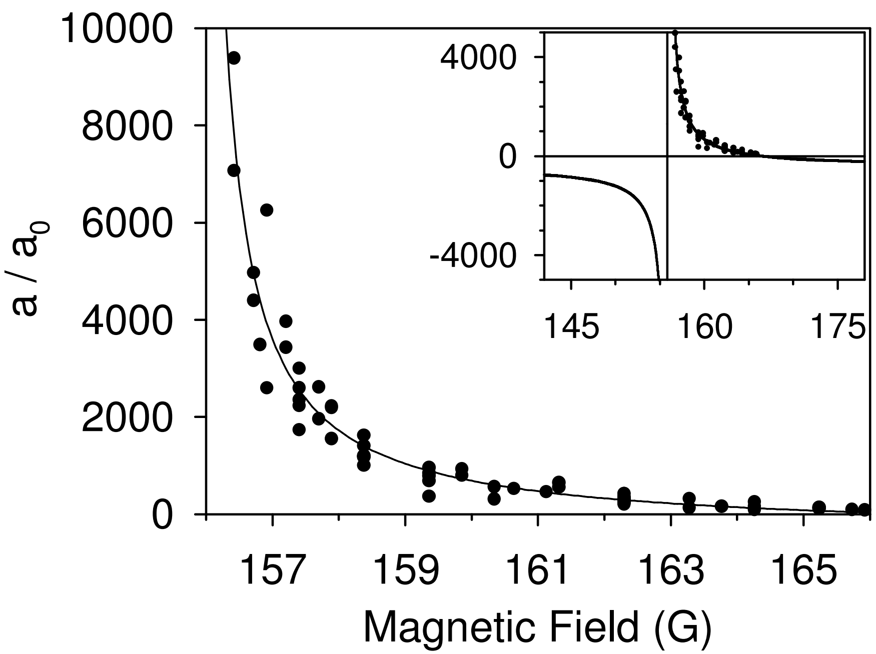
Tuning the Interaction Strength

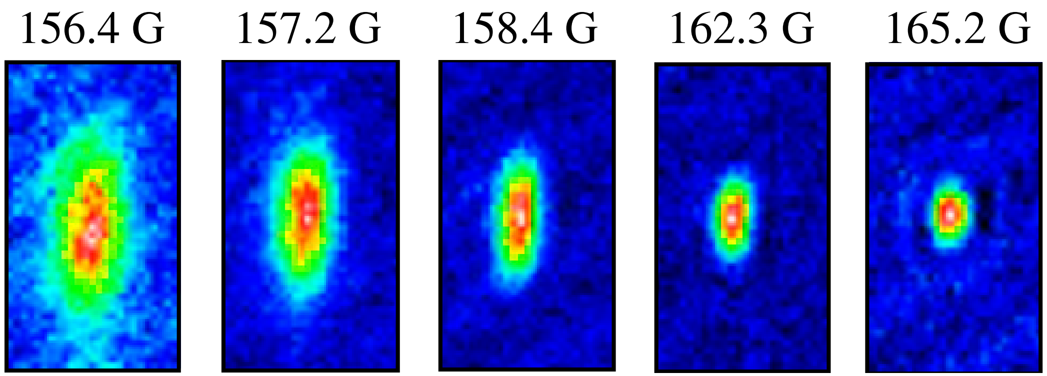
Feshbach Resonance

- virtual admixture of a bound state with a different m_F modifies the net interaction strength (scattering length a)



- scattering length depends on energy difference, i.e., the strength of the magnetic field (Zeeman effect)
- ^{85}Rb: attractive interaction in weak magnetic fields ($a \approx -400\,a_0$); Feshbach resonance at $B \approx 155$G





JILA, NIST, U Colorado; *S.L. Cornish, N.R. Claussen, et al.* [Phys. Rev. Lett. 85 (2000) 1796]

