Pairing and Transport Properties in 1D Bose-Fermi & Fermi-Fermi Mixtures

Q 49 Stark korrelierte Systeme

Felix Schmitt   Markus Hild   Robert Roth

Institut für Kernphysik
TU Darmstadt

Frühjahrstagung
Einsteinjahr 2005

Overview

1. Working Environment / Hubbard Model
2. Fermion-Fermion Systems
   - Correlations In Coordinate Space
   - Correlations In Momentum Space
   - Flow Properties
3. Boson-Fermion Systems
   - Correlations In Coordinate And Momentum Space
4. Summary
5. Outlook

The Hubbard Model

- 1D lattice with I sites
- binary fermion-fermion mixture
- single-particle states: localised Wannier functions
- restrict Hilbert space to lowest energy band; $T = 0\,K$

Fermi-Fermi-Hubbard Hamlitonian

$$\hat{H} = -J \sum_{i=1}^{I} \left(\hat{a}_{i+1}^{\dagger} \hat{a}_i + \hat{b}_{i+1}^{\dagger} \hat{b}_i + \ h.a. \right) + V_{ab} \sum_{i=1}^{I} \hat{n}_i^{(a)} \ \hat{n}_i^{(b)}$$

J hopping matrix element

V_{ab} on-site two-body interaction energy

- V_{ab}/J as control parameter

Exact Diagonalisation

- exact diagonalisation of high-dimensional Hamilton matrix leads to ground state
- many-body basis: occupation number representation

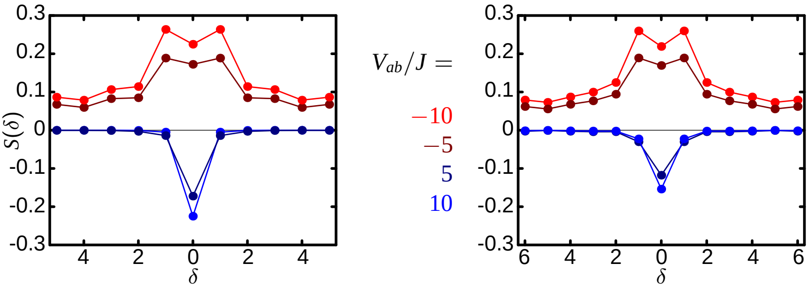
$$| \psi \rangle = \sum_{\alpha=1}^{D_a} \sum_{\beta=1}^{D_b} C_{\alpha\beta} \left| \{n_1^{(a)}, \dots, n_I^{(a)}\}_\alpha \right\rangle \otimes \left| \{n_1^{(b)}, \dots, n_I^{(b)}\}_\beta \right\rangle$$

- $I = 10 \quad N_a = N_b = 5$ basis dimension: $D_a \cdot D_b = 63504$
- $I = 12 \quad N_a = N_b = 5$ basis dimension: $D_a \cdot D_b = 627264$

Pair-Coherence Function : $S(\delta)$

$$
\begin{aligned}
S(\delta) \;=\;& \langle\, \psi \mid \hat{a}_1^\dagger \hat{b}_1^\dagger \hat{b}_{1+\delta} \hat{a}_{1+\delta} \mid \psi \,\rangle \\
&- \langle\, \psi \mid \hat{a}_1^\dagger \hat{a}_{1+\delta} \mid \psi \,\rangle \langle\, \psi \mid \hat{b}_1^\dagger \hat{b}_{1+\delta} \mid \psi \,\rangle
\end{aligned}
$$

$I = 10 \quad N_a = 5 \quad N_b = 5$ $\qquad\qquad$ $I = 12 \quad N_a = 5 \quad N_b = 5$



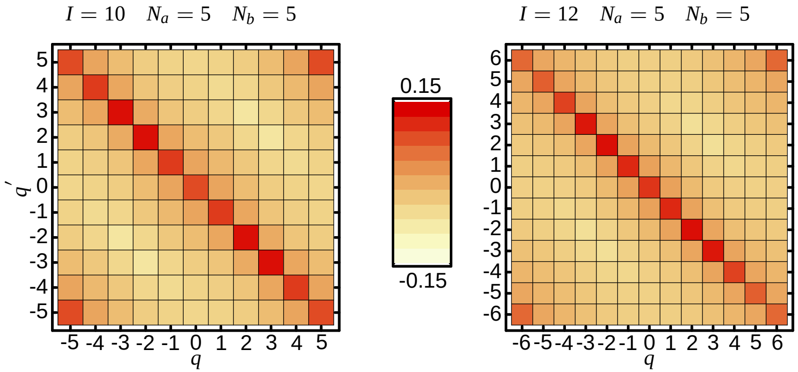
- 'long range pair-coherence' at $V_{ab}/J < 0$

