Ultracold Atoms in Optical Lattices

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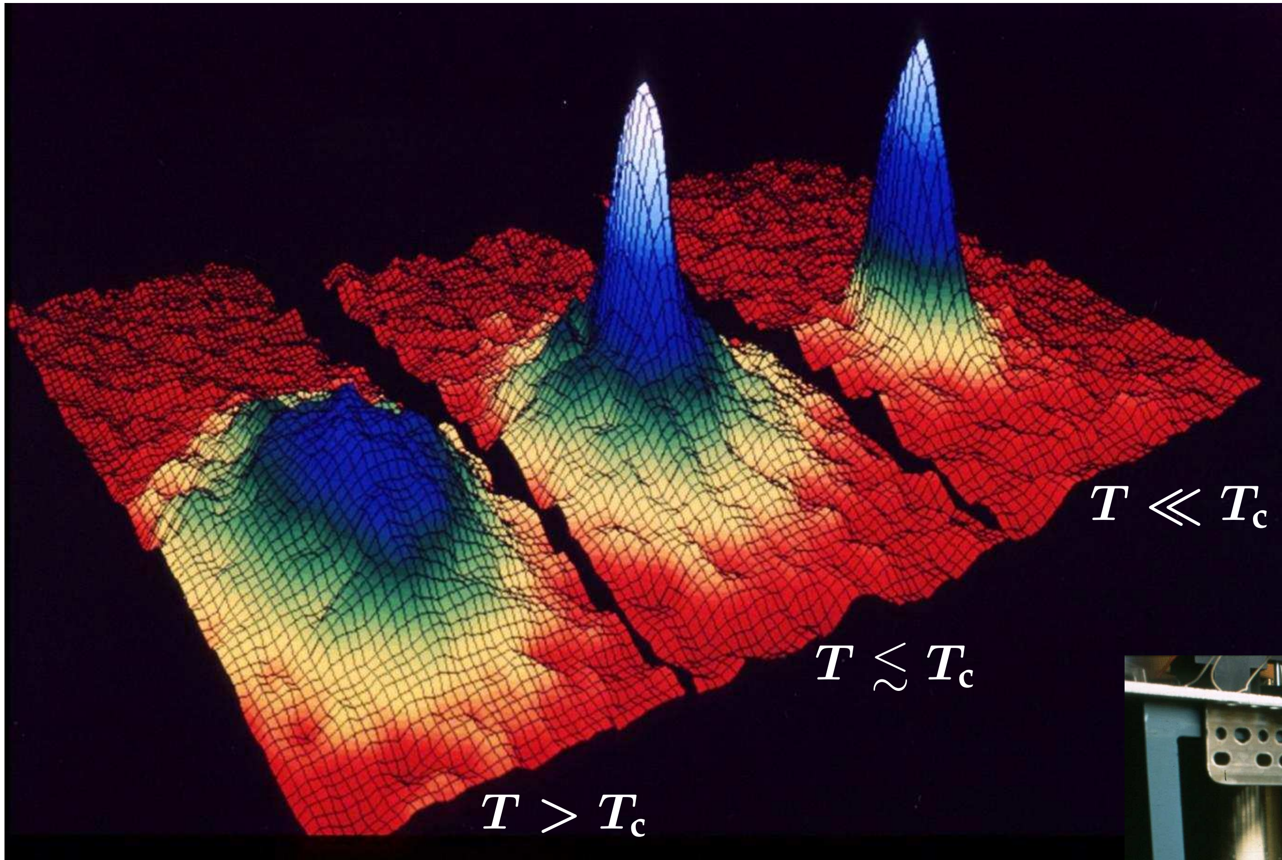
Overview

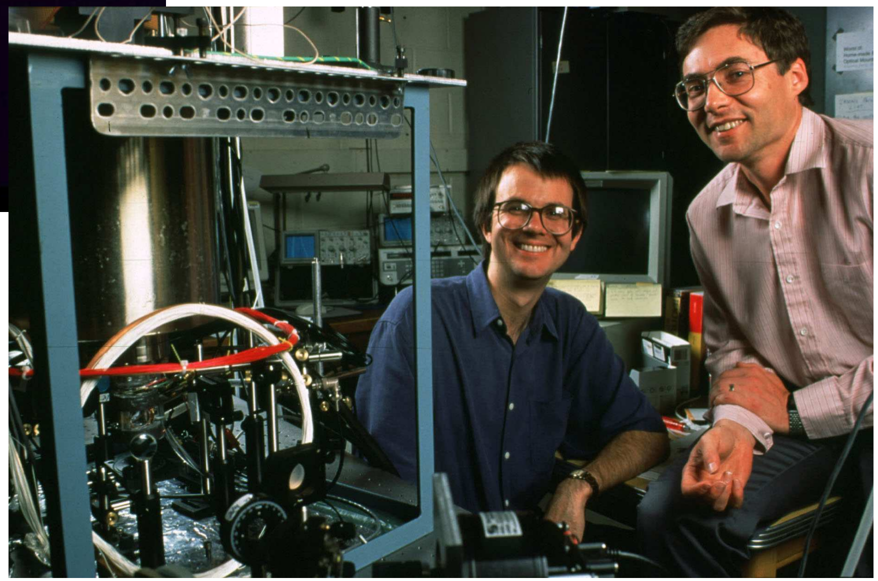
- Ultracold Atomic Gases
- The Lattice Experiment
- Bose-Hubbard Model
- Condensate & Superfluid
- Superfluid to Mott-Insulator Transition
- Two-Colour Superlattices
- Boson-Fermion Mixtures in Lattices

Boulder / Colorado — June 5th, 1995 — 10:54 am

BEC of Rubidium Atoms



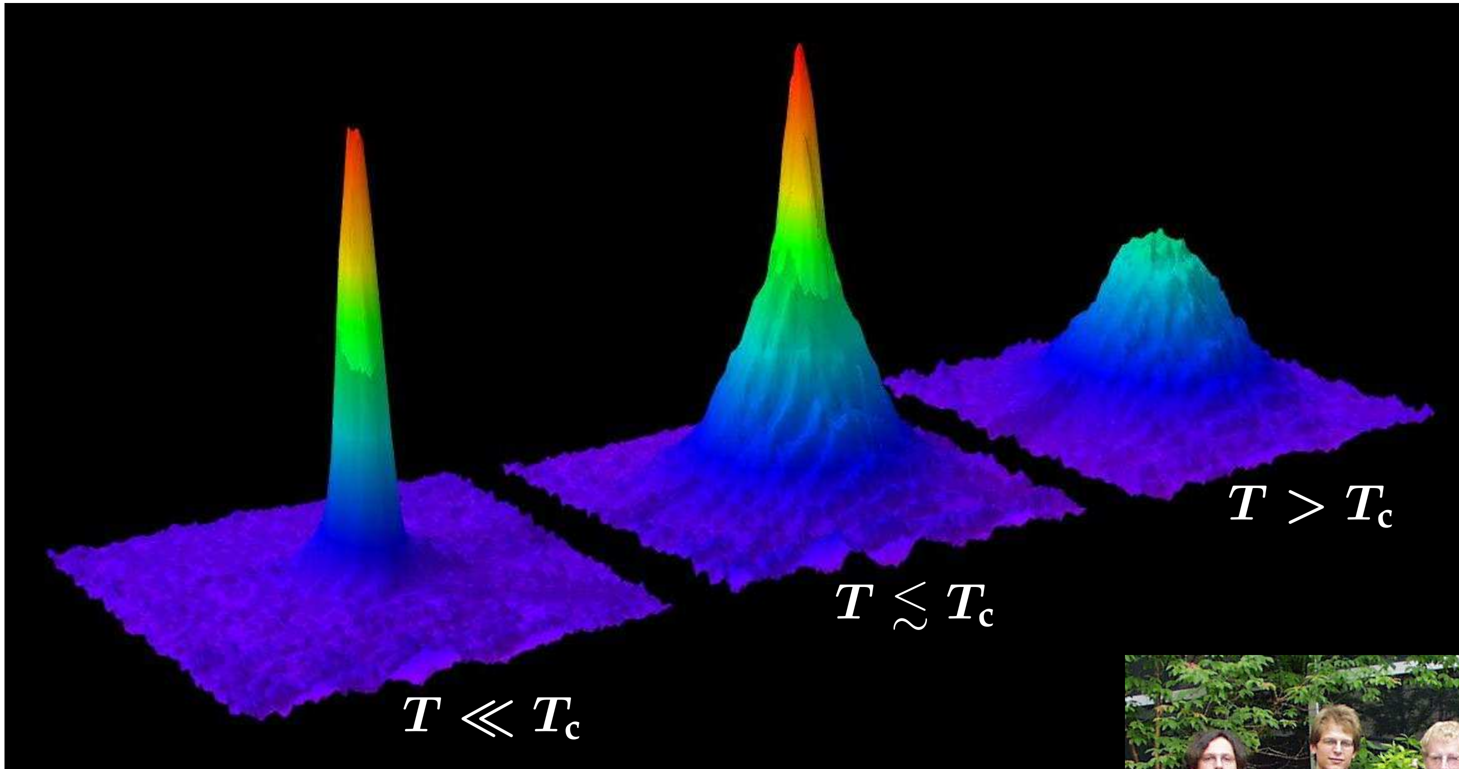
- ^{87}Rb $(F = 2, m_F = 2)$
- $N_{\text{initial}} \approx 10^6$
- $N_{\text{BEC}} \approx 2000$
- $T_c \approx 170\text{nK}$
- absorption image after 60 ms expansion
- $0.2\text{mm} \times 0.27\text{mm}$