Stirring a Condensate

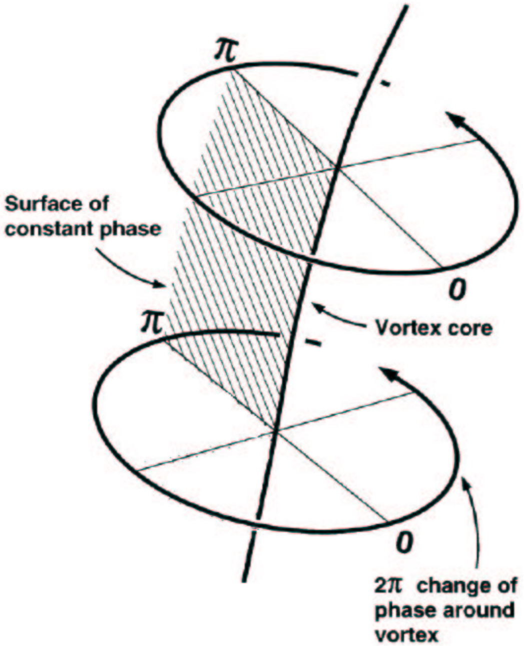
Abrikosov Vortex Lattice

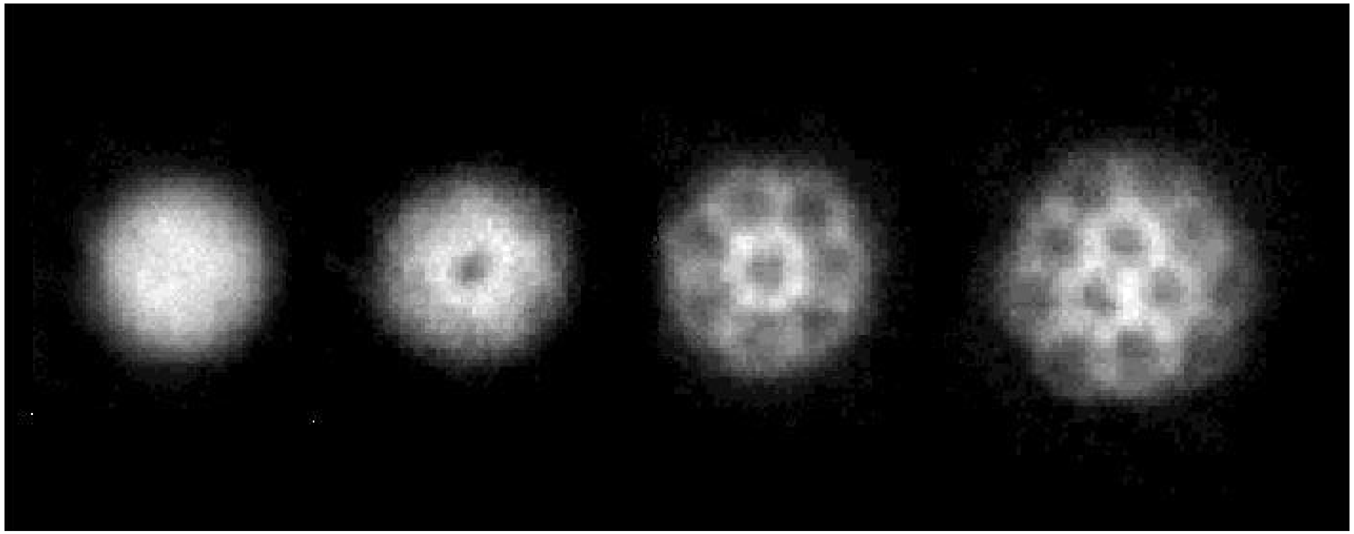
Rotating A Bucket

- if you start to rotate a bucket filled with a superfluid slowly then the superfluid stays in absolute rest
- if you rotate faster than some critical velocity a vortex will be created at the wall
- around a vortex the phase of the complex order parameter changes by 2π (stable topological structure)

