Ultracold Atomic Gases in Optical Lattices

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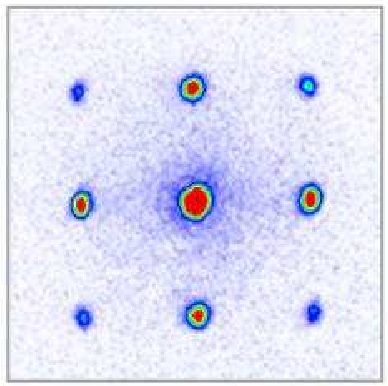
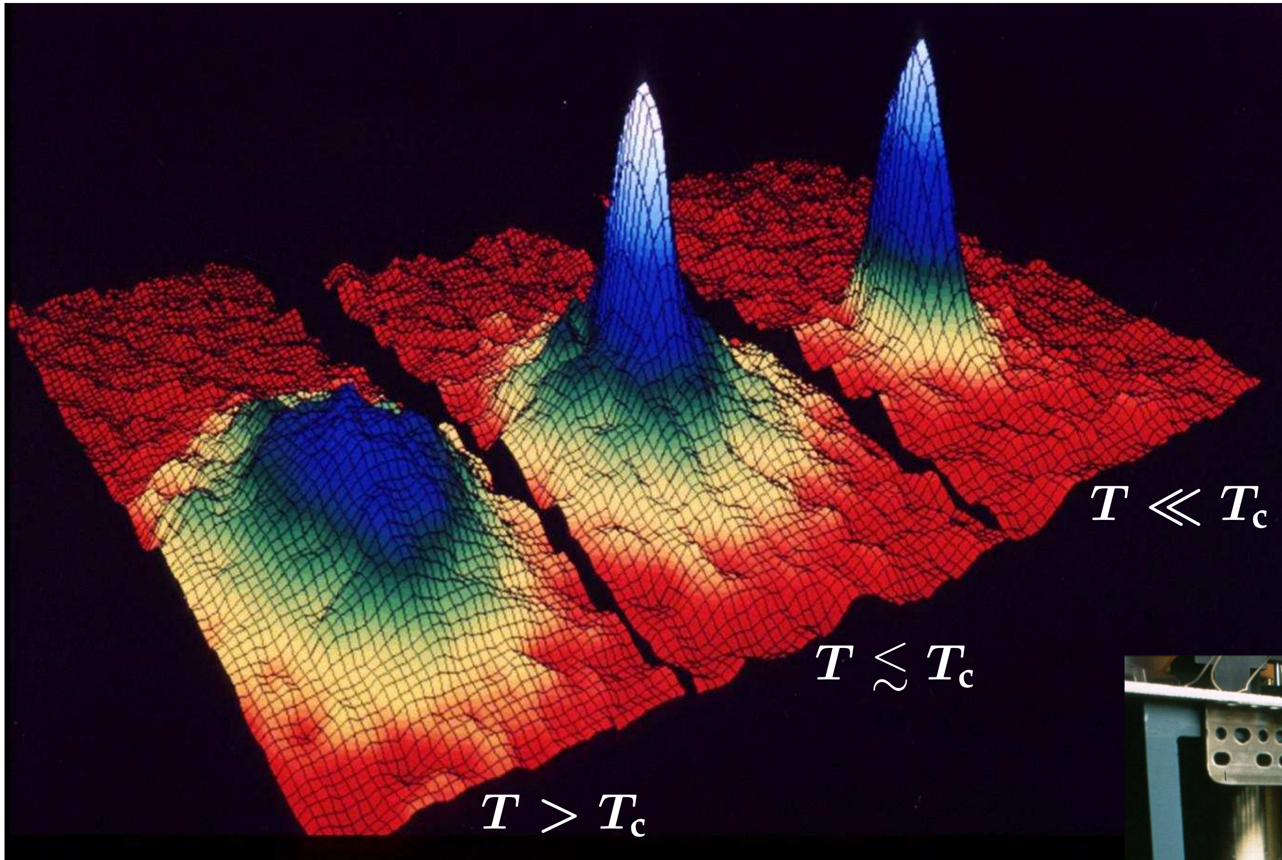


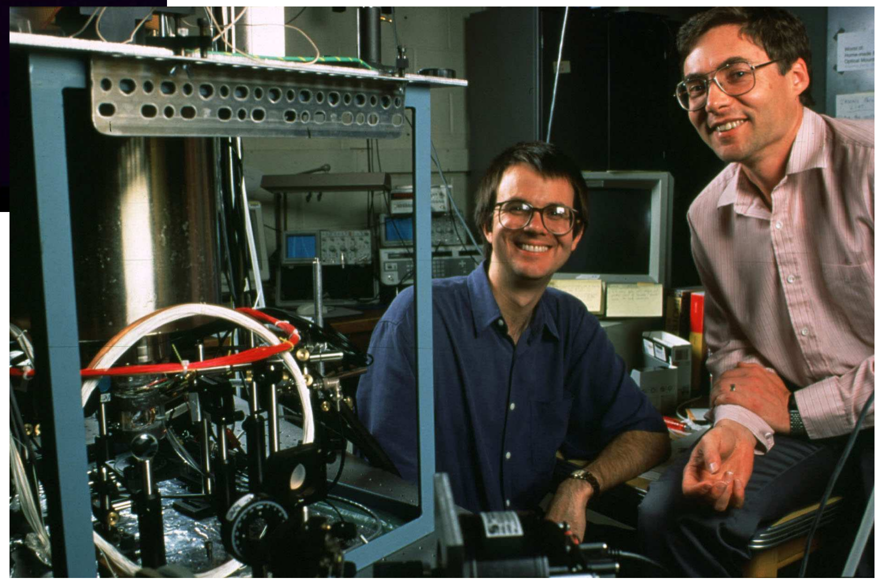
Overview

- Ultracold Atomic Gases
- The Lattice Experiment
- Bose-Hubbard Model
- Condensation & Superfluidity
- Two-Color Superlattices
- Boson-Fermion Mixtures in Lattices