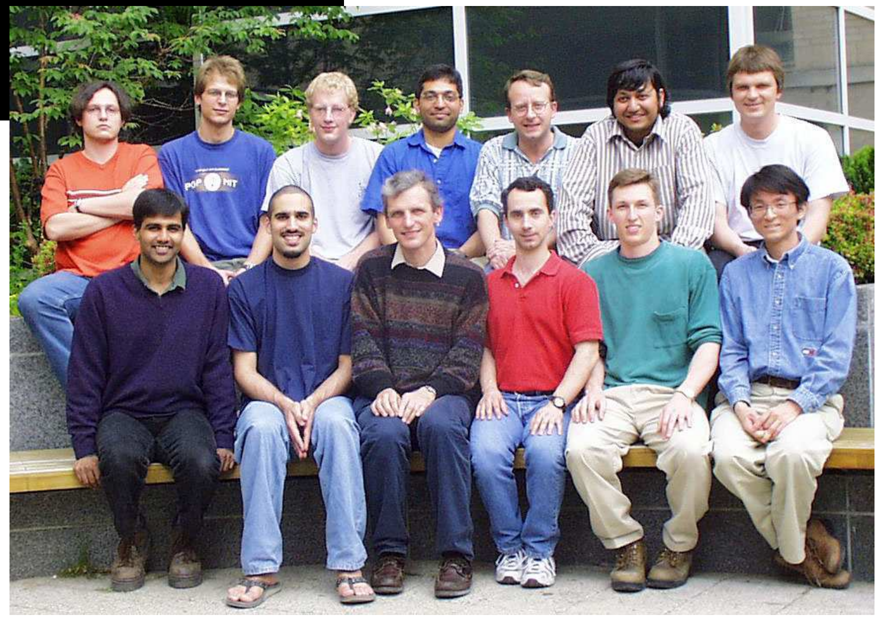
E. Cornell, C. Wieman, et al.
(JILA, NIST, U of Colorado)

Nobel Prize in Physics 2001

BEC with Sodium Atoms



- ^{23}Na $(F = 1, m_F = -1)$
- $N_{\text{initial}} \approx 10^9$
- $N_{\text{BEC}} \approx 5 \times 10^5$
- $T_c \approx 2\,\mu$K
- absorption image after $60\,$ms expansion

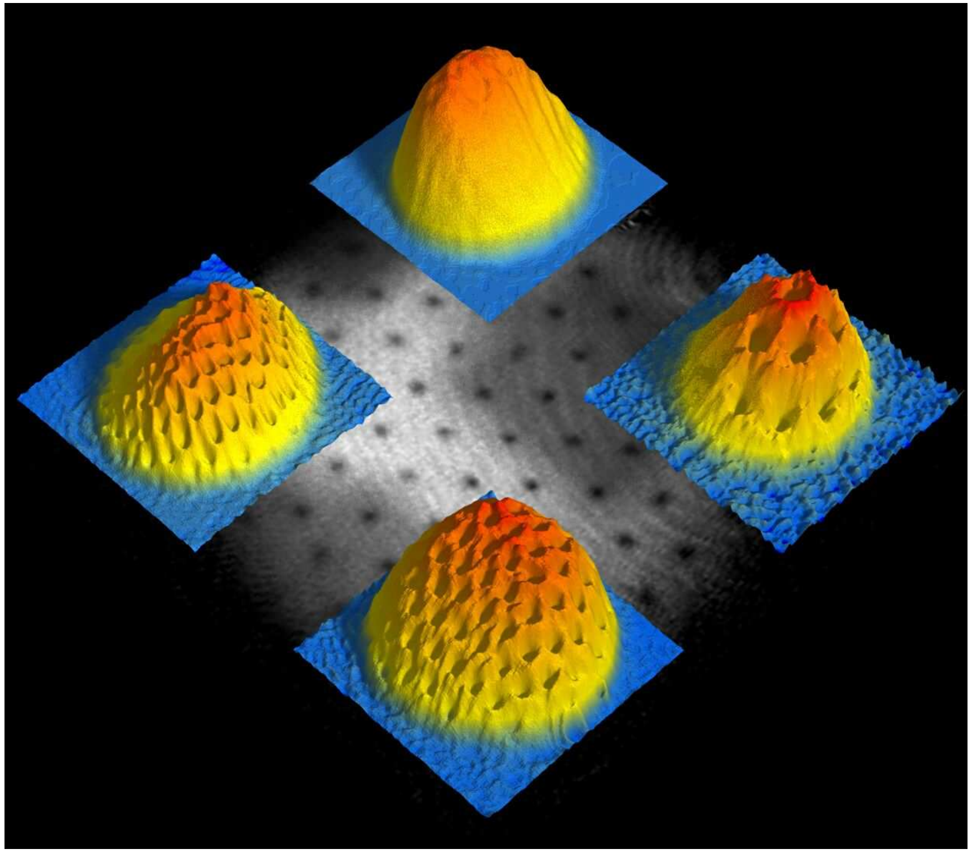


W. Ketterle, et al.
(MIT)

Nobel Prize in Physics 2001

Dynamics of Dilute Quantum Gases

- amazing experimental achievements
  - condensates of ^{1}H, ^{4}He*, ^{7}Li, ^{23}Na, ^{41}K, ^{85}Rb, ^{87}Rb, ^{133}Cs, ^{174}Yb
  - vortices, vortex lattices and their dynamics
  - bright and dark solitons and soliton trains
  - collective excitations and collapse
  - boson-fermion mixtures and ultracold fermions



[W. Ketterle *et al.*; Science 292 (2001) 476]

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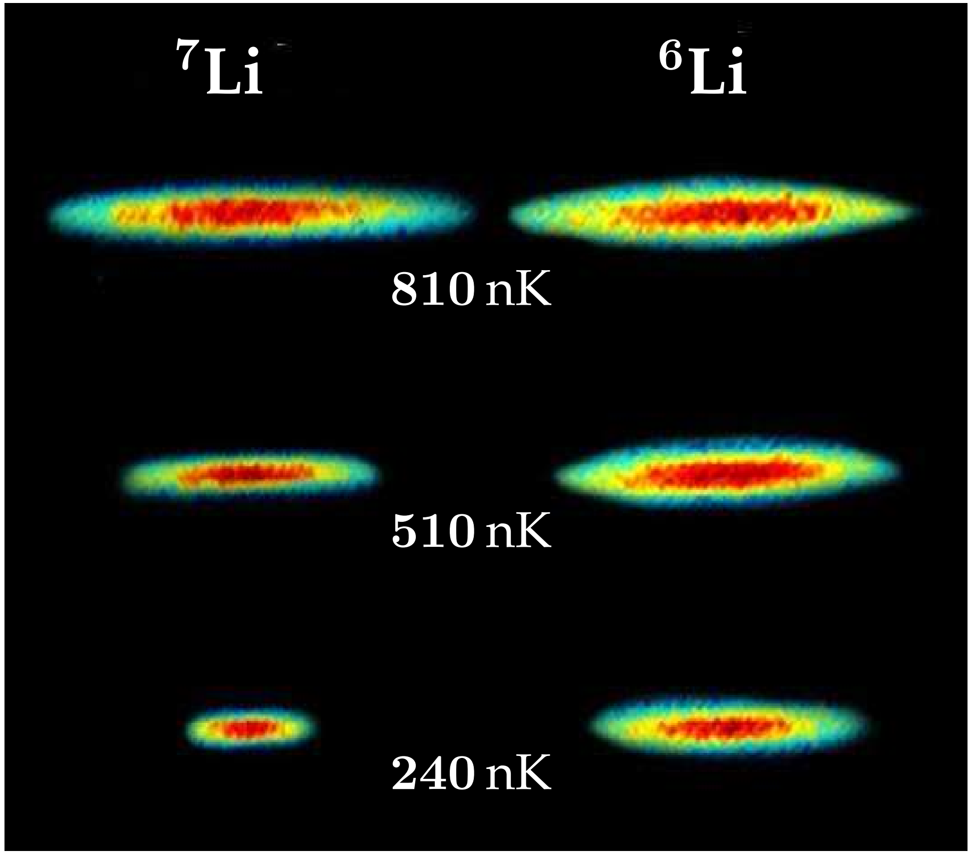

[R. Hulet *et al.*; Science 291 (2001) 2570]

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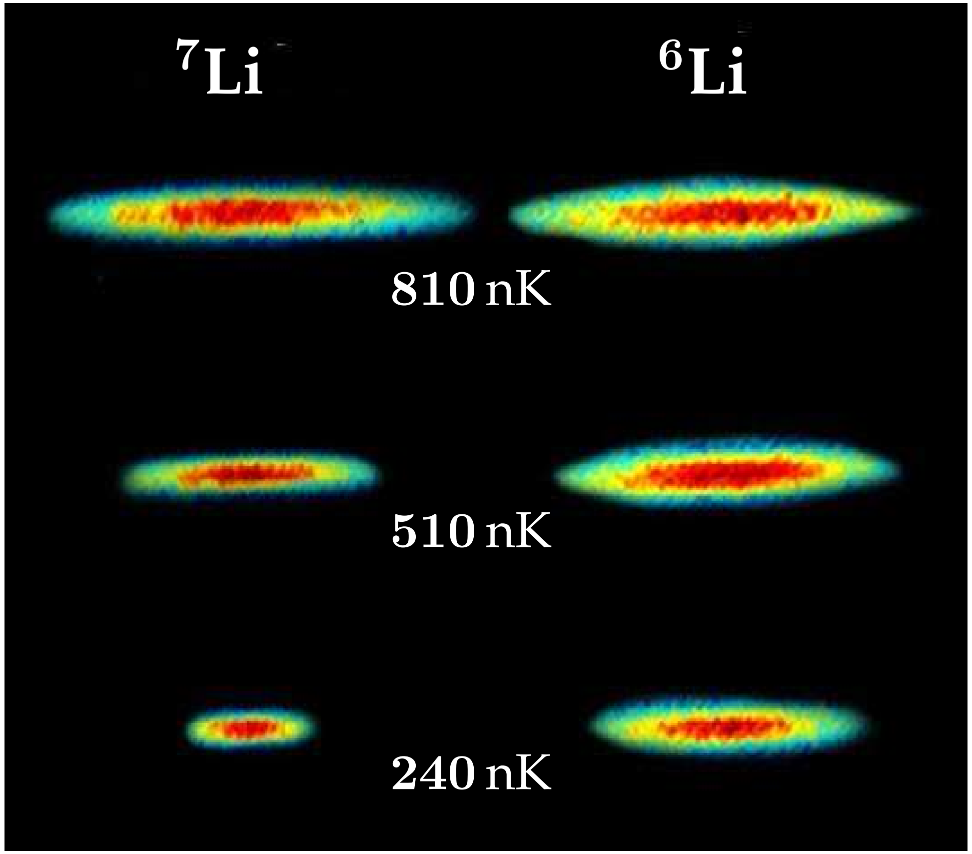
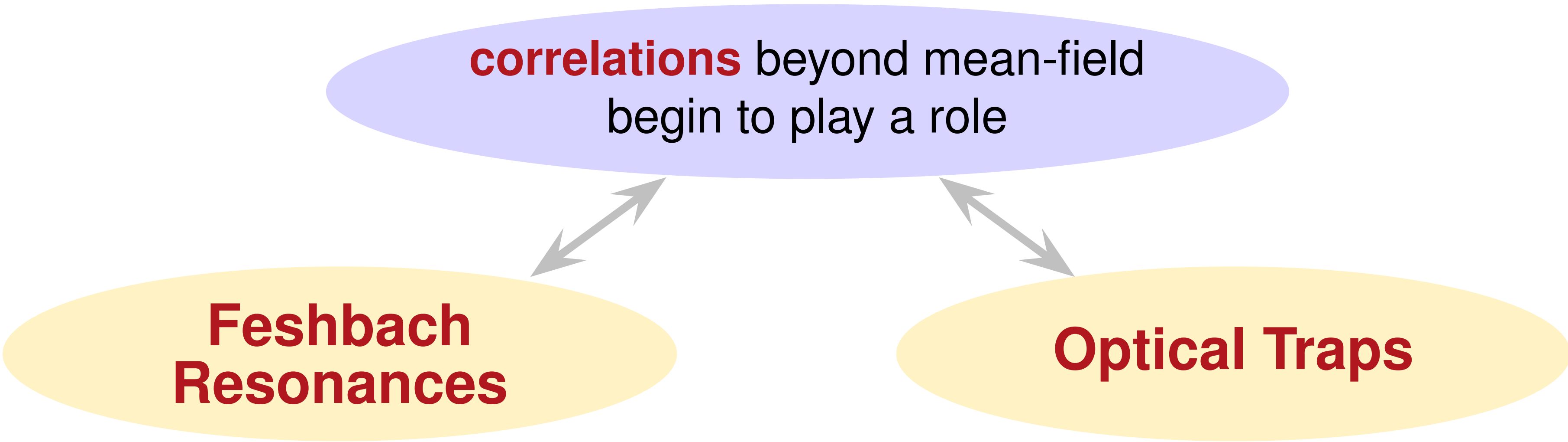


