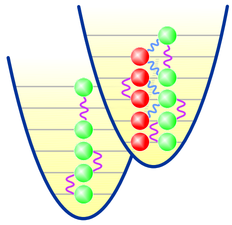

Mean-Field Instability of Trapped Degenerate Fermi Gases with Effective s- and p-Wave Interactions

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Overview

Effective Interaction

- want a simple effective interaction suited for a mean-field description of very dilute systems
- effective contact interaction for all partial waves that reproduces the two-body energy spectrum when used in a mean-field framework
- for single-component (one m_F state) Fermi gas s-wave interactions are prohibited by Pauli principle ➠ p-wave is leading contribution
- for two-component (two m_F states) Fermi gas s- and p-wave interactions compete

Mean-Field Instability

- dilute ultracold atomic Fermi (Bose) gases are in a metastable state, true ground state is a solid
- for weak attractive interactions the system is stabilized by the kinetic energy
- if the s-wave or p-wave attraction becomes too strong then the attractive mean-field cannot be stabilized any more ➠ mean-field instability
- for given scattering lengths there is a maximum density or particle number up to which the metastable state exists ➠ stability condition

Connection to Cooper-Pairing

- attraction between fermionic atoms may lead to the formation of correlated pairs that behave like bosons ➠ Cooper pairs, BCS transition
- s-wave pairing of atoms in different m_F states; p-wave pairing of atoms in identical states
- temperature of BCS phase transition increases with density and strength of the attractive s- or p-wave interaction
- one has to look for optimum with regard to mechanical stability and transition temperature

Results

- p-wave interaction can have significant influence on the stability of ultracold single- and multi-component Fermi gases
- attractive p-wave interaction induces instability in single-component Fermi gases
- competition between s- and p-wave in two-component gases; repulsive p-wave stabilizes against collapse induced by attractive s-wave
- **It is not appropriate to neglect the p-wave interaction from the outset!**

Formalism

Effective Contact Interaction (ECI)

What is a proper effective interaction to describe properties of dilute ultracold quantum gases?

- system is dilute and cold

  mean particle distance $\gg$ interaction range
  relative wavelength $\gg$ interaction range

- particles experience an average interaction and do not probe details of the potential
- the many-body problem can be treated in a mean-field approach with a suitable effective interaction instead of the full atom-atom potential

Concept of the ECI

- ► consists of a non-local zero-range interaction for each partial wave
- ► interaction strength is chosen such that the expectation value of the ECI in the two-body system (calculated with free angular-momentum eigenstates) equals the energy-shift induced by the full interaction

$$\langle nlm_l|\,\mathrm{v}^{\mathrm{ECI}}\,|nlm_l\rangle \stackrel{!}{=} \Delta E_{nl} \qquad (\star)$$

Construction of the ECI

- two particles interacting via a potential $v(r)$ with phase shifts $\eta_l(q)$ for the l-th partial wave
- impose a boundary condition $R_{nl}(\Lambda)=0$ for the relative wave-function
- energy-shift ΔE_{nl} of the n-th positive energy state of the interacting spectrum $\bar{E}_{nl}$ compared to the n-th level of the free spectrum E_{nl}

$$\frac{\Delta E_{nl}}{E_{nl}} = -\frac{2}{q_{nl}\Lambda}[\eta_l(q_{nl}) - \pi n_{\mathrm{bs}}]$$

- ansatz for the ECI for the l-th partial wave

$$\mathrm{v}_l^{\mathrm{ECI}} = \int d^3r\; |\vec{r}\rangle \frac{\overleftarrow{\partial}^l}{\partial r^l}\, g_l\, \frac{\delta(r)}{4\pi r^2}\, \frac{\overrightarrow{\partial}^l}{\partial r^l} \langle\vec{r}|$$

- interaction strength g_l obtained from evaluation of the energy-shift condition ($\star$)

$$g_l = -\frac{4\pi}{m}\left[\frac{(2l+1)!!}{l!}\right]^2 \frac{[\eta_l(q)-\pi n_{\mathrm{bs}}]}{q^{2l+1}}$$

- parameterization in terms of scattering lengths

$$g_l \approx \frac{4\pi}{m}\frac{(2l+1)!}{(l!)^2}\, a_l^{2l+1} + \mathcal{O}(q^2)$$