BEC of Rubidium Atoms



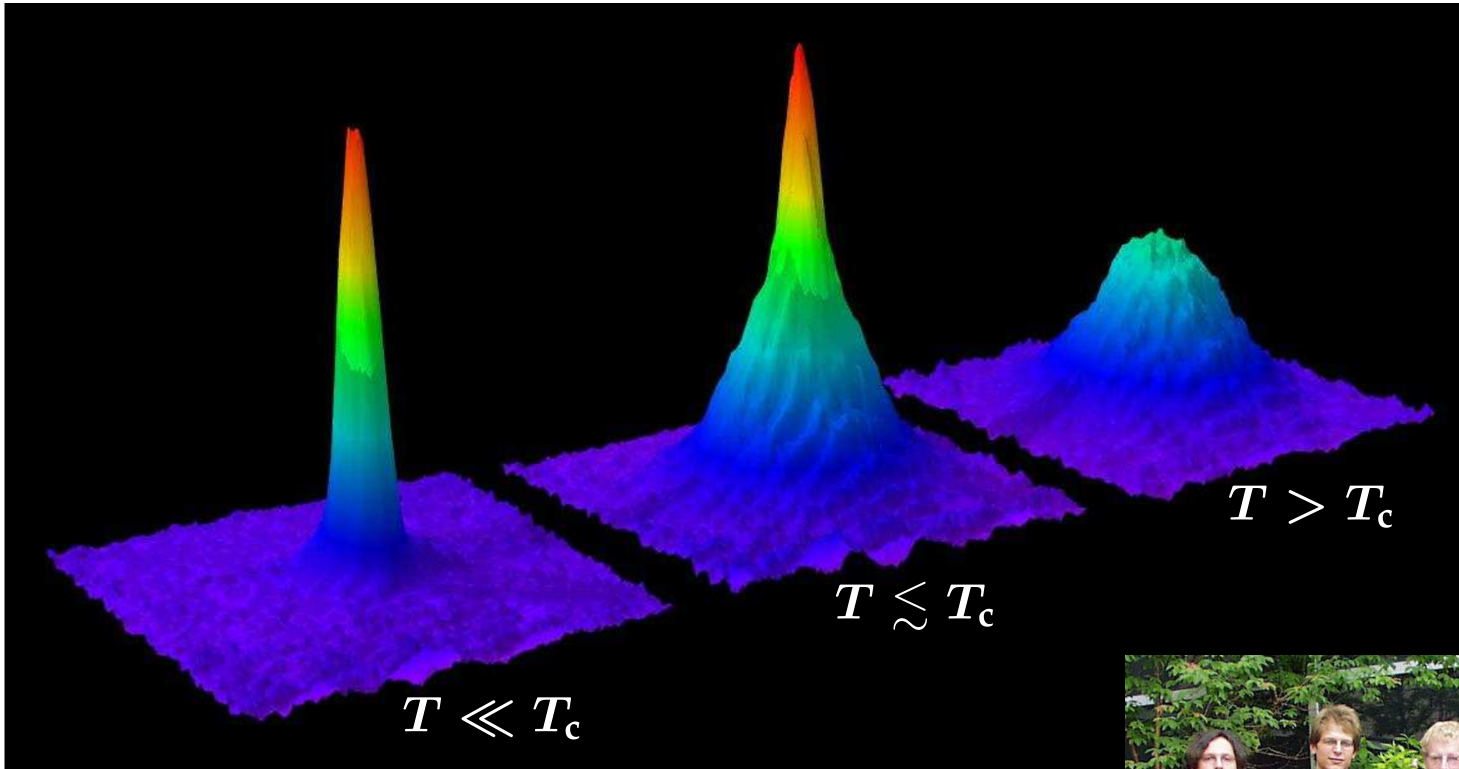
- ^{87}Rb
- $N_{\text{initial}} \approx 10^6$
- $N_{\text{BEC}} \approx 2000$
- $T_c \approx 170$nK
- absorption image after 60 ms expansion



E. Cornell, C. Wieman, et al.
(JILA, NIST, U of Colorado)

Nobel Prize in Physics 2001

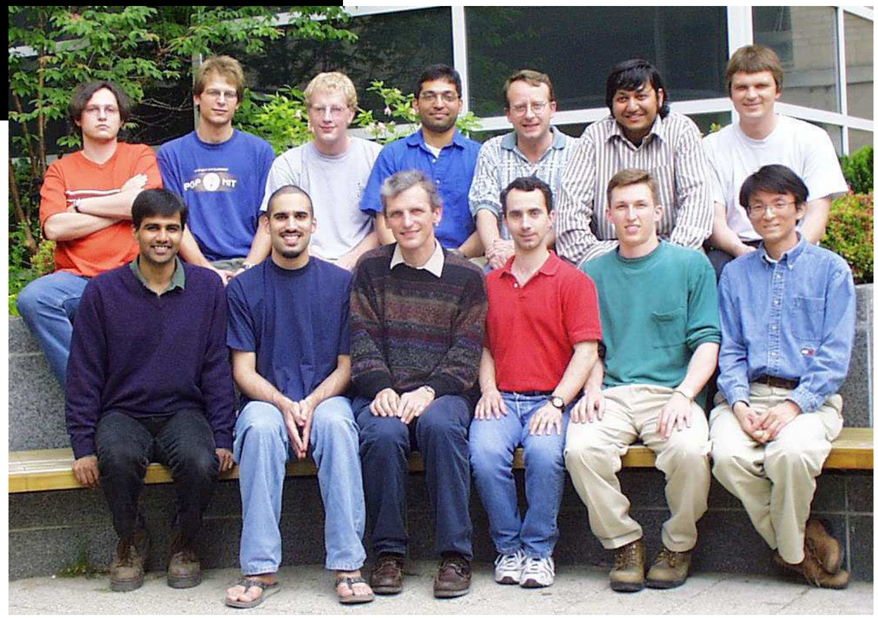
BEC with Sodium Atoms



- ^{23}Na
- $N_{\text{initial}} \approx 10^9$
- $N_{\text{BEC}} \approx 5 \times 10^5$
- $T_c \approx 2\,\mu$K
- absorption image after 60 ms expansion