[R. Hulet *et al.*; Science 291 (2001) 2570]

► all these phenomena are well described in the framework of a **mean-field theory** (Gross-Pitaevskii equation for bosons)

...Today

The Advent of Correlations



The Advent of Correlations

correlations beyond mean-field begin to play a role

Feshbach Resonances

- tuning of the scattering length over several orders of magnitude
- **strong interaction regime**: mean-field collapse, three-body losses,...
- **coherent molecule formation**: molecular condensates, ultracold chemistry

Optical Traps

- tightly confining traps with a multitude of geometries
- **quasi 1D and 2D traps**: quantum gases in low dimensions
- **optical lattices in 1D, 2D and 3D**: band structure, quantum phase transitions, disorder, ...

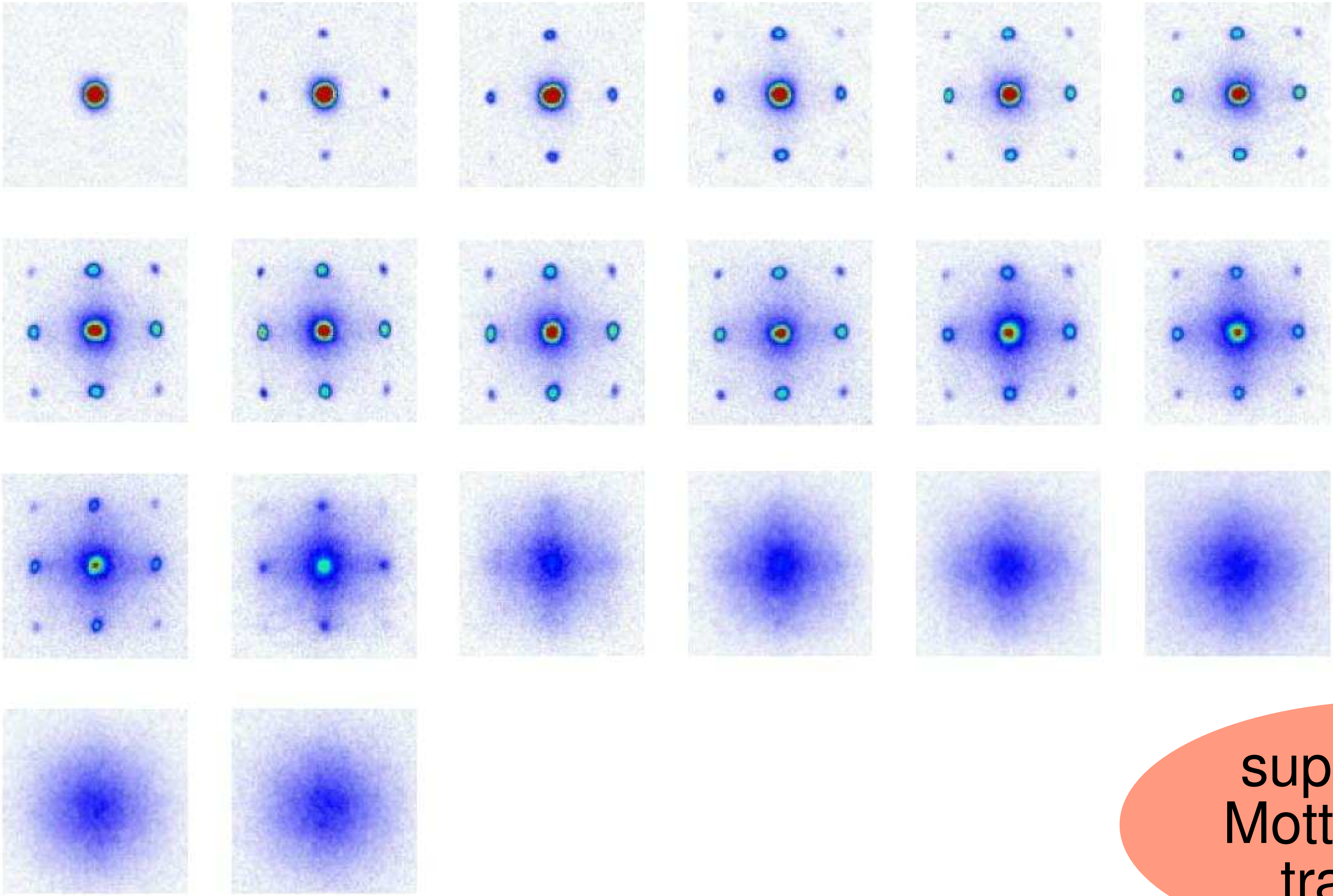
A Theoreticians' View of

The Lattice Experiment

- produce a **Bose-Einstein condensate** of atoms in a magnetic trap
- load the condensate into an **optical standing-wave lattice** created by counter-propagating laser beams
- in a 3D lattice one ends up with **few atoms per lattice site** (favourable to study quantum phase transitions) in a 1D lattice one can have thousands of atoms
- probe different physical regimes by varying **lattice depth** and **interaction strength**
- switch off the lattice, let the gas expand, and observe the **matter-wave interference pattern**

Interference Pattern

increasing lattice depth ⟶



characteristic interference pattern of array of coherent BECs

incoherent background emerges and peaks vanish

superfluid to Mott-insulator transition

[M. Greiner *et al.*; Nature 415 (2002) 39]

Questions...

- How to describe ultracold bosons in a lattice?
- How to define **superfluid** and **condensate**?
- What is the **superfluid to Mott-insulator transition**?
- Are there **other quantum-phases** one can investigate?
- What happens if the lattice is **irregular**?
- What about **fermions**?

Bose-Hubbard Model

Bose-Hubbard Model

- one-dimensional lattice with N particles and I lattice sites at $T = 0\mathrm{K}$
- restrict Hilbert space to the **lowest energy band**
- localised Wannier wavefunctions $w_i(x)$ with associated **occupation numbers** n_i for the individual sites $i = 1...I$

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- represent N-boson state in complete basis of **Fock states** $|\{n_1, ..., n_I\}_\alpha\rangle$

$$|\Psi\rangle = \sum_{\alpha=1}^{D} C_\alpha \, |\{n_1, ..., n_I\}_\alpha\rangle$$

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- basis dimension D **grows dramatically** with I and N

I	6	8	10	12
D	462	6435	92 378	1 352 078

for $N/I = 1$

Bose-Hubbard Hamiltonian

- second quantised Hamiltonian with respect to Wannier basis [Fisher *et al.* (1989); Jaksch *et al.* (1998)]

$$\hat{\mathrm{H}}_0 = -J\sum_{i=1}^{I}(\hat{\mathrm{a}}_{i+1}^{\dagger}\hat{\mathrm{a}}_i + \mathrm{h.a.}) + \sum_{i=1}^{I}\epsilon_i\,\hat{\mathrm{n}}_i + \frac{V}{2}\sum_{i=1}^{I}\hat{\mathrm{n}}_i(\hat{\mathrm{n}}_i - 1)$$

tunnelling between adjacent lattice sites

single-particle energy

on-site two-body interaction

- assumptions: (a) only lowest band, (b) constant nearest-neighbour hopping, (c) only short-range interactions

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▸ Bose-Hubbard model is able to describe **strongly correlated systems** as well as **pure condensates**

▸ **exact solution**: compute the lowest eigenstates of $\hat{H}_0$ using iterative Lanczos algorithms

Simple Physical Quantities

- consider a regular lattice ➔ $\epsilon_i = 0$
- solve eigenproblem for various V/J

