Exact time evolution of atomic gases in modulated 1D optical lattices

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Overview

- Framework
  - Bose Hubbard model
  - Oscillating lattice amplitude
  - Time evolution
- Results
  - Bose gas
  - Fermi/Fermi mixture
- Summary / Outlook

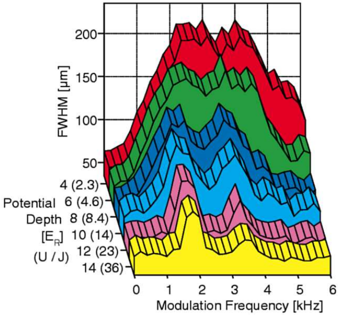
Experimental motivation

Setup

- trapped Bose gas in an 1D optical lattice
- lattice potential is fixed at a specific depth
- depth is modulated by a frequency ω

Results

- broad excitation spectrum in superfluid phase
- sharp resonance peaks in Mott phase



Esslinger *et al*, Phys. Rev. Lett. **92**,130403 (2004)

Framework

Bose Hubbard Hamiltonian ...

$$\mathbf{H} = -\mathcal{J} \sum_{i}^{I} \left(\mathbf{a}_i^\dagger \mathbf{a}_{i+1} + \mathbf{a}_{i+1}^\dagger \mathbf{a}_i \right) + \frac{\mathcal{U}}{2} \sum_{i}^{I} \mathbf{n}_i \left(\mathbf{n}_i - 1 \right)$$

$\mathcal{J}$	tunneling strength
$\mathcal{U}$	interaction strength
I	# of lattice sites
D	dimensions of system
n_i	# of atoms at i-th site

... and its groundstate

$$|\Psi_0\rangle = \sum_{\alpha}^{D} c_\alpha^{(0)} |\{n_1 n_2 \ldots n_I\}_\alpha\rangle$$

Lattice amplitude modulation

oscillating lattice amplitude

$$V_{\text{lattice}}(x,t) = V_{0,\text{lattice}}\left(1 + \mathcal{F}\sin\left(\omega t\right)\right)\sin^2\left(kx\right)$$

amplitude $\mathcal{F}$, frequency ω

$\downarrow$

How do $\mathcal{J}(t)$ and $\mathcal{U}(t)$ look like ?

$$\begin{aligned}
\mathcal{J}(t) &\approx \mathcal{J}_0 \exp\left(-\mathcal{F}\sin\left(\omega t\right)\right) \\
\mathcal{U}(t) &\approx \mathcal{U}_0\left(1 + \mathcal{F}\sin\left(\omega t\right)\right)^{1/4}
\end{aligned}$$

Time evolution

how to evolute a system...

... described by a high dimensional, time-dependent Hamilton matrix ?

$\Longrightarrow$ **Crank-Nicholson scheme**:

$$U(t, t + \Delta t) = \exp(-i\mathbf{H}(t)\Delta t) \approx \frac{1 - i\mathbf{H}(t)\Delta t/2}{1 + i\mathbf{H}(t)\Delta t/2}$$

nice features

- unitary
- unconditionally stable
- 2nd order in time

not so nice features

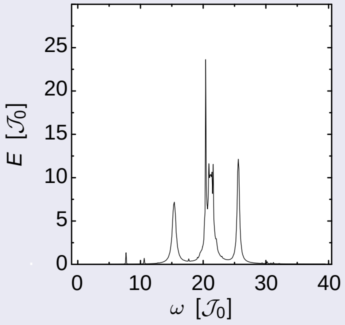
- solve a (sparse) linear equations set in each timestep

Bose gas
system setup

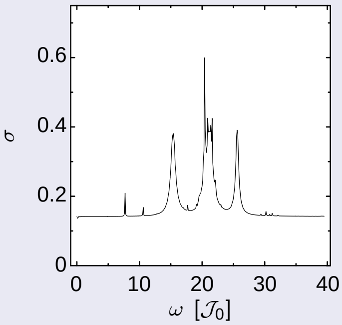
parameters

- I=6 lattice sites
- N=6 bosonic atoms
- $\mathcal{J}_0$=1 tunneling strength
- $\mathcal{U}_0$=20 interaction strength

energy transfer



number fluctuations

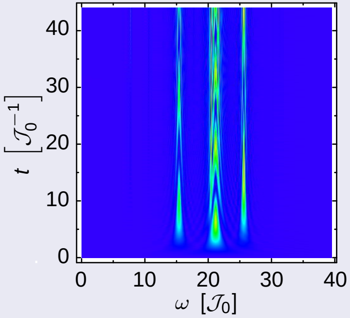


Bose gas
time evolution of energy and fluctuations

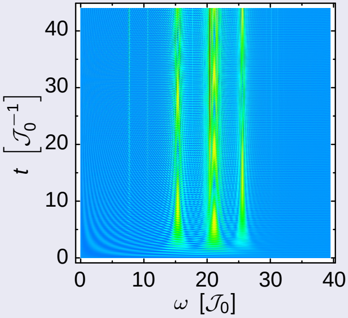
parameters

- I=6 lattice sites
- N=6 bosonic atoms
- $\mathcal{J}_0$=1 tunneling strength
- $\mathcal{U}_0$=20 interaction strength