Stirring A Condensate

- a blue-detuned focused laser beam "repels" the BEC and can be used as a stirrer
- if you stir slowly there is a depletion of the density at the position of the laser
- if you stir above a critical frequency vortices are created and move through the BEC
- multiple vortices arrange in a regular lattice





ENS Paris; *K.W. Madison, J. Dalibard, et al.* [Phys. Rev. Lett. 84 (2000) 806]

Construction of an Atom Laser

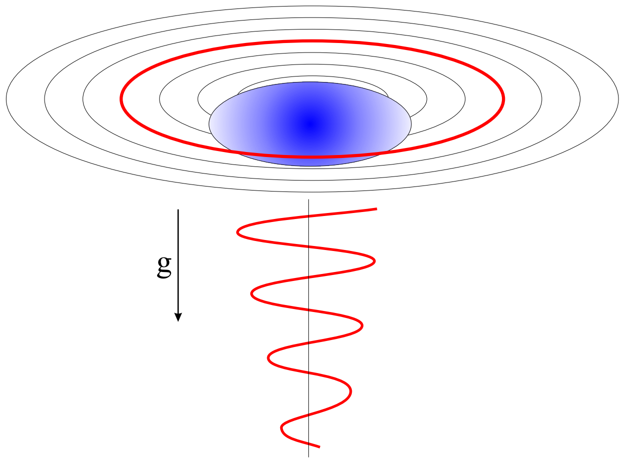
Pragmatic Definition

A "laser" is a source of an intense, coherent and directional wave

Output Coupler

Q: how to extract atoms coherently out of a trapped BEC ?

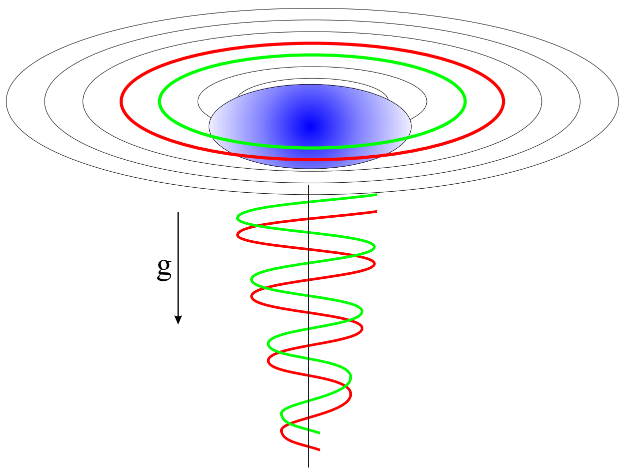
- need to extract atoms out of a well defined slice to ensure coherence
- apply a radio frequency to induce transition to an untrapped m_F-state and let the atoms fall
- due to Zeeman shift the resonance condition is fulfilled only on an iso-field slice