- **mean occupation numbers**

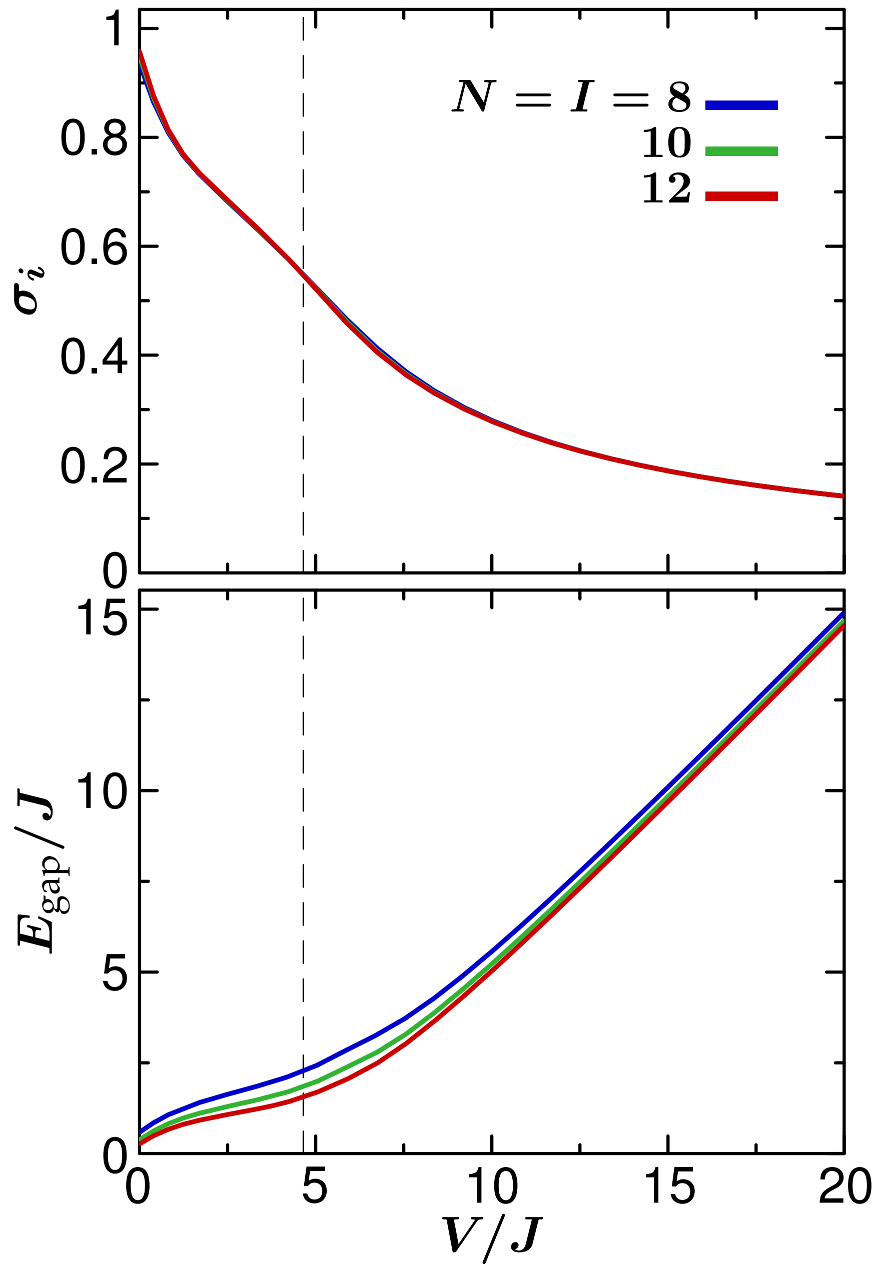
$$\bar{n}_i = \langle \Psi_0 | \hat{n}_i | \Psi_0 \rangle$$

- **number fluctuations**

$$\sigma_i = \sqrt{\langle \Psi_0 | \hat{n}_i^2 | \Psi_0 \rangle - \langle \Psi_0 | \hat{n}_i | \Psi_0 \rangle^2}$$

- **energy gap**

$$E_{\text{gap}} = E_{\text{1st excited}} - E_0$$



Condensate & Superfluidity

- eigensystem of the **one-body density matrix**

$$\rho_{ij}^{(1)} = \langle \Psi_0 | \hat{a}_j^\dagger \hat{a}_i | \Psi_0 \rangle$$

defines natural orbitals and the corresponding occupation numbers

Bose-Einstein Condensate

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- Onsager-Penrose criterion: **Bose-Einstein condensate** is present if one of the eigenvalues of $\rho_{ij}^{(1)}$ is of order N (in the thermodynamic limit)

eigenvalue	$\longrightarrow$	N_0	:	number of condensed particles
eigenvector	$\longrightarrow$	$\phi_{0,i}$	:	condensate wave function

General Definition of the Bose-Einstein Condensate

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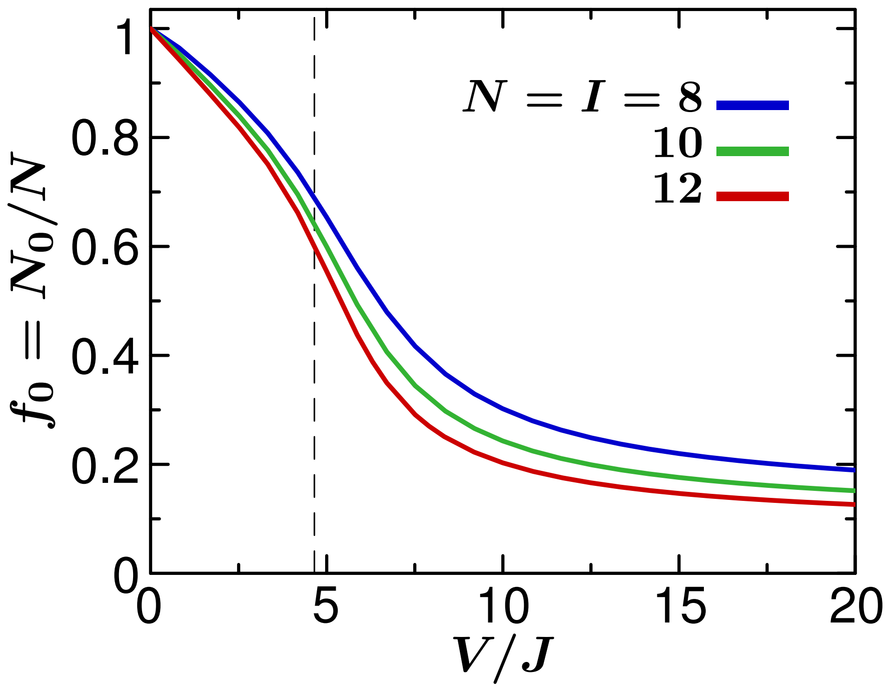
- in a regular lattice the natural orbitals are **quasimomentum eigenstates**

Condensate & Quasimomentum Distribution



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- rapid depletion of the condensate with increasing V/J
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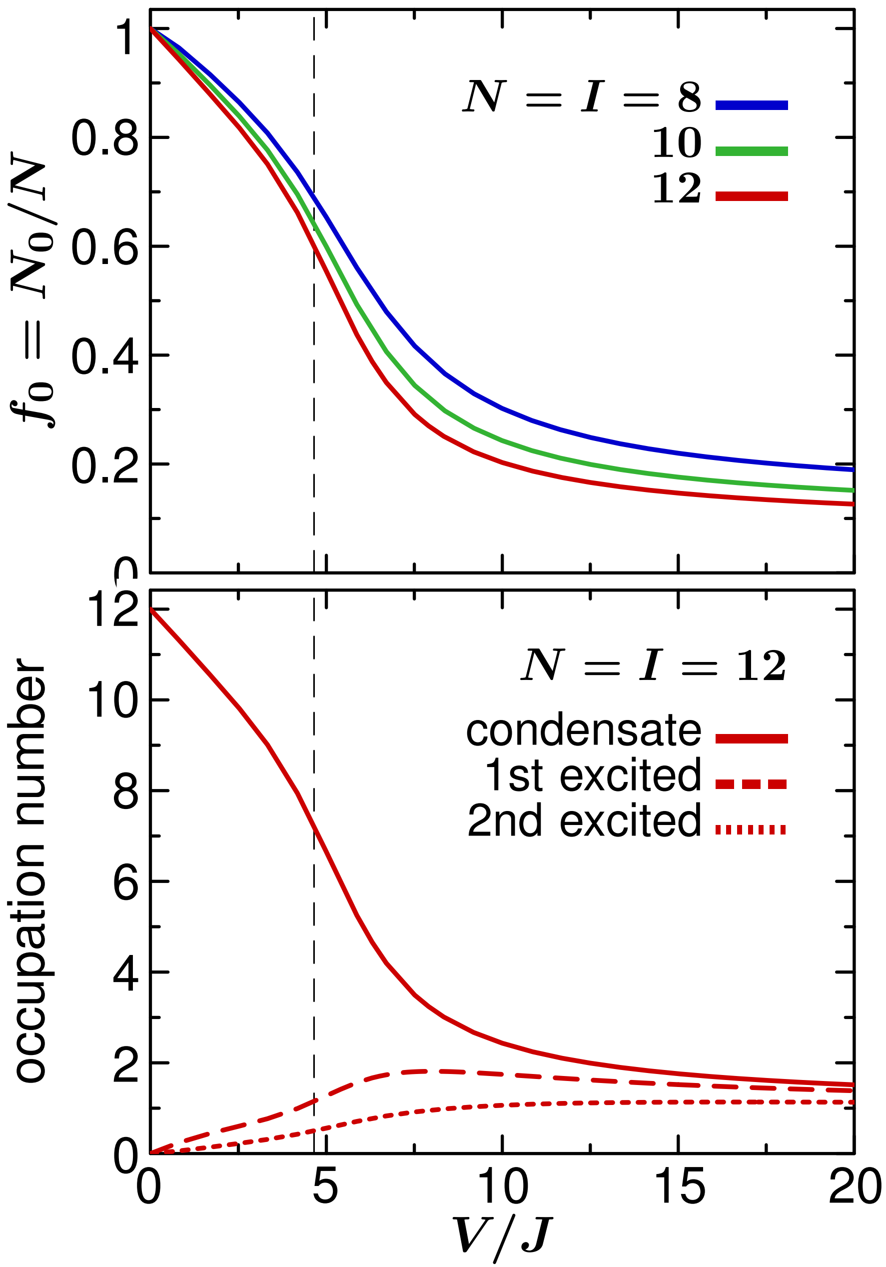

- pure condensate for $V/J = 0$
- rapid depletion of the condensate with increasing V/J
- finite size effect: condensate fraction in a finite lattice always $\geq 1/I$
- states with larger quasimomentum are populated successively
- homogeneous occupation of the band in the limit of large V/J

What is Superfluidity?