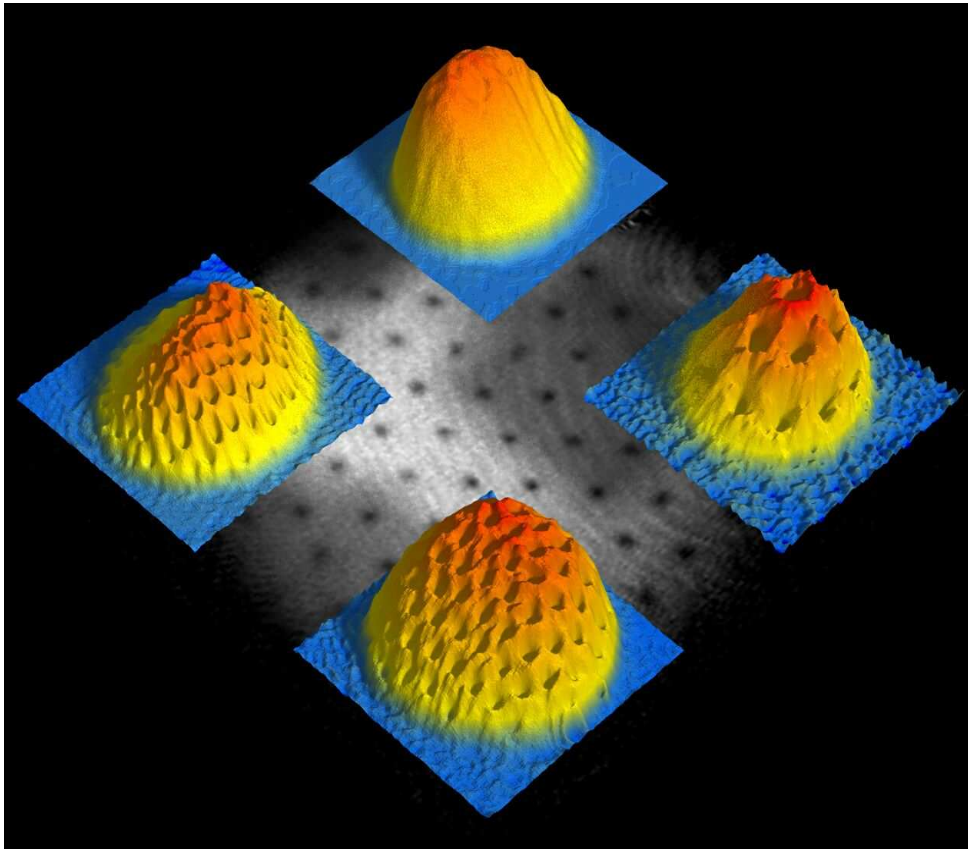
W. Ketterle, et al.
(MIT)

Nobel Prize in Physics 2001



Dynamics of Dilute Quantum Gases

- amazing experimental achievements
  - condensates of ^{1}H, ^{4}He*, ^{7}Li, ^{23}Na, ^{41}K, ^{85}Rb, ^{87}Rb, ^{133}Cs, ^{174}Yb
  - vortices, vortex lattices and their dynamics
  - bright and dark solitons and soliton trains
  - collective excitations and collapse
  - boson-fermion mixtures and ultracold fermions



[W. Ketterle *et al.*; Science 292 (2001) 476]

...over the Intervening Years

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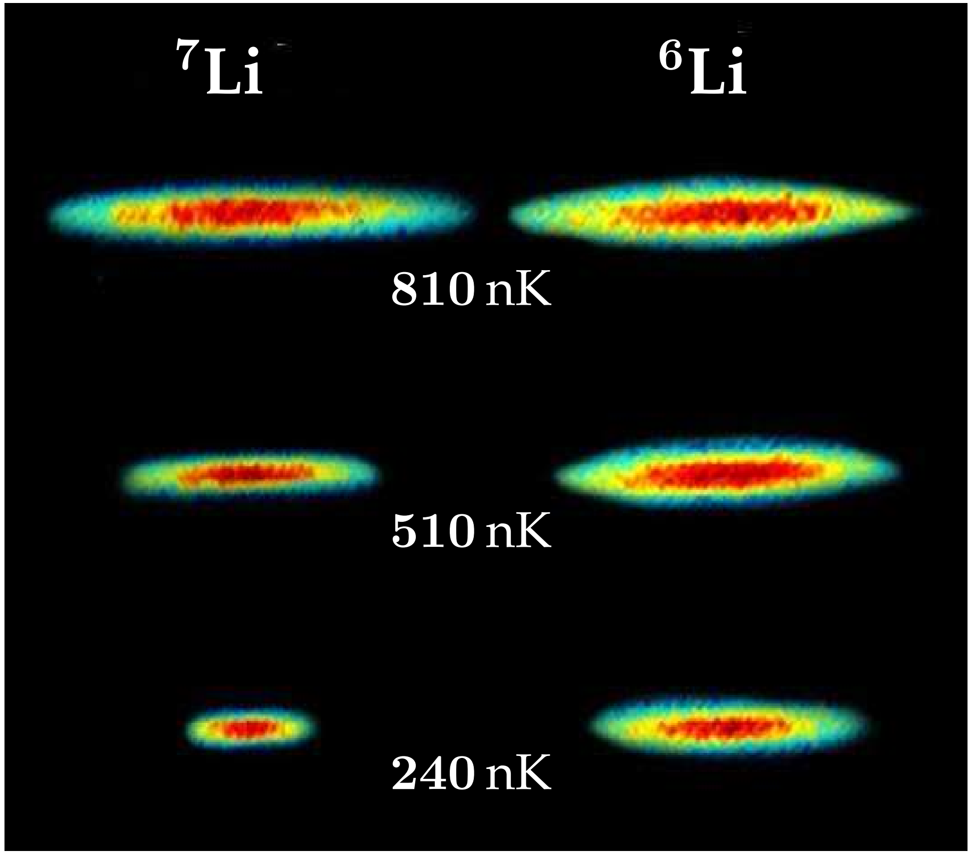


[R. Hulet *et al.*; Science 291 (2001) 2570]

everything well described within
mean-field theory (Gross-Pitaevskii equation)

The Advent of Correlations

correlations beyond mean-field begin to play a role

Feshbach Resonances

- tuning of the scattering length over several orders of magnitude
- **coherent molecule formation**: molecular condensates, ultracold chemistry
- **BEC-BCS crossover**: generalized Cooper pairing, fermionic superfluidity

Optical Traps

- tightly confining traps with a multitude of geometries
- **quasi 1D and 2D traps**: quantum gases in low dimensions
- **optical lattices in 1-3D**: band structure, quantum phase transitions, disorder, ...