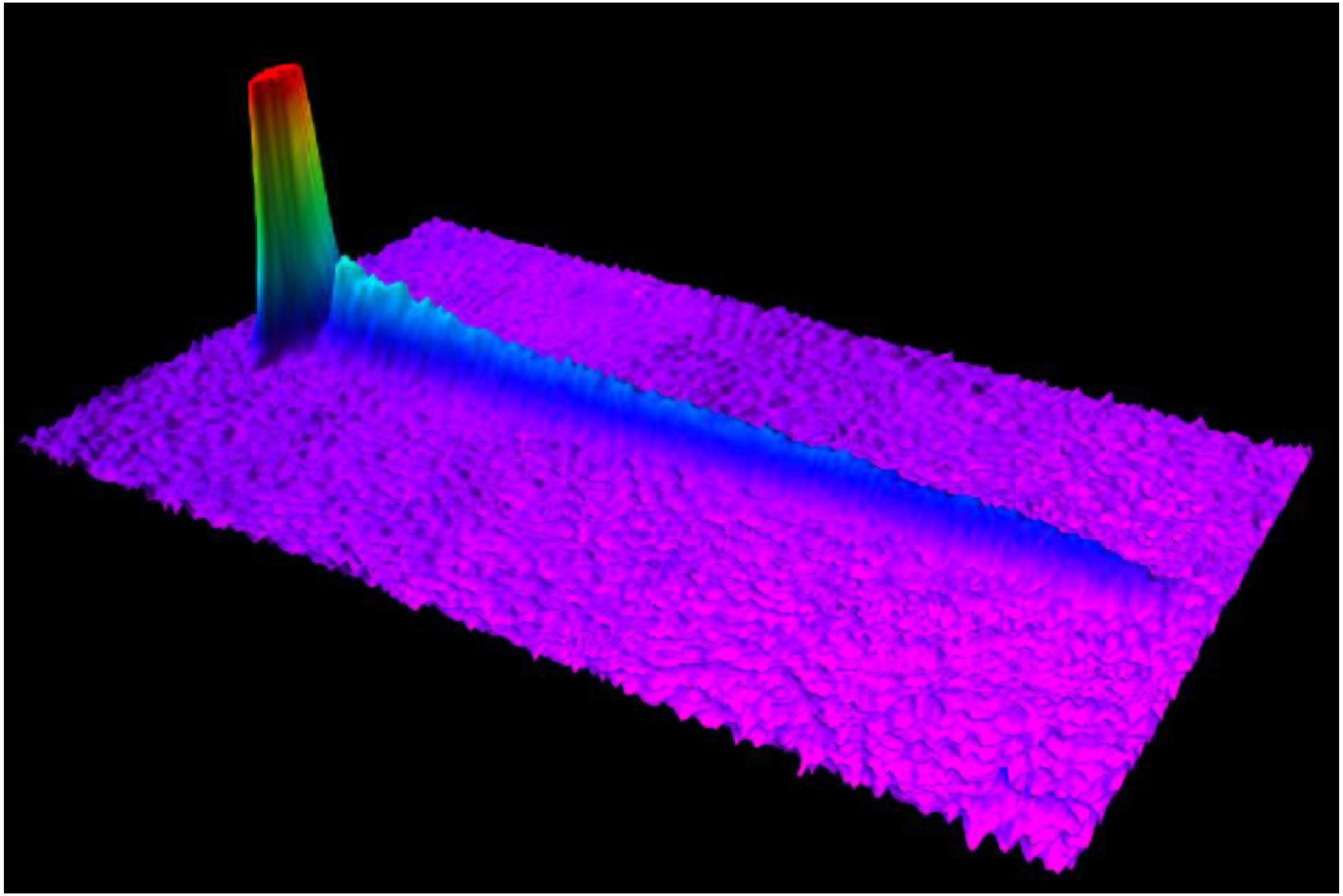
Double Slit Experiment

Q: how to realize a double slit experiment with atom lasers?

- use an output coupler with two frequencies, i.e., extract two slices
- distance between the slices gives a well defined phase offset and the two beams interfere