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- macroscopically the superfluid flow is **non-dissipative** and **irrotational**, i.e., it is described by the gradient of a scalar field

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- energy in the comoving frame differs from ground state energy in the rest frame by the **kinetic energy of the superflow**

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► these two ideas are basis for the **microscopic definition of superfluidity**

Definition of Superfluidity

- the velocity field of the superfluid is defined by the **gradient of the phase** of the condensate wavefunction $\phi_0(\vec{x})$

$$\vec{v}_{\mathrm{SF}} = \frac{\hbar}{m} \vec{\nabla} \theta(\vec{x}) \qquad \phi_0(\vec{x}) = \mathrm{e}^{\mathrm{i}\theta(\vec{x})} \, |\phi_0(\vec{x})|$$

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- employ **twisted boundary conditions** to impose a linear phase variation

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- **superfluid fraction** $=$ stiffness with respect to phase variations

$$F_{\text{SF}} = \frac{N_{\text{SF}}}{N} = \frac{2m\, L^2}{\hbar^2 N} \, \frac{E_\Theta - E_0}{\Theta^2} \qquad \Theta \ll \pi$$

Superfluidity on the Lattice

- express $\boldsymbol{F}_{\text{SF}}$ in terms of the parameters of the Bose-Hubbard model

$$\boldsymbol{F}_{\text{SF}} = \frac{\boldsymbol{m}}{\boldsymbol{m}^\star} \, \boldsymbol{f}_{\text{SF}} \qquad \boldsymbol{f}_{\text{SF}} = \frac{\boldsymbol{I}^2}{\boldsymbol{J}\boldsymbol{N}} \, \frac{\boldsymbol{E}_{\boldsymbol{\Theta}} - \boldsymbol{E}_0}{\boldsymbol{\Theta}^2}$$

$$I = L/a$$
$$J = \hbar^2/(2m^\star a^2)$$

  - $\frac{m}{m^\star}$: reduction of flow by the lattice itself
  - f_{SF} : interaction-induced depletion relevant for quantum phase transitions

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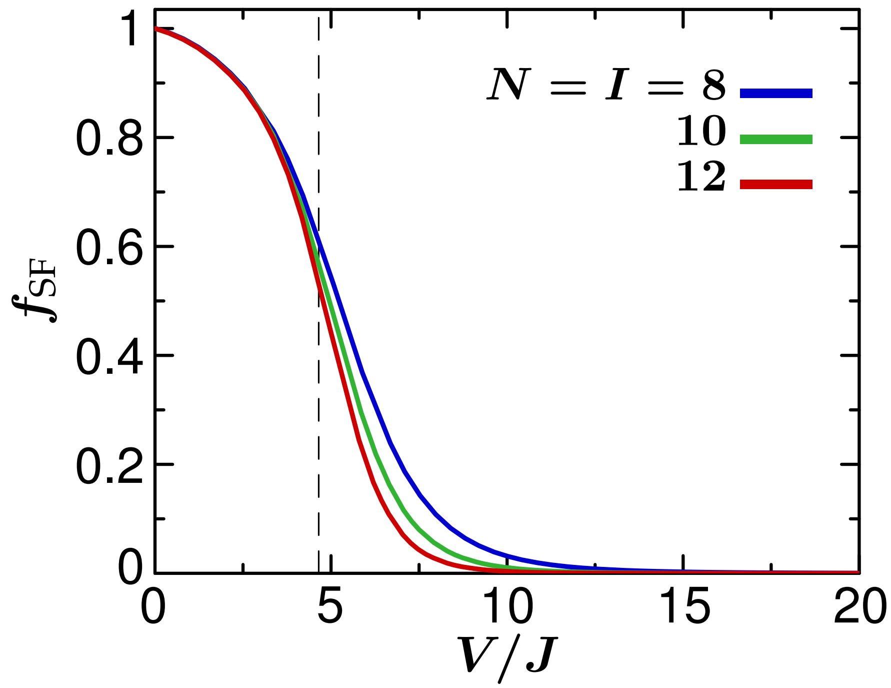
- closely related to helicity modulus [Fisher, Barber, Jasnow (1973)] and winding number [Pollock, Ceperley (1987)]

- this is not the Landau picture of superfluidity ➔ we do not consider the stability of the superflow (critical velocity)

Superfluid Fraction



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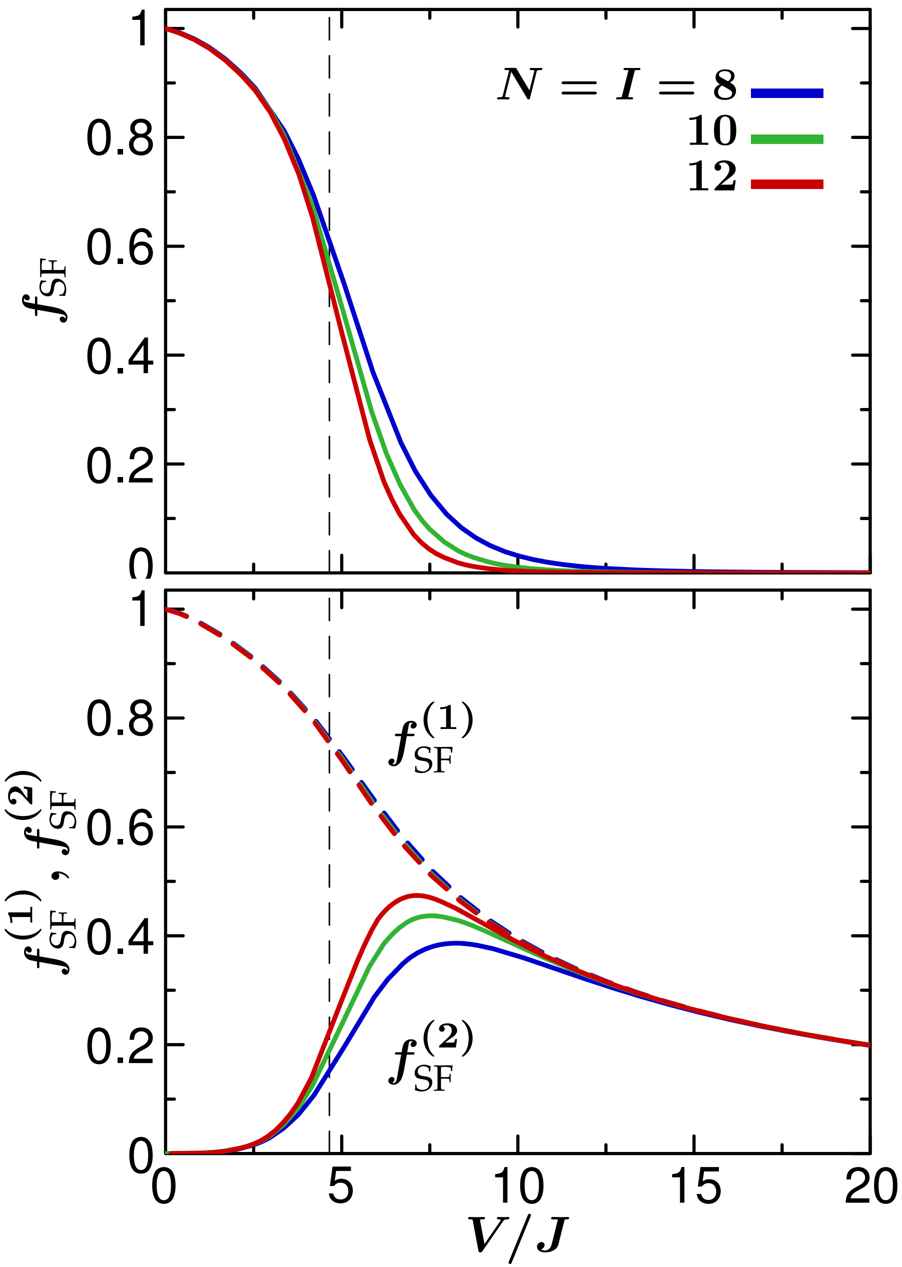

- superfluid fraction is the natural **order parameter** for the superfluid-insulator transition
- rapid decrease of f_{SF} in a narrow window in V/J already for small systems
- $f_{SF}^{(1)}$ decreases only very slowly
- vanishing of f_{SF} is due to a cancellation between $f_{SF}^{(1)}$ and $f_{SF}^{(2)}$
- coupling to **excited states** is crucial for the vanishing of f_{SF} in the insulating phase

Condensate -vs- Superfluid

Condensate

- largest eigenvalue of the one-body density matrix
- involves only the ground state
- measure for off-diagonal long-range order / coherence

Superfluid

- response of the system to an external perturbation
- depends crucially on the excited states of the system
- measures a flow property

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$\neq$

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$f_0 < f_{SF}$

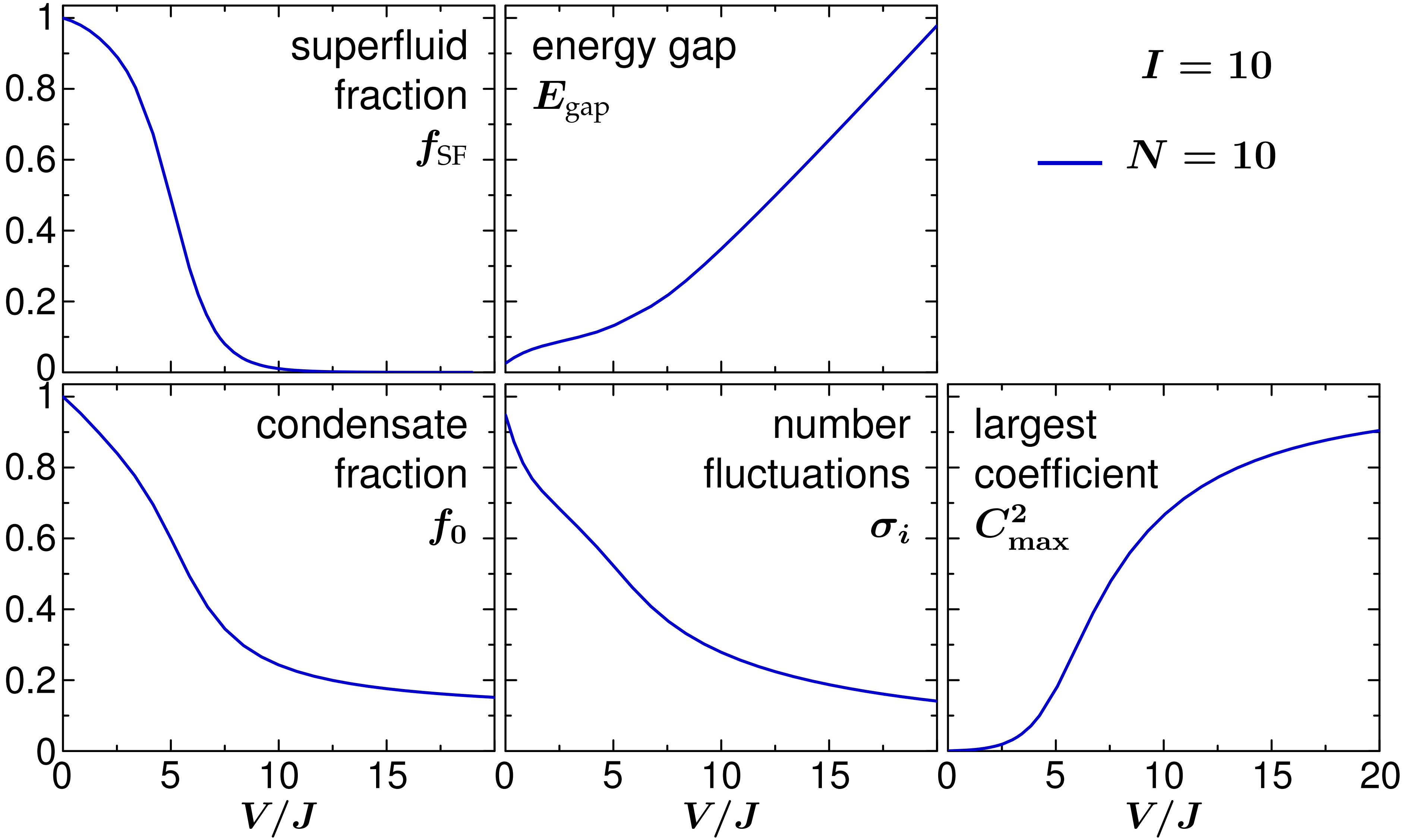
- non-condensed particles are dragged along with condensate
- liquid ^{4}He at $T = 0$K:

$$f_0 \approx 0.1, \quad f_{SF} = 1$$

$f_0 > f_{SF}$

- part of the condensate has a reduced stiffness under phase variations
- seems to occur in systems with defects or disorder

Superfluid to Mott-Insulator Transition



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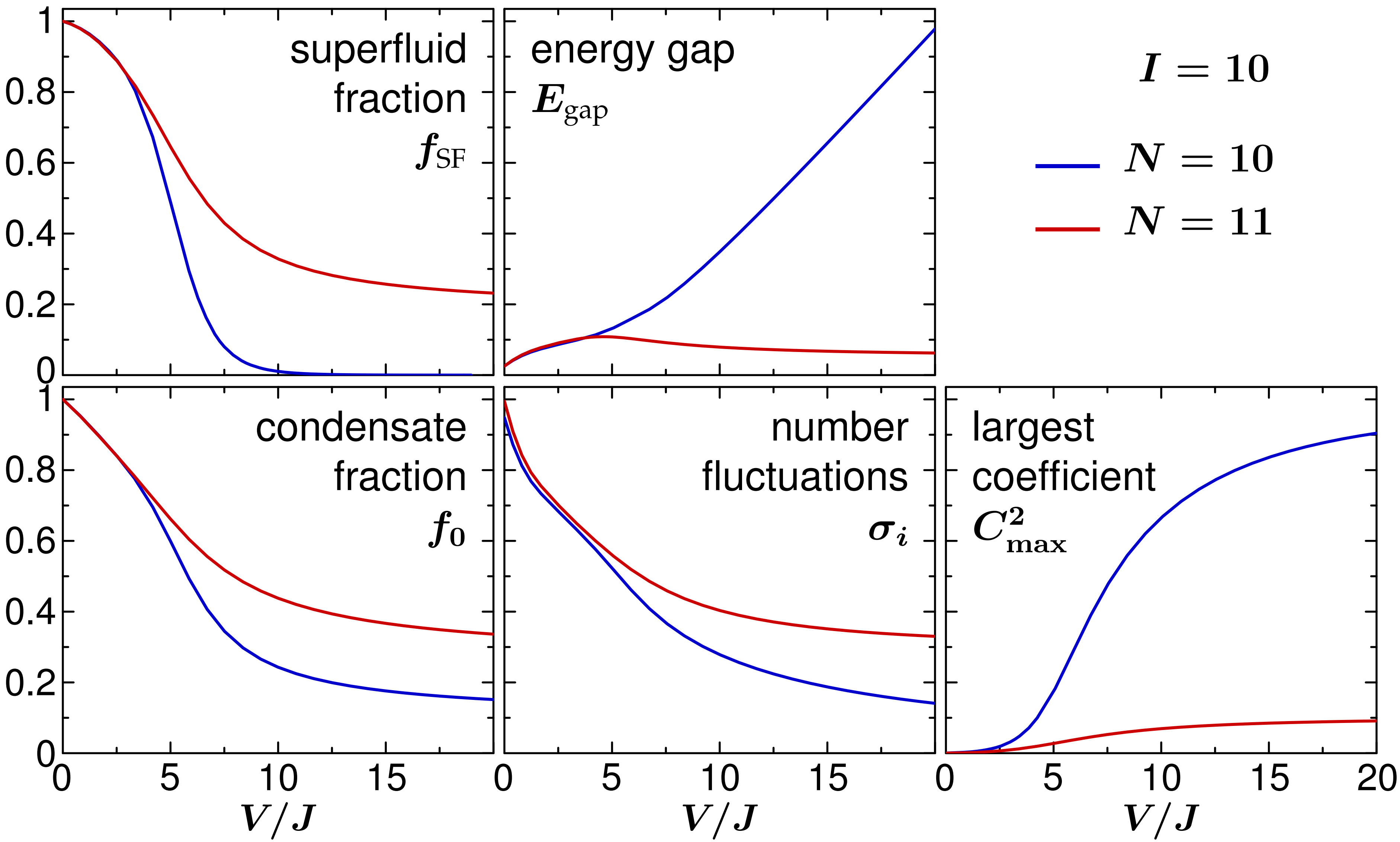

Two-Colour Superlattices

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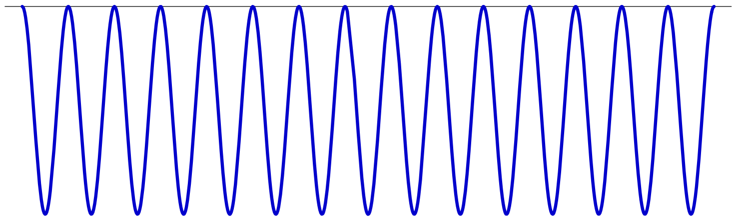
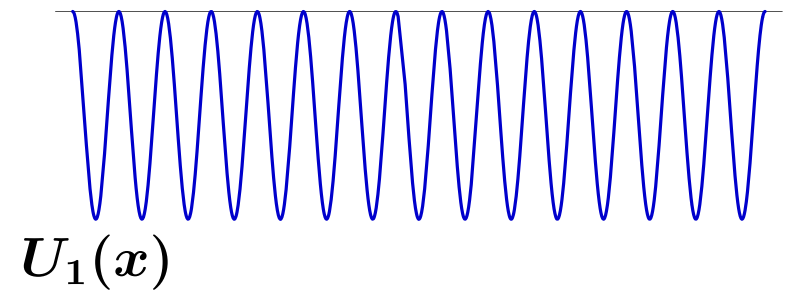
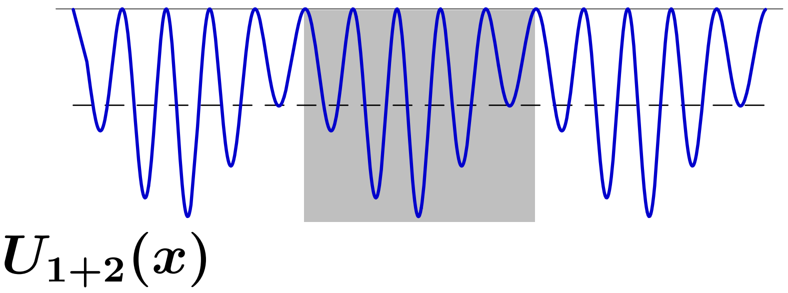


$U_1(x)$

- start with a standing wave created by a laser with wavelength λ_1

Two-Colour Superlattices





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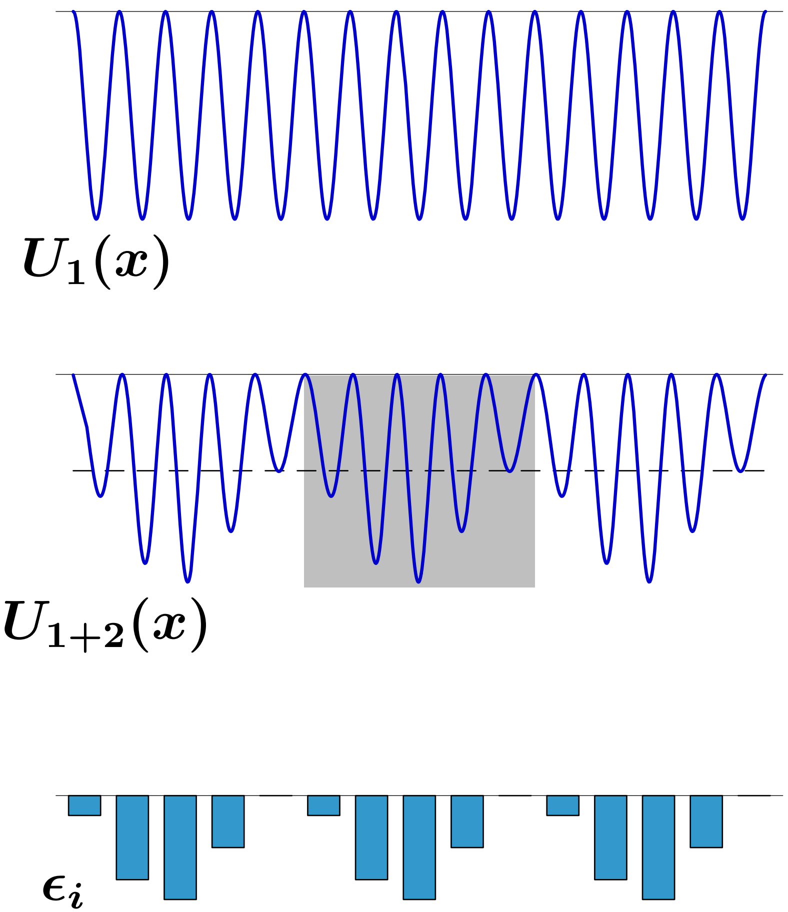
- add a second standing wave created by a laser with wavelength $\lambda_2 = \frac{5}{7}\lambda_1$ and much smaller intensity (here 4%)
- potential exhibits a periodic modulation of the well-depth with a period of 5 sites