Quasi-Momentum Correlation Function : $C_{ab}(q, q^{'})$

$$C_{ab}(q, q^{'}) = \langle \psi \mid \hat{n}_q^{(a)} \hat{n}_{q'}^{(b)} \mid \psi \rangle - \langle \psi \mid \hat{n}_q^{(a)} \mid \psi \rangle \langle \psi \mid \hat{n}_{q'}^{(b)} \mid \psi \rangle$$

$\hat{n}_q^{(a,b)}$: occupation number operator for Bloch states

$$V_{ab}/J = -10$$

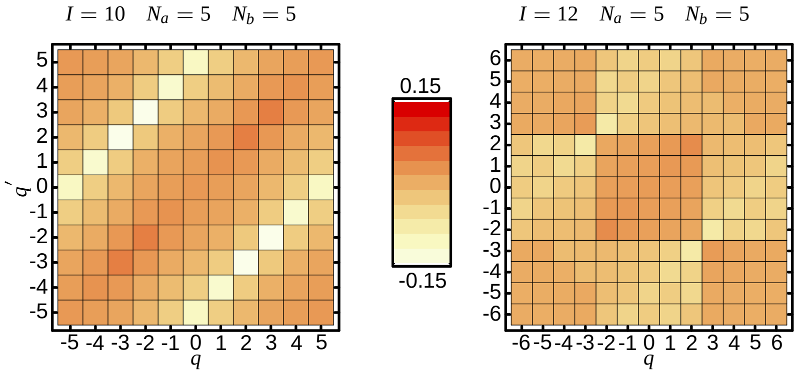


Quasi-Momentum Correlation Function : $C_{ab}(q, q^{'})$

$$C_{ab}(q,q^{'}) = \langle \psi \mid \hat{n}_q^{(a)} \hat{n}_{q'}^{(b)} \mid \psi \rangle - \langle \psi \mid \hat{n}_q^{(a)} \mid \psi \rangle \langle \psi \mid \hat{n}_{q'}^{(b)} \mid \psi \rangle$$

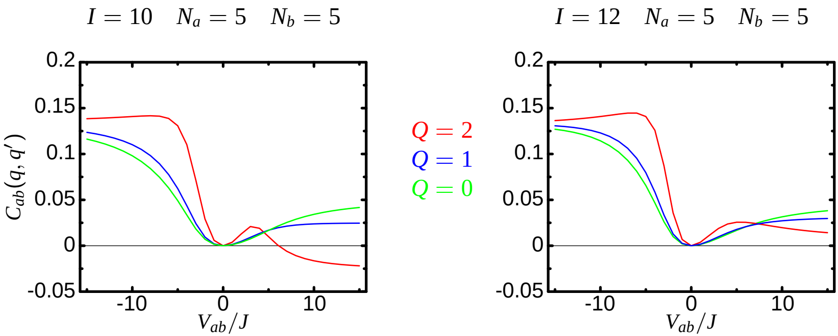
$\hat{n}_q^{(a,b)}$: occupation number operator for Bloch states

$$V_{ab}/J = +10$$



Quasi-Momentum Correlation Function : $C_{ab}(q, q')$

$C_{ab}(q, q')$ with fixed $q = -q' = Q$ plotted over V_{ab}/J



- $(q\,,\ -q')$- pairing at $V_{ab}/J < 0$
- dominant pairing at Fermi surface ($Q = 2$) at $V_{ab}/J \lesssim 0$

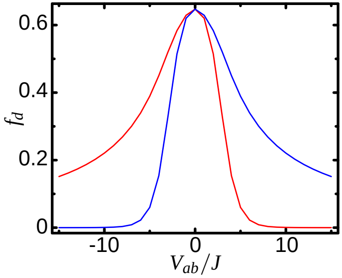
Flow Properties

- impose velocity field via phase gradient
- $|\,\psi\,\rangle \xrightarrow{\quad\text{flow}\quad} |\,\psi_\Theta\,\rangle$