Energy-Density for Trapped Fermions

- assume a gas of Ξ different fermionic components at $T=0\,\mathrm{K}$ interacting via the s-wave and the p-wave part of the ECI
- the energy-density of the inhomogeneous Fermi gas is calculated in a mean-field approach using the Thomas-Fermi approximation
- groundstate is described by the density that minimizes the energy under the constraint of given particle numbers N_ξ
- functional variation leads to the extremum condition that has to be solved for each point $\vec{x}$

One Component		Two Components
$\mathcal{E}[\kappa](\vec{x}) =$		$\mathcal{E}[\kappa_1,\kappa_2](\vec{x}) =$
$= \frac{1}{6\pi^2}\,U(\vec{x})\,\kappa^3(\vec{x})$	← **trap** →	$= \frac{1}{6\pi^2}\,[U_1(\vec{x})\,\kappa_1^3(\vec{x}) + U_2(\vec{x})\,\kappa_2^3(\vec{x})]$
$+ \frac{1}{20\pi^2 m}\,\kappa^5(\vec{x})$	← **kinetic** →	$+ \frac{1}{20\pi^2 m}\,[\kappa_1^5(\vec{x}) + \kappa_2^5(\vec{x})]$
✗	← **s-wave** →	$+ \frac{a_0}{9\pi^3 m}\,\kappa_1^3(\vec{x})\,\kappa_2^3(\vec{x})$
$+ \frac{a_1^3}{30\pi^3 m}\,\kappa^8(\vec{x})$	← **p-wave** →	$+ \frac{a_1^3}{30\pi^3 m}\,[\kappa_1^8(\vec{x}) + \kappa_2^8(\vec{x}) + \frac{1}{2}\kappa_1^3(\vec{x})\kappa_2^5(\vec{x}) + \frac{1}{2}\kappa_1^5(\vec{x})\kappa_2^3(\vec{x})]$

- local Fermi mom. $\kappa_\xi(\vec{x}) = \sqrt[3]{6\pi^2\,\rho_\xi(\vec{x})}$
- energy $E = \int d^3x\; \mathcal{E}[\kappa_1,\dots,\kappa_\Xi](\vec{x})$
- particle number $N_\xi = \frac{1}{6\pi^2}\int d^3x\; \kappa_\xi^3(\vec{x})$
- trap potential $U(\vec{x}) = x^2/(2m\ell^4)$, $\ell = (m\omega)^{-1/2}$

Application I

Single-Component Fermi Gas — p-Wave Induced Instability

- s-wave contact interaction does not contribute due to the Pauli principle; p-wave is the leading interaction term
- the groundstate densities [*see Fig. A*] are obtained by point-wise solution of the extremum condition

$$m[\mu - U(\vec{x})] = \frac{1}{2}\kappa^2(\vec{x}) + \frac{8\,a_1^3}{15\pi}\kappa^5(\vec{x})$$

- for attractive p-wave interactions ($a_1 < 0$) the r.h.s. of the extremum condition exhibits a maximum at $(\kappa_{\max}, \mu_{\max})$

Figure A

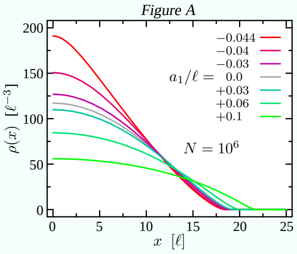
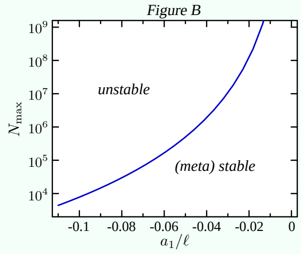


Figure B



- ► the metastable state exist only for densities, chemical potentials and particle numbers below the maximum values [*see Fig. B*]

$$\kappa_{\max} = \frac{(3\pi)^{1/3}}{2\,|a_1|} \qquad \rho_{\max} = \frac{1}{16\pi\,|a_1|^3}$$

$$\mu_{\max} = \frac{3(3\pi)^{3/2}}{40m\,|a_1|^2} \qquad N_{\max} = \frac{(0.445\,\ell)^6}{|a_1|^6}$$