Two-Colour Superlattices



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- Bose-Hubbard model: varying on-site energies $\epsilon_i \in [0, -\Delta]$

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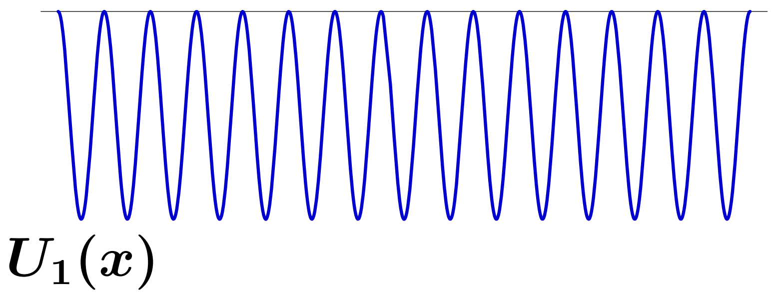
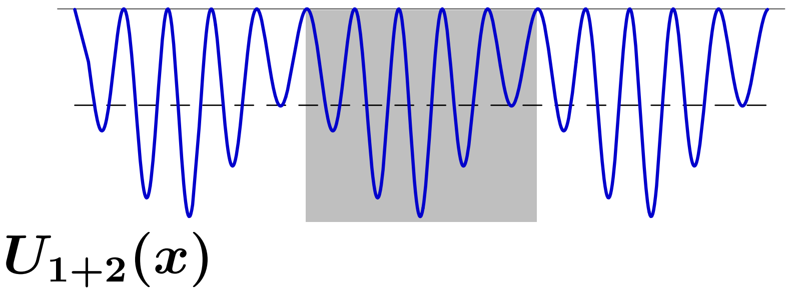
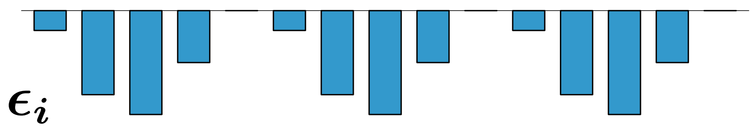


- start with a standing wave created by a laser with wavelength λ_1



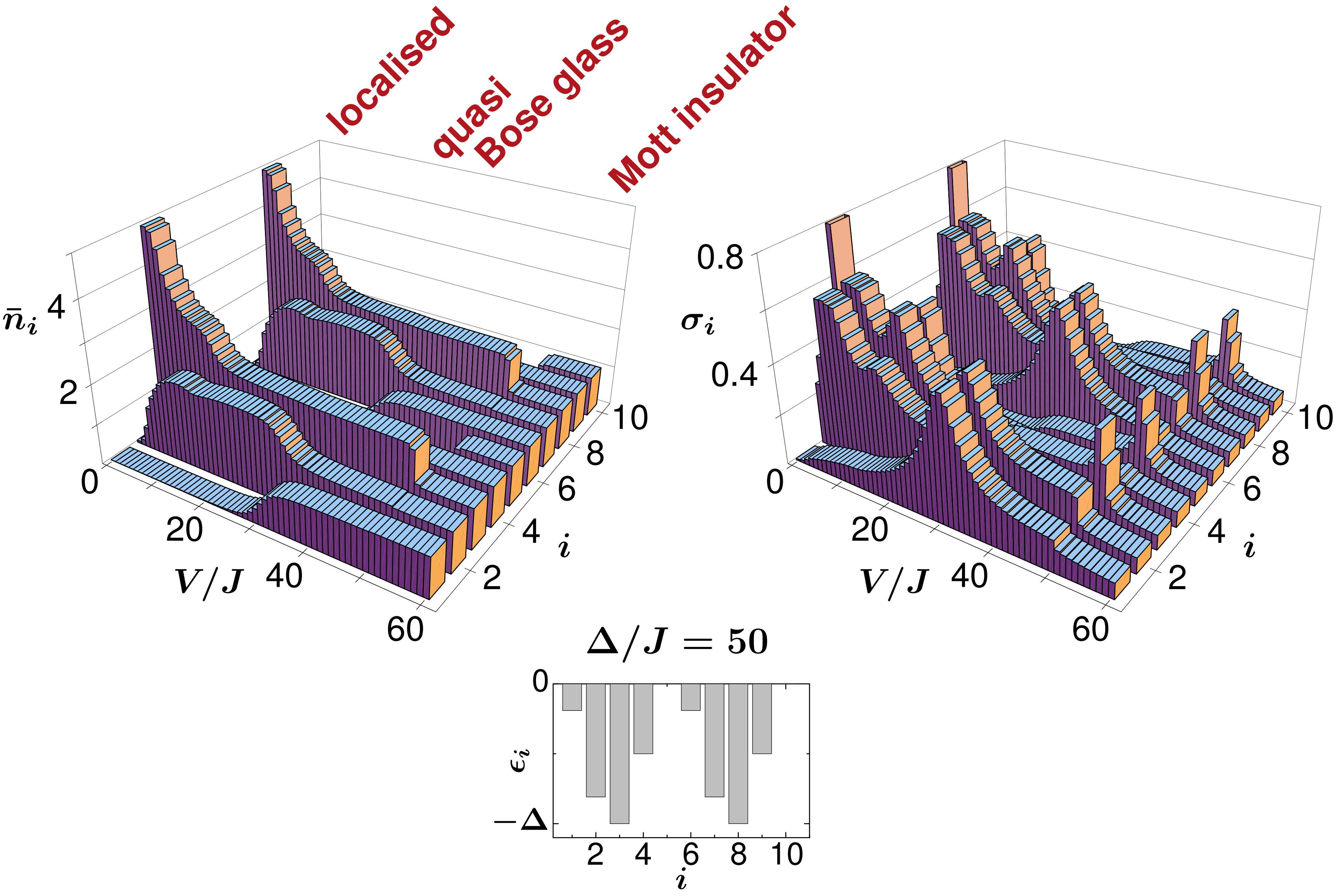
- add a second standing wave created by a laser with wavelength $\lambda_2 = \frac{5}{7}\lambda_1$ and much smaller intensity (here 4%)
- potential exhibits a periodic modulation of the well-depth with a period of 5 sites



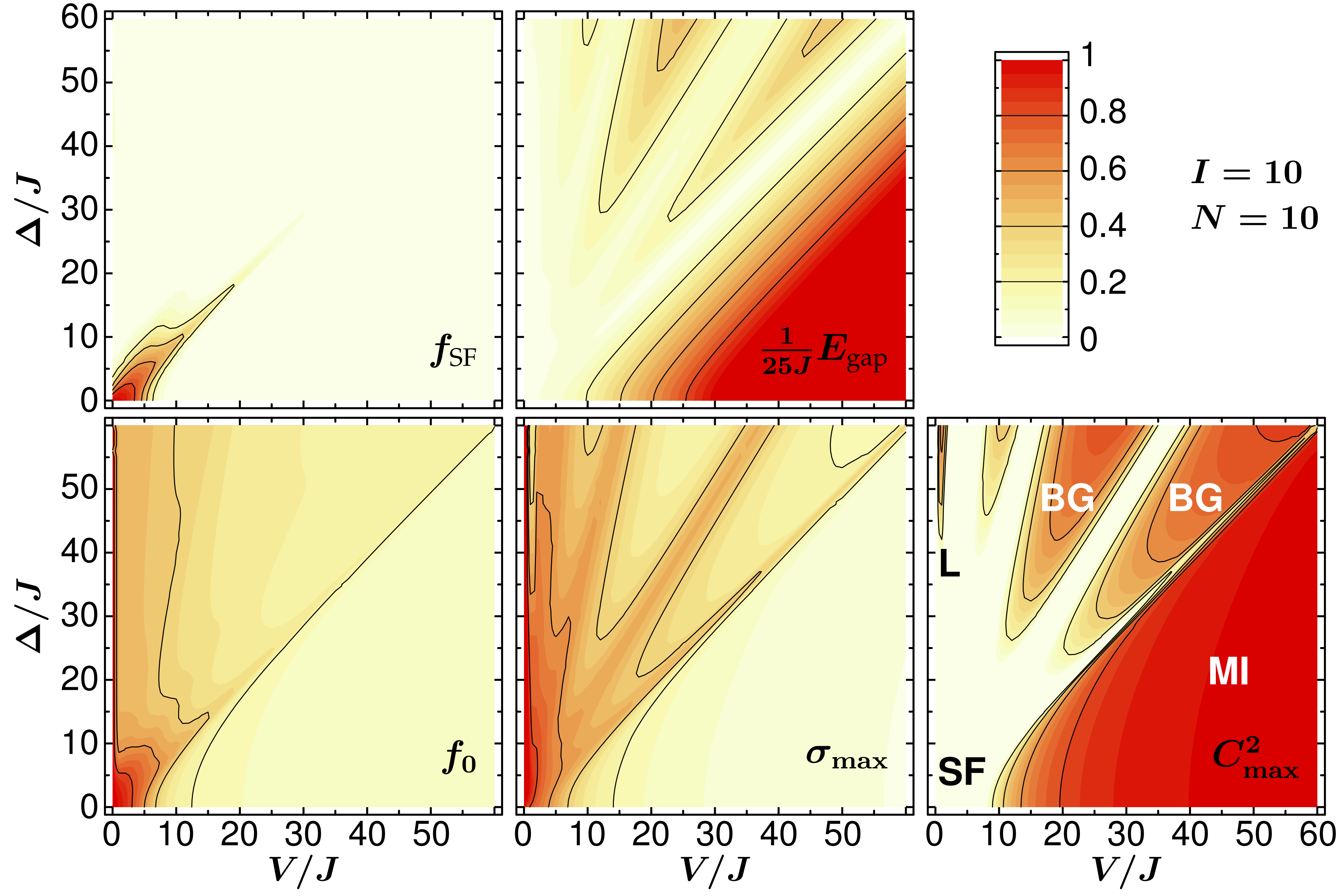
- Bose-Hubbard model: varying on-site energies $\epsilon_i \in [0, -\Delta]$

► **controlled lattice irregularities** open novel possibilities to study “disorder” related effects; more complex topologies easily possible