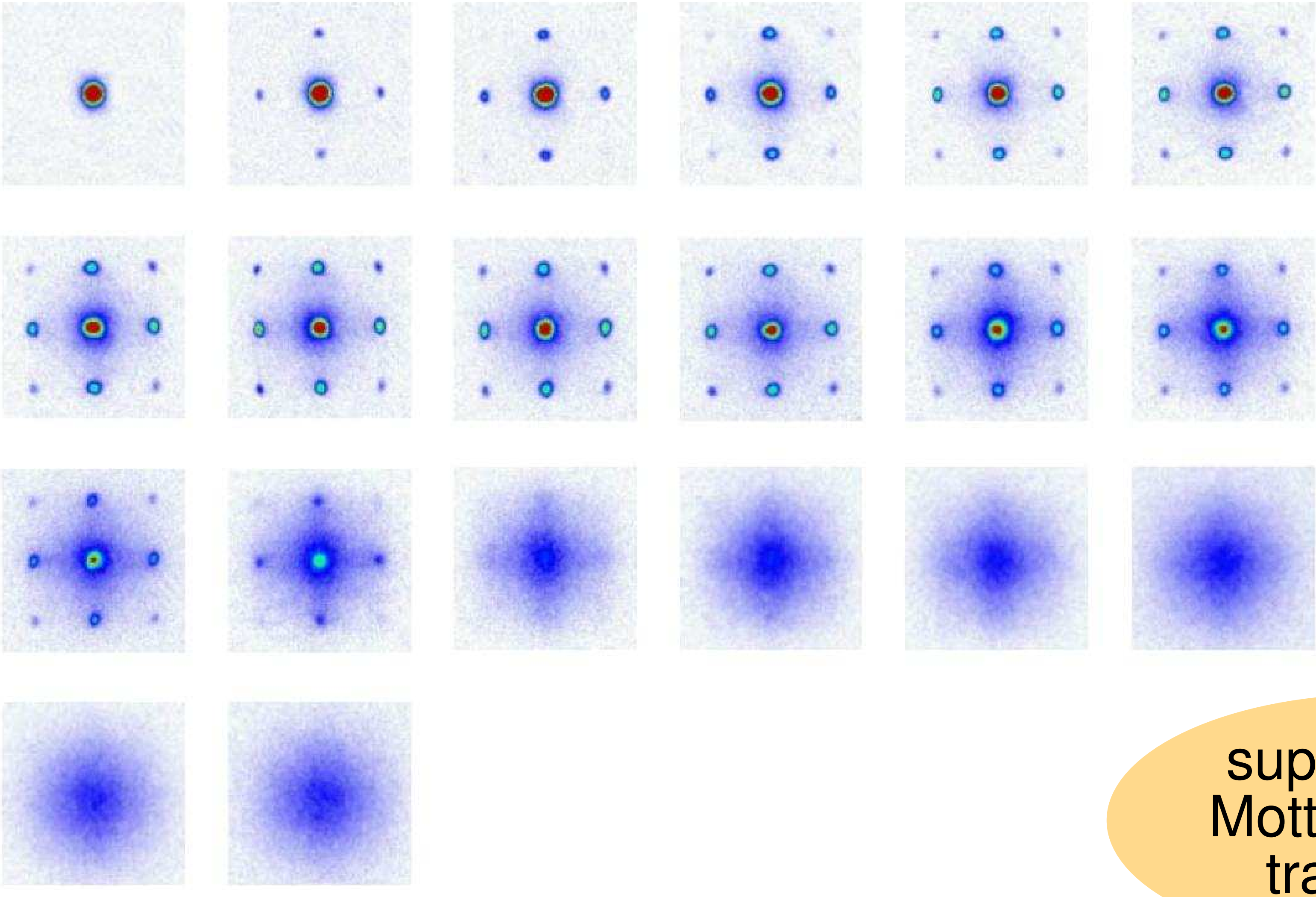
The Lattice Experiment

- produce a **Bose-Einstein condensate** of atoms in a magnetic trap
- load the condensate into an **optical standing-wave lattice** created by counter-propagating laser beams
- in a 3D lattice one ends up with **few atoms per lattice site** in a 1D lattice one can have thousands of atoms
- probe different physical regimes by varying **lattice depth** and **interaction strength**
- switch off the lattice, let the gas expand, and observe the **matter-wave interference pattern**

Munich Experiment

Interference Pattern

increasing lattice depth ⟶



characteristic interference pattern of array of coherent BECs

incoherent background emerges and peaks vanish

[M. Greiner *et al.*; Nature 415 (2002) 39]

superfluid to Mott-insulator transition

Questions...

- How to describe ultracold bosons in a lattice?
- How to define **superfluid** and **condensate**?
- What is the **superfluid to Mott-insulator transition**?
- Are there **other quantum-phases** one can investigate?
- What happens if the lattice is **irregular**?
- What about **fermions**?

Bose-Hubbard Model

Bose-Hubbard Model

- one-dimensional lattice with N particles and I lattice sites at $T = 0\text{K}$
- restrict Hilbert space to the **lowest energy band**
- localized Wannier wavefunctions $w_i(x)$ with associated **occupation numbers** n_i for the individual sites $i = 1...I$
- represent N-boson state in complete basis of **Fock states** $|\{n_1, ..., n_I\}_\alpha\rangle$

$$|\Psi\rangle = \sum_{\alpha=1}^{D} C_\alpha \, |\{n_1, ..., n_I\}_\alpha\rangle$$

- basis dimension D **grows dramatically** with I and N

I	6	8	10	12
D	462	6435	92 378	1 352 078

for $N/I = 1$

Bose-Hubbard Hamiltonian

- second quantized Hamiltonian with respect to Wannier basis [Fisher *et al.* (1989); Jaksch *et al.* (1998)]

$$\mathbf{H}_0 = -J \sum_{i=1}^{I} (\mathbf{a}_{i+1}^\dagger \mathbf{a}_i + \mathbf{h.a.}) + \sum_{i=1}^{I} \epsilon_i \, \mathbf{n}_i + \frac{V}{2} \sum_{i=1}^{I} \mathbf{n}_i (\mathbf{n}_i - 1)$$

tunneling between adjacent lattice sites — single-particle energy — on-site two-body interaction

- assumptions: (a) only lowest band, (b) constant nearest-neighbor hopping, (c) only short-range interactions

▶ describes **strongly correlated systems** as well as **pure condensates**

▶ **exact solution**: compute lowest eigenstates using Lanczos algorithms

Simple Physical Quantities

- consider a regular lattice ➔ $\epsilon_i = 0$
- solve eigenproblem for various V/J