energy transfer

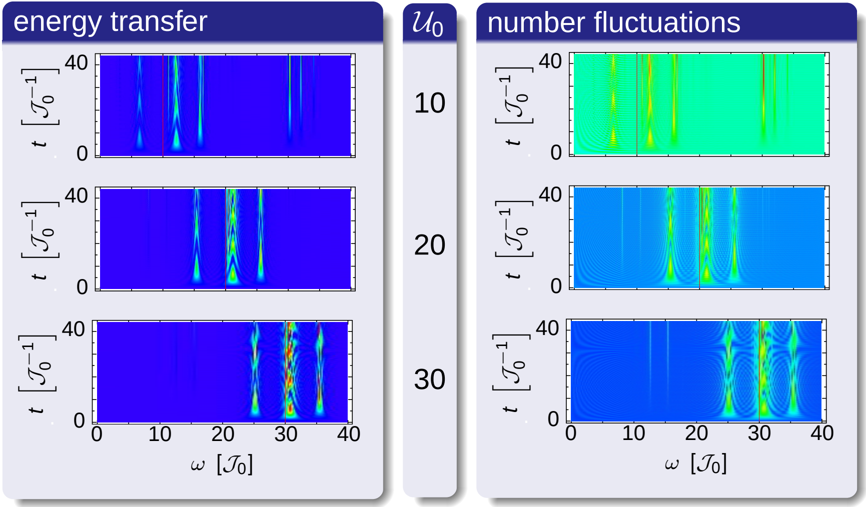


number fluctuations



Bose gas

varying interaction strength $\mathcal{U}_0$, fixed tunneling strength $\mathcal{J}_0 = 1$



Fermi/Fermi mixture
Framework

Fermi Hubbard Hamiltonian ...

$$\mathbf{H} = - \mathcal{J} \sum_i^I \left(\mathbf{a}_i^\dagger \mathbf{a}_{i+1} + \mathbf{a}_{i+1}^\dagger \mathbf{a}_i \right) - \mathcal{J} \sum_i^I \left(\mathbf{b}_i^\dagger \mathbf{b}_{i+1} + \mathbf{b}_{i+1}^\dagger \mathbf{b}_i \right) + \mathcal{U}_{ab} \sum_i^I \mathbf{n}_i^{(a)} \mathbf{n}_i^{(b)}$$

$\mathcal{J}$	tunneling strength	$\mathcal{U}_{ab}$	interaction strength
I	# of lattice sites	D	dimension of basis

$n_i^{(a)}$ # of atoms of species a at i-th site

... and its groundstate

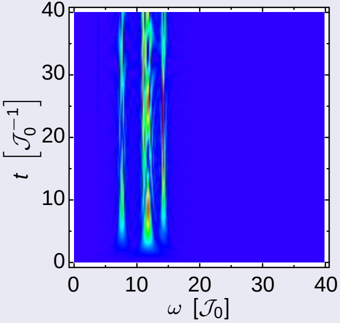
$$|\Psi_0\rangle = \sum_\alpha^{D_a} \sum_\beta^{D_b} c_{\alpha\beta}^{(0)} |\{n_1^{(a)} n_2^{(a)} \dots n_I^{(a)}\}_\alpha\rangle \otimes |\{n_1^{(b)} n_2^{(b)} \dots n_I^{(b)}\}_\beta\rangle$$

Fermi/Fermi mixture
Setup

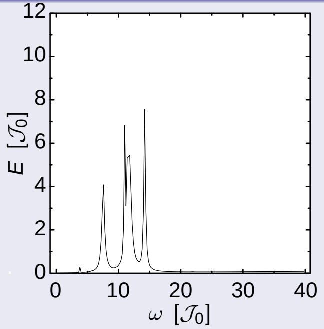
parameters

- I=6 lattice sites
- $N_a = N_b = 3$ fermions
- $\mathcal{J}_0$=1 tunneling strength
- $\mathcal{U}_0^{ab}$=10 interaction strength

energy transfer



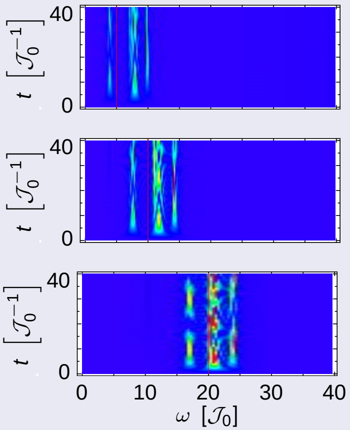
average energy transfer



Fermi/Fermi mixture

varying interaction strength $\mathcal{U}_0$, fixed tunneling strength $\mathcal{J}_0 = 1$

energy transfer



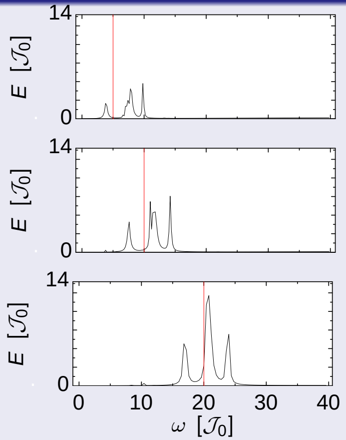
$\mathcal{U}_0$

5

10

20

average energy transfer



Summary / Outlook

- Bosons
  - sharp resonance peaks occur in the Mott phase in agreement with the experiment
  - the main resonance slides onto the frequency $\omega = \mathcal{U}_0$ for larger ratio $\mathcal{U}_0/\mathcal{J}_0$
- Fermi/Fermi mixtures
  - fermions show similar effects

• • •

- calculate larger systems (10 lattice sites and more)
- work on fermions is in progress