Interaction -vs- Lattice Irregularity



V-Δ Phase Diagrams



Boson-Fermion Mixtures in Lattices

Bose-Fermi-Hubbard Hamiltonian

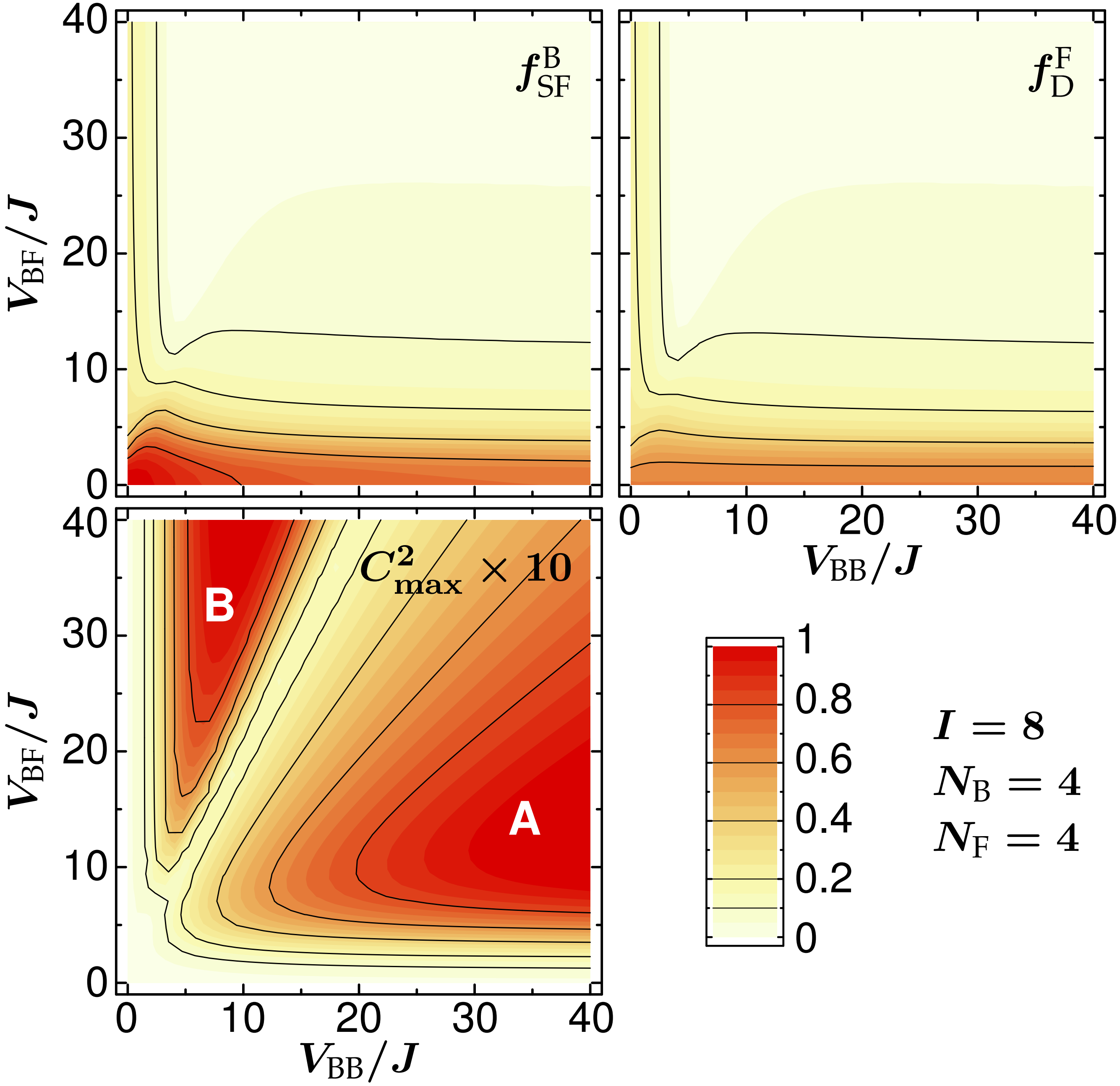
- second quantised Hamiltonian containing boson (B) and fermion (F) operators [Albus et al. (2003)]

$$\hat{H}_0 = -J_{B} \sum_{i=1}^{I} (\hat{a}_{i+1}^{B\dagger} \hat{a}_i^{B} + \text{h.a.}) + \frac{V_{BB}}{2} \sum_{i=1}^{I} \hat{n}_i^{B} (\hat{n}_i^{B} - 1)$$
$$- J_{F} \sum_{i=1}^{I} (\hat{a}_{i+1}^{F\dagger} \hat{a}_i^{F} + \text{h.a.}) + V_{BF} \sum_{i=1}^{I} \hat{n}_i^{B} \hat{n}_i^{F}$$

- exact solution of eigenvalue problem in combined Fock-state representation

- in addition to ground state observables we employ two stiffnesses to characterise the various phases
  - **bosonic phase stiffness** ➔ boson superfluid fraction
  - **fermionic phase stiffness** ➔ Drude weight, fermionic conductivity

V_{BB}-V_{BF} Phase Diagrams



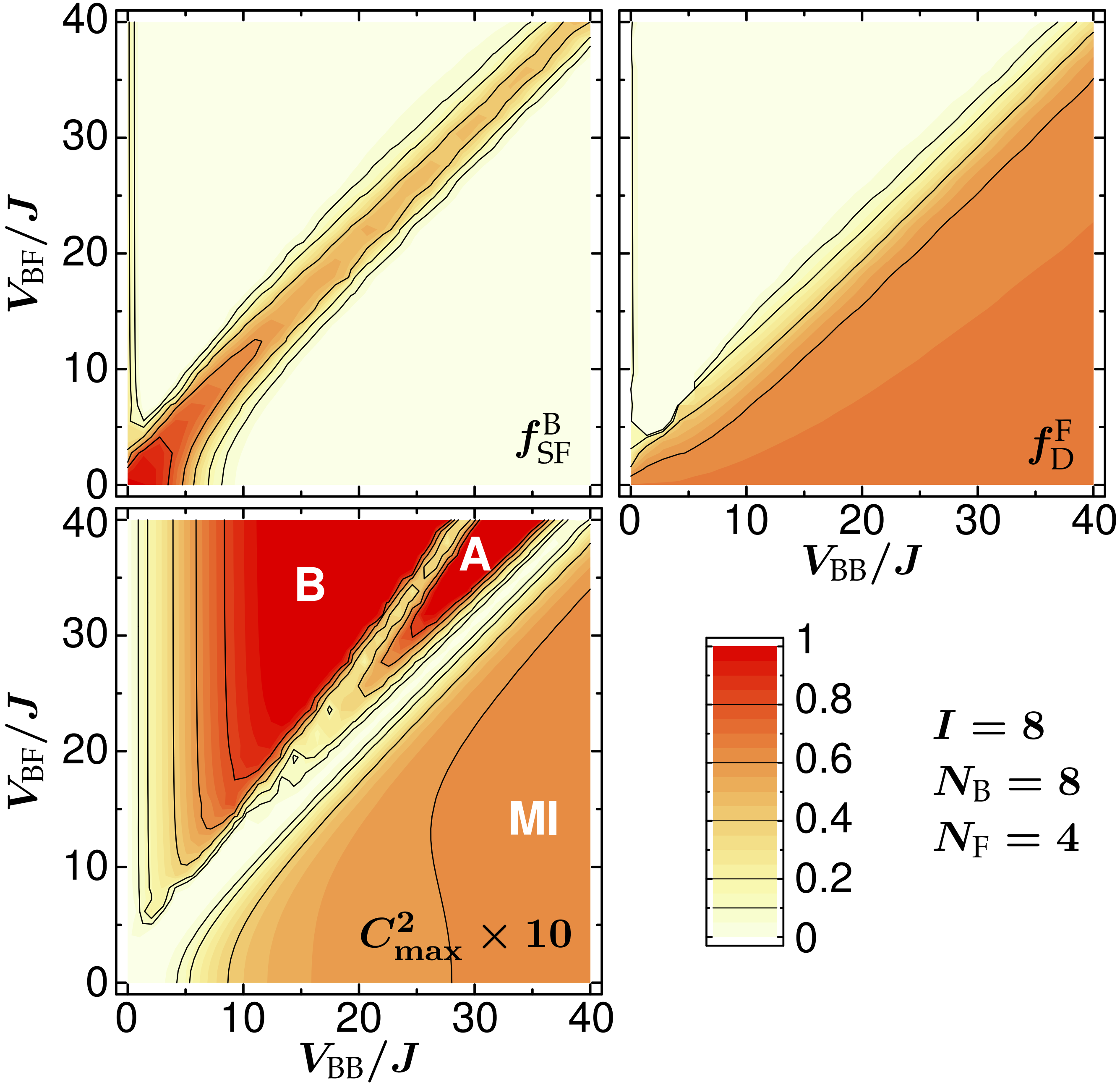
A: alternating boson-fermion occupation

➔ crystalline diagonal long-range order

B: continuous boson and fermion blocks

➔ component separation

V_{BB}-V_{BF} Phase Diagrams



A: alternating boson-fermion occupation

➔ crystalline diagonal long-range order

B: continuous boson and fermion blocks

➔ component separation

MI: bosonic Mott insulator

➔ fermions not affected

Conclusions

- **Superfluidity**
  - stiffness under phase twists; depends crucially on the excitation spectrum
- **Condensate & Coherence**
  - property of the one-body density matrix of the ground state
  - ground state quantities (interference pattern, fluctuations, etc.) cannot give direct information on the superfluid fraction
- **Two-Colour Superlattices**
  - rich phase diagram with several insulating phases: localised, quasi Bose-glass, Mott-insulator
- **Boson-Fermion Mixtures in Lattices**
  - novel class of lattice systems with largely unexplored phase diagram

Epilogue

- unique degree of **experimental control** makes ultracold atomic gases in optical lattices...
  - ideal model systems to study strong correlation effects (quantum phase transitions) and other solid-state questions
  - promising “hardware” for quantum information processing

- many **fascinating questions** still open...
  - fermions and boson-fermion mixtures in lattices, spinor Bose gases
  - long-range interactions, Cooper pairs, molecules, dynamics,...

Epilogue

- **References**

  - R. Roth, K. Burnett; cond-mat/0310114
  - R. Roth, K. Burnett; Phys. Rev. A 68 (2003) 023604
  - R. Roth, K. Burnett; Phys. Rev. A 67 (2003) 031692(R)
  - A. M. Rey, et al.; J. Phys. B 36 (2003) 825
  - R. Roth, K. Burnett; J. Opt. B 5 (2003) S50

- **€ / £**

  - DFG, UK EPSRC, EU