- **mean occupation numbers**

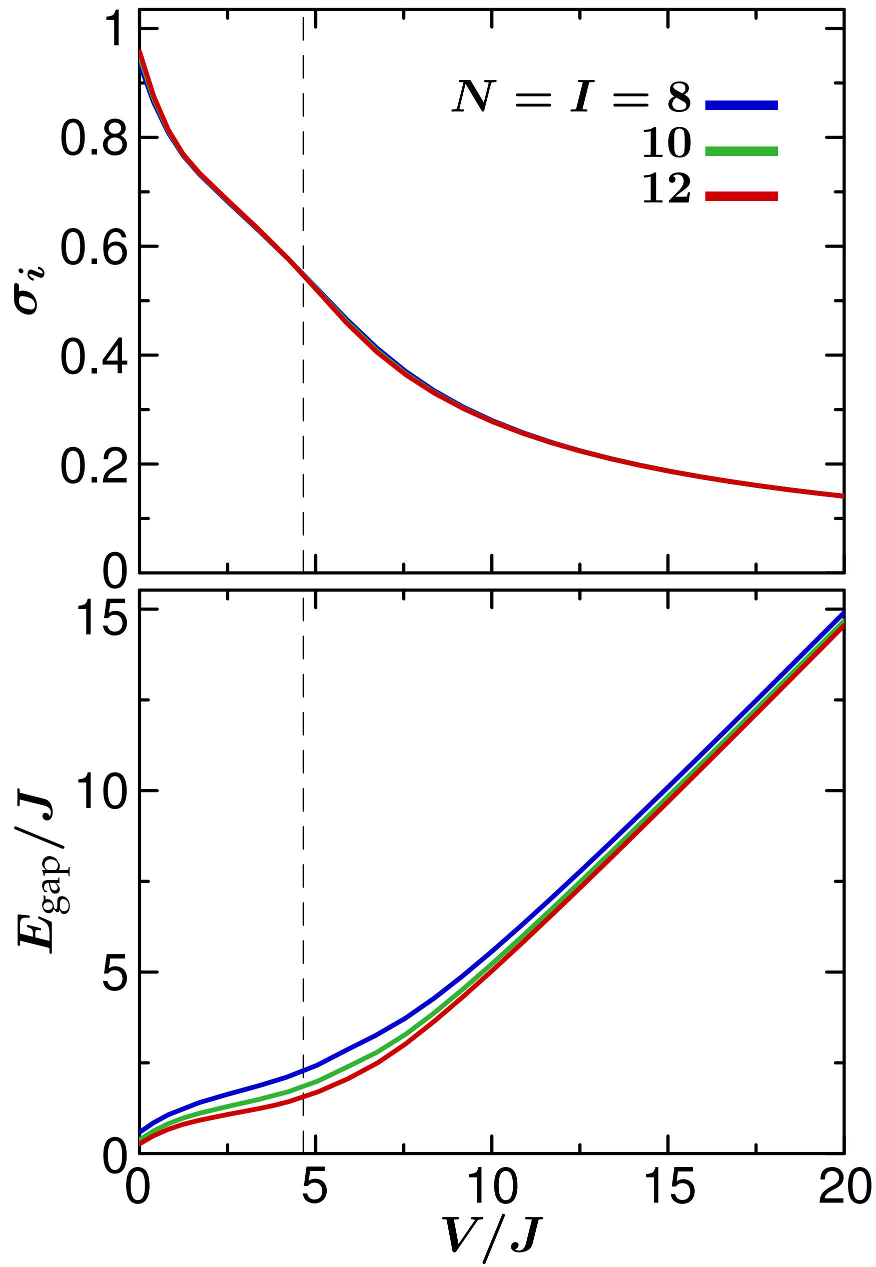
$$\bar{n}_i = \langle \Psi_0 | \mathrm{n}_i | \Psi_0 \rangle$$

- **number fluctuations**

$$\sigma_i = \sqrt{\langle \Psi_0 | \mathrm{n}_i^2 | \Psi_0 \rangle - \langle \Psi_0 | \mathrm{n}_i | \Psi_0 \rangle^2}$$

- **energy gap**

$$E_{\text{gap}} = E_{\text{1st excited}} - E_0$$



Condensate & Superfluidity

Bose-Einstein Condensation

- eigensystem of the **one-body density matrix**

$$\rho^{(1)}_{ij} = \langle \Psi_0 | \, \mathbf{a}^\dagger_j \mathbf{a}_i \, | \Psi_0 \rangle$$

  defines natural orbitals and the corresponding occupation numbers

- Onsager-Penrose criterion: **Bose-Einstein condensate** is present if one of the eigenvalues of $\rho^{(1)}_{ij}$ is of order N (in the thermodynamic limit)

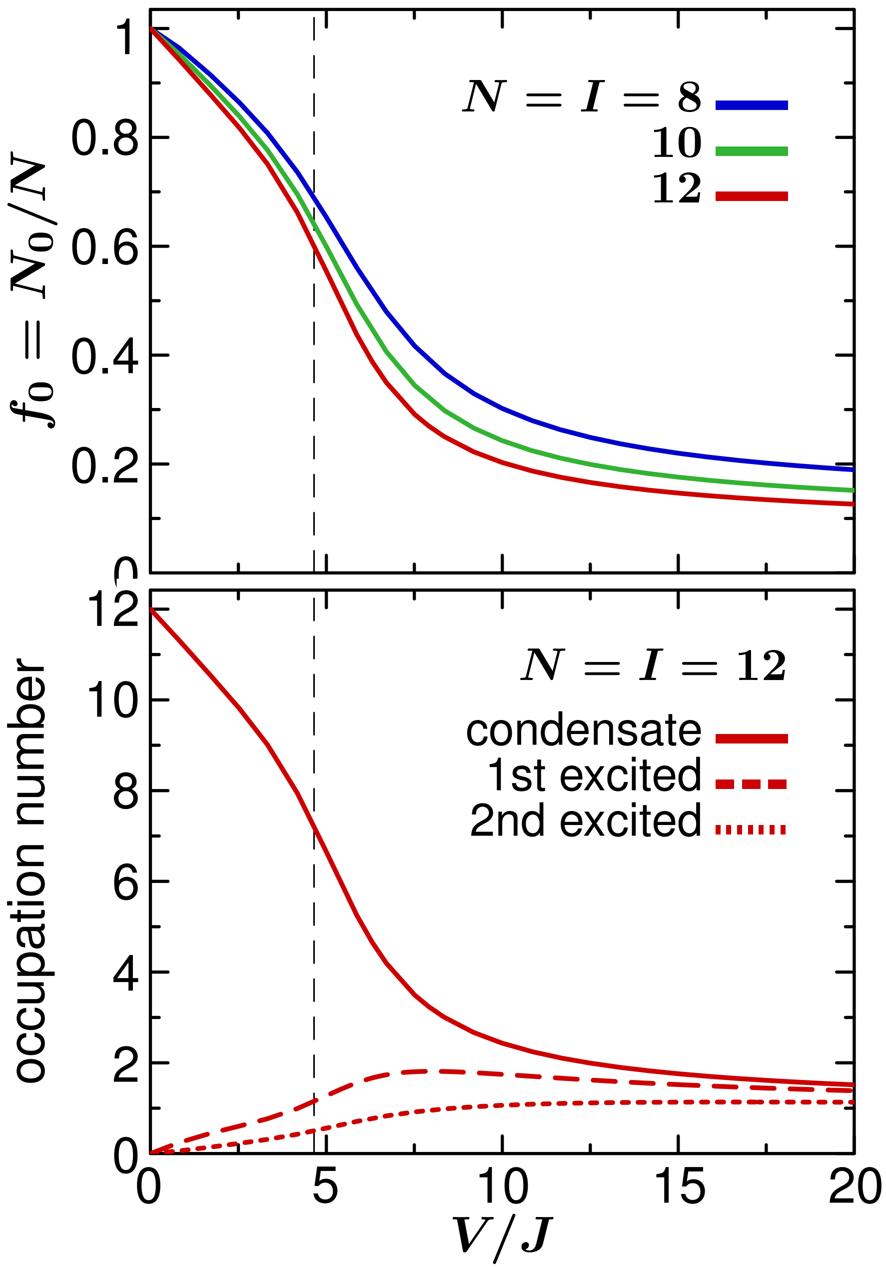
$$\begin{array}{rcccl} \text{eigenvalue} & \longrightarrow & N_0 & : & \text{number of condensed particles} \\ \text{eigenvector} & \longrightarrow & \phi_{0,i} & : & \text{condensate wave function} \end{array}$$

- existence of a condensate implies **off-diagonal long range order**

$$\rho^{(1)}_{ij} \not\to 0 \quad \text{as} \quad |i-j| \to \infty$$

- in a regular lattice the natural orbitals are **quasimomentum eigenstates**

Condensate & Quasimomentum Distribution



- pure condensate for $V/J = 0$
- rapid depletion of the condensate with increasing V/J
- finite size effect: condensate fraction in a finite lattice always $\geq 1/I$
- states with larger quasimomentum are populated successively
- homogeneous occupation of the band in the limit of large V/J

What is Superfluidity?