Drude Weight : f_d — E_Θ energy of $|\,\psi_\Theta\,\rangle$

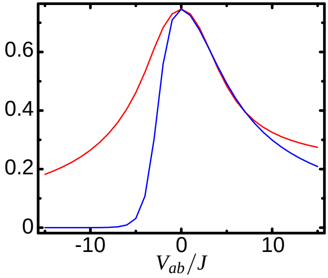
$$f_d = \frac{I^2}{J(N_a + N_b)} \frac{E_\Theta - E}{\Theta^2}$$

$I = 10 \quad N_a = 5 \quad N_b = 5$



twist
counter twist

$I = 12 \quad N_a = 5 \quad N_b = 5$



Bose-Fermi-Hubbard Hamiltonian

$$
\begin{aligned}
\hat{H} \;=\; & -J\sum_{i=1}^{I}\left(\hat{a}_{i+1}^{\dagger}\hat{a}_i+\hat{b}_{i+1}^{\dagger}\hat{b}_i+\;h.a.\right)\\
& +V_{ab}\sum_{i=1}^{I}\hat{n}_i^{(a)}\,\hat{n}_i^{(b)}+\frac{V_{aa}}{2}\sum_{i=1}^{I}\hat{n}_i^{(a)}(\hat{n}_i^{(a)}-1)
\end{aligned}
$$

J hopping matrix element
V_{ab} boson fermion interaction engergy
V_{aa} boson boson interaction energy

- V_{ab}/J and V_{aa}/J as control parameters

Exact Diagonalisation

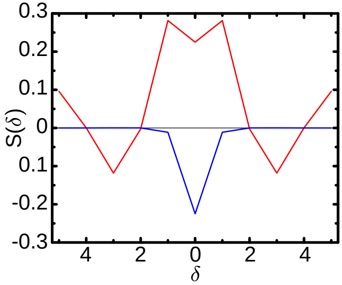
- the whole machinery can be applied to boson-fermion mixtures
- exact diagonalisation of high-dimensional Hamilton matrix leads to ground state
- many-body basis: occupation number representation

$$| \, \psi \, \rangle = \sum_{\alpha=1}^{D_a} \sum_{\beta=1}^{D_b} C_{\alpha\beta} \; \Big| \; \{n_1^{(a)}, \dots, n_I^{(a)}\}_\alpha \; \Big\rangle \otimes \Big| \; \{n_1^{(b)}, \dots, n_I^{(b)}\}_\beta \; \Big\rangle$$

- $I = 10 \quad N_a = N_b = 5$ dimension: $D_a \cdot D_b = 504504$

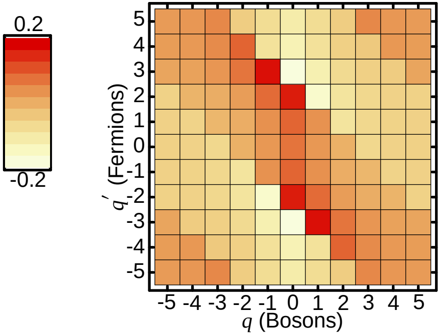
Correlations In Coordiante And Momentum Space

pair coherence $S(\delta)$ at $V_{aa}/J = 10$



$V_{ab}/J = -10$ $V_{ab}/J = 10$

momentum correlation $C(q, q^{'})$ at $V_{aa}/J = 10$ $V_{ab}/J = -10$



- no 'long range pair coherence'
- different from pairing in fermion-fermion systems

Summary

- Fermion-Fermion Systems
  - observed a pairing behaviour of attractive fermion-fermion systems in coordinate space with ’long range pair coherence’
  - as well as pairing in momentum space where particles show a strong $(q^a, -q^b)$- pairing near the Fermi surface
  - flow properties show a transition at small attractive interaction strength too
- Boson-Fermion Systems
  - there is no ’long range pair coherence’ in boson-fermion systems
  - pairing behaviour in momentum space differs from fermion-fermion systems

Outlook

- try to find observables for conductivity to distinguish superconducting, normal conducting and isolating phases
- understand the pairing properties of bose-fermi mixtures
- bigger, better, faster...