ℓ: mean oscillator length

- ► beyond the critical values the mean-field attraction is not stabilized by the kinetic energy contribution any more and the system collapses

Application II

Two-Component Fermi Gas — Competition between s-Wave and p-Wave

- s-wave and p-wave terms of the ECI contribute, their interplay has a strong influence on the stability
- extremum condition under the assumption of equal $[\mu - U(\vec{x})]$ and equal $\kappa(\vec{x})$ for both components

$$m[\mu - U(\vec{x})] = \frac{1}{2}\kappa^2(\vec{x}) + \frac{2\,a_0}{3\pi}\kappa^3(\vec{x}) + \frac{4\,a_1^3}{5\pi}\kappa^5(\vec{x})$$

- metastable states exist only for local Fermi momenta below $\kappa_{\max}$ given by [*see Fig. C*]

$$-a_0\,\kappa_{\max} - 2(a_1\,\kappa_{\max})^3 = \pi/2$$

- for given trap geometry this can be translated to a maximum particle number $N_{\max}$ [*see Fig. D*]

Figure C

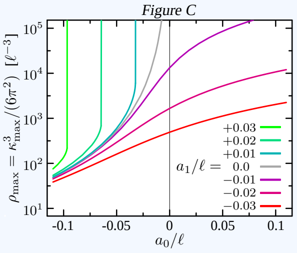
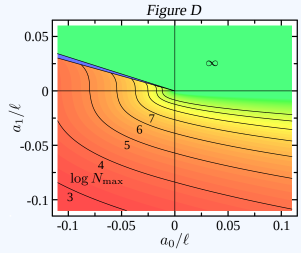


Figure D



- ► the mean-field instability can occur if the s-wave and/or the p-wave interaction is attractive
- ► if one of the interactions is repulsive then it stabilizes the system, i.e., it increases the maximum density
- ► a repusive p-wave interaction leads to an absolute stabilization ($\rho_{\max} \to \infty$) of the Fermi gas against s-wave induced collapse if

$$a_1/|a_0| > 2/(3\pi^{2/3}) \approx 0.311$$

- ► may be interesting for Cooper pairing: mechanically stable systems with strongly attractive s-wave interactions possible if p-wave is sufficiently repulsive

Two-Component Fermi Gas — p-Wave Stabilized High-Density Phase

- given a system with attractive s-wave and repulsive p-wave interaction, such that

$$0.274 < a_1/|a_0| < 0.311$$

- the r.h.s. of the extremum condition shows two separated branches at low and high densities, respectively [*see Fig. E*]
- for $[\mu - U(\vec{x})] > \mu_{\mathrm{trans}}$ the high-density solution is energetically favored otherwise the low-density solution is preferred (Maxwell construction)

Figure E

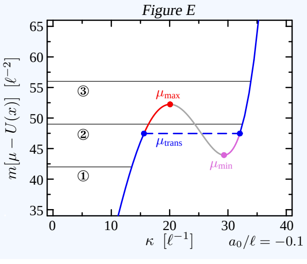
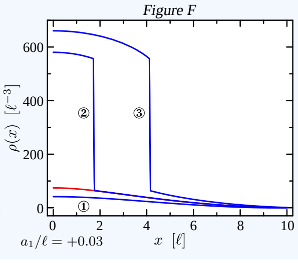


Figure F



- ► depending on the value of $\mu(N)$ the solutions exhibit different density profiles [*see Fig. F*]
  - ① $\mu < \mu_{\mathrm{trans}}$: smooth low density profile
  - ② $\mu_{\mathrm{trans}} < \mu < \mu_{\max}$: the energetically preferred configuration has a region of high density in the center of the trap; the smooth low-density solution may occur as metastable state
  - ③ $\mu_{\max} < \mu$: no low-density solution exists for the central region of the trap; high-density phase must occur
- ► central density is increased by at least one order of magnitude; life-time is reduced significantly but may be still sufficient to observe the phenomenon

References: R. Roth, H. Feldmeier; *J. Phys. B 33 (2000) 787*; e-print: cond-mat/0003266
R. Roth, H. Feldmeier; *submitted to Phys. Rev. A*; e-print: cond-mat/0102416

Web-Resources: http://theory.gsi.de/˜trap/ http://theory.gsi.de/˜rroth/

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