- macroscopically the superfluid flow is **non-dissipative** and **irrotational**, i.e., it is described by the gradient of a scalar field

$$\vec{v}_{\mathrm{SF}} \propto \vec{\nabla}\theta(\vec{x})$$

- **classical two-fluid picture**: only normal component responds to an imposed velocity field $\vec{v}$ (moving walls), the superfluid stays at rest

- energy in the comoving frame differs from ground state energy in the rest frame by the **kinetic energy of the superflow**

$$E(\text{imposed } \vec{v},\ \text{comoving frame}) = E(\text{at rest}) + \frac{1}{2} M_{\mathrm{SF}}\, \vec{v}^2$$

▶ these two ideas are basis for the **microscopic definition of superfluidity**

Definition of Superfluidity

- the velocity field of the superfluid is defined by the **gradient of the phase** of the condensate wavefunction $\phi_0(\vec{x})$

$$\vec{v}_{\mathrm{SF}} = \frac{\hbar}{m}\vec{\nabla}\theta(\vec{x}) \qquad\qquad \phi_0(\vec{x}) = \mathrm{e}^{\mathrm{i}\theta(\vec{x})}\,|\phi_0(\vec{x})|$$

- employ **twisted boundary conditions** to impose a linear phase variation

$$\Psi(\vec{x}_1,...,\vec{x}_i + L\vec{e}_1,...,\vec{x}_N) = \mathrm{e}^{\mathrm{i}\Theta}\,\Psi(\vec{x}_1,...,\vec{x}_i,...,\vec{x}_N) \qquad \forall i$$

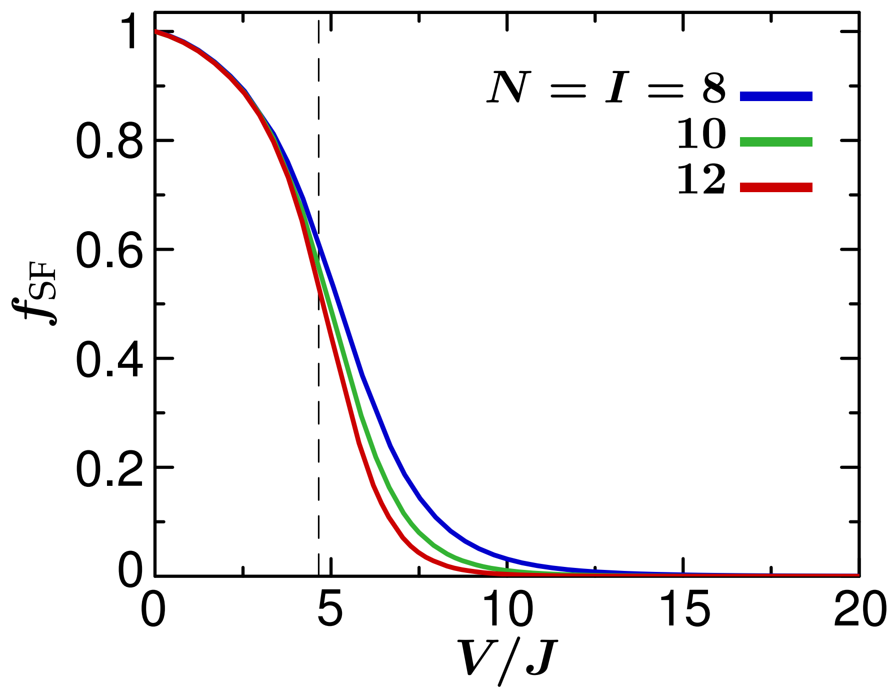
- the change in energy $E_\Theta - E_0$ due to the phase twist is for small Θ identified with the **kinetic energy of the superflow**

$$E_\Theta - E_0 = \tfrac{1}{2}M_{\mathrm{SF}}\,v_{\mathrm{SF}}^2 = \tfrac{1}{2}mN_{\mathrm{SF}}\,v_{\mathrm{SF}}^2$$

- **superfluid fraction** = stiffness with respect to phase variations

$$F_{\mathrm{SF}} = \frac{N_{\mathrm{SF}}}{N} = \frac{2m\,L^2}{\hbar^2 N}\,\frac{E_\Theta - E_0}{\Theta^2} \qquad\qquad \Theta \ll \pi$$

Superfluid Fraction



- solve eigenvalue problem with & without imposed phase twist and directly compute $E_\Theta - E_0$ and f_{SF}
- f_{SF} is the natural **order parameter** for the superfluid-insulator transition
- rapid decrease of f_{SF} in a narrow window in V/J already for small systems
- coupling to **excited states** is crucial for the vanishing of f_{SF} in the insulating phase

Condensate -vs- Superfluid

Condensate

- largest eigenvalue of the one-body density matrix
- involves only the ground state
- measure for off-diagonal long-range order / coherence

$\neq$

Superfluid

- response of the system to an external perturbation
- depends crucially on the excited states of the system
- measures a flow property