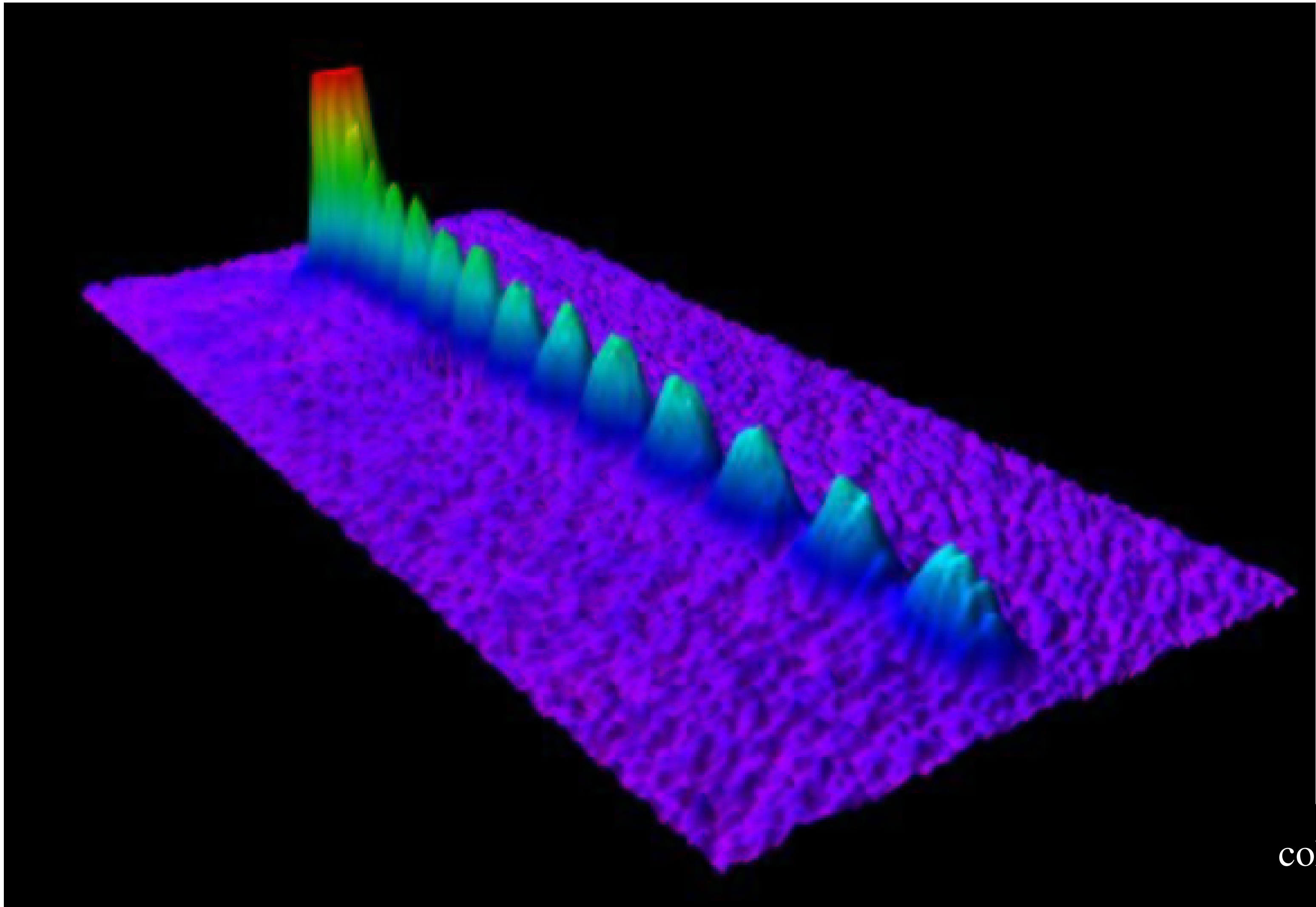
A Coherent Matter Wave



MPI Quantenoptik / Garching; *T. Hänsch, T. Esslinger, et al.*

- ^{87}Rb BEC
  $(F=2, m_F=1)$
- rf output coupler
  $(m_F=1 \rightarrow m_F=0)$
- absorption image
- $N_{\text{trap}} \approx 10^9$
- $l_{\text{beam}} \approx 2\text{mm}$

Double-Slit Experiment with Matter Waves



MPI Quantenoptik / Garching; *T. Hänsch, T. Esslinger, et al.*

- ^{87}Rb BEC
  $(F=2, m_F=1)$
- rf output coupler
  (two frequencies)
  $(m_F=1 \rightarrow m_F=0)$
- absorption image
- $N_{\text{trap}} \approx 10^9$
- $l_{\text{beam}} \approx 2$mm

coherence length
$\approx$
size of BEC