$f_0 < f_{\mathrm{SF}}$

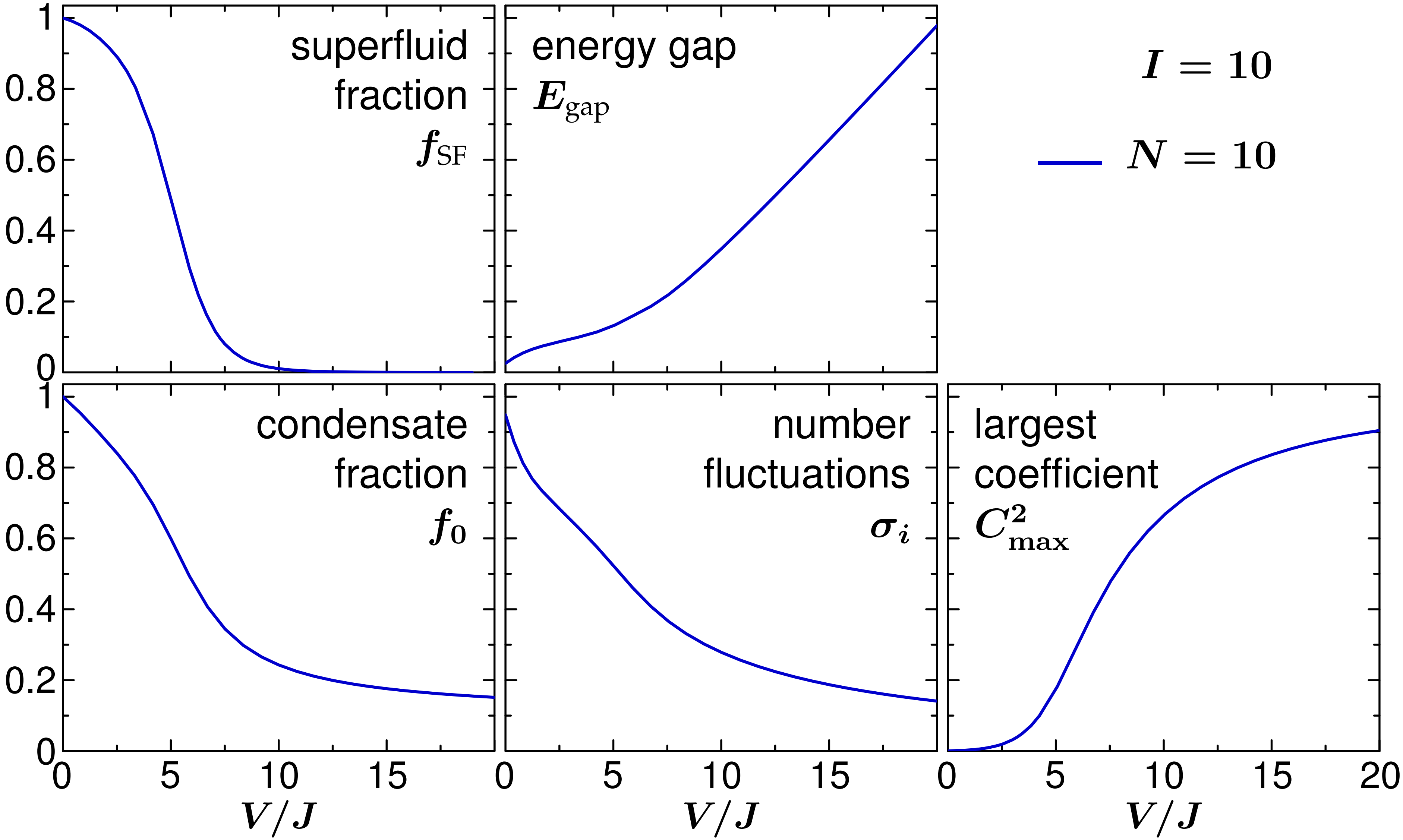
- non-condensed particles are dragged along with condensate
- liquid ^{4}He at $T = 0$K:

$$f_0 \approx 0.1, \quad f_{\mathrm{SF}} = 1$$

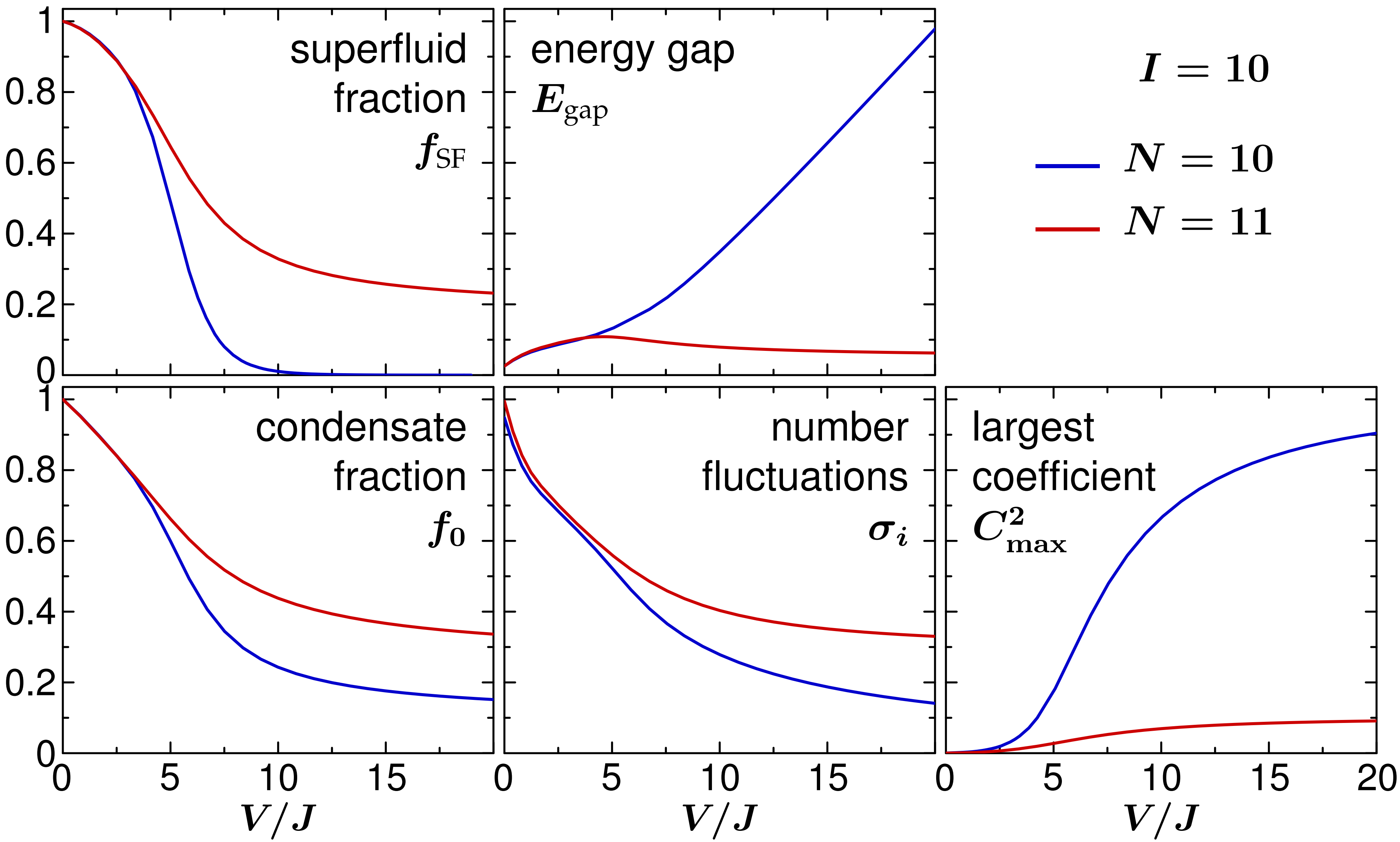
$f_0 > f_{\mathrm{SF}}$

- part of the condensate has a reduced stiffness under phase variations
- seems to occur in systems with defects or disorder

Superfluid to Mott-Insulator Transition



Superfluid to Mott-Insulator Transition



Two-Color Superlattices

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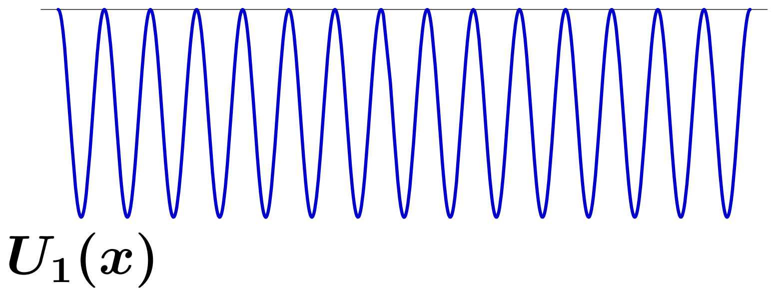
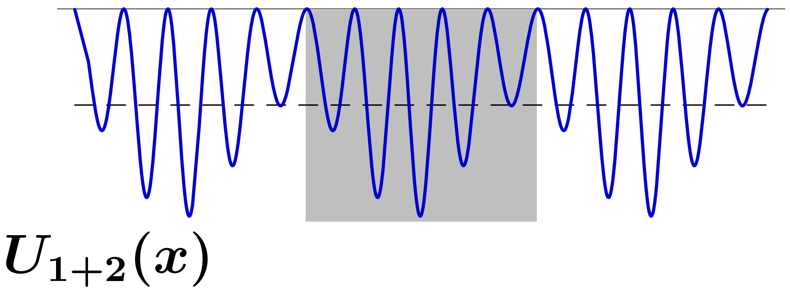
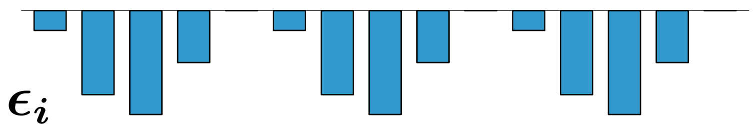


- start with a standing wave created by a laser with wavelength λ_1



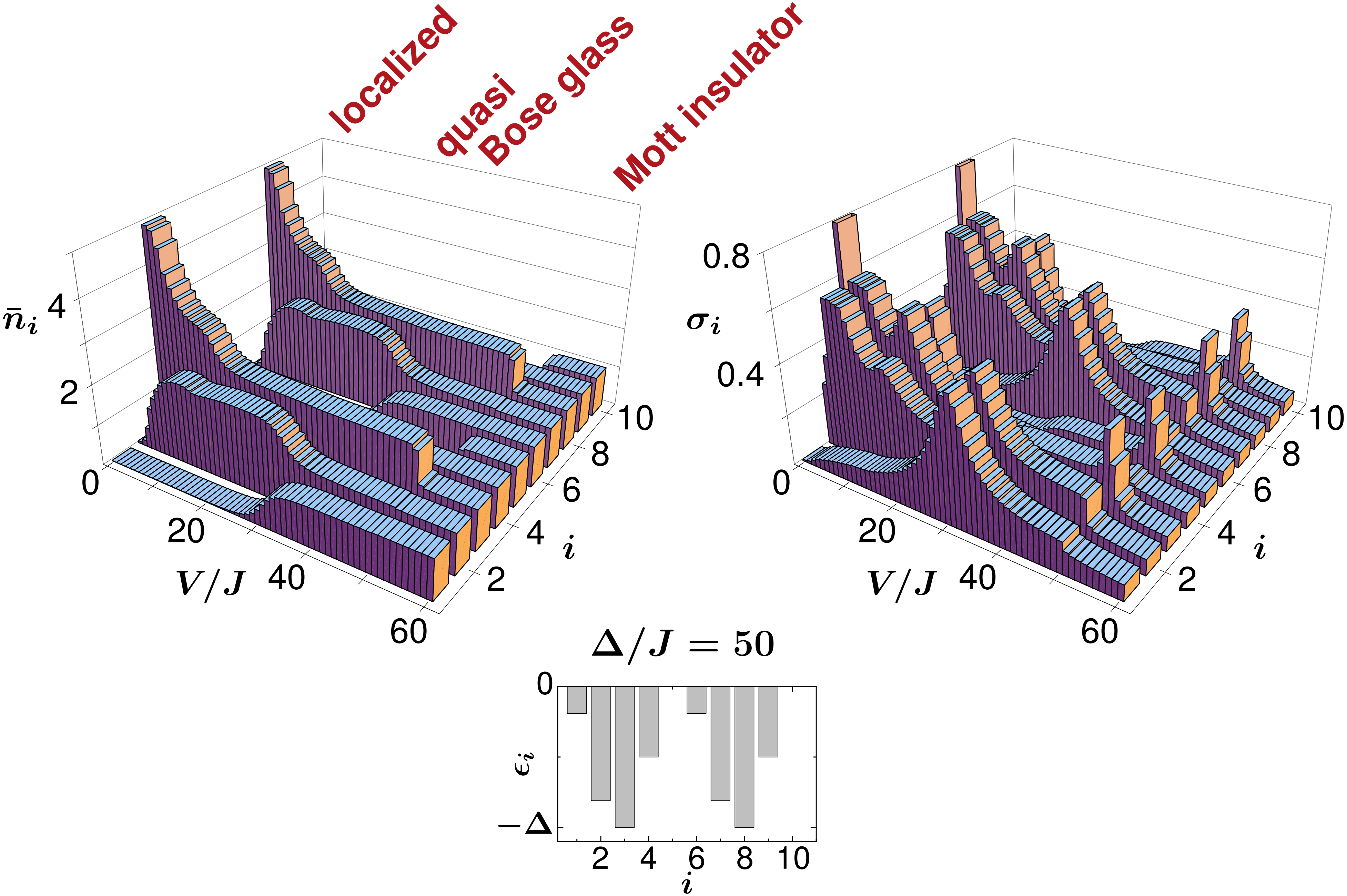
- add a second standing wave created by a laser with wavelength $\lambda_2 = \frac{5}{7}\lambda_1$ and much smaller intensity (here 4%)
- potential exhibits a periodic modulation of the well-depth with a period of 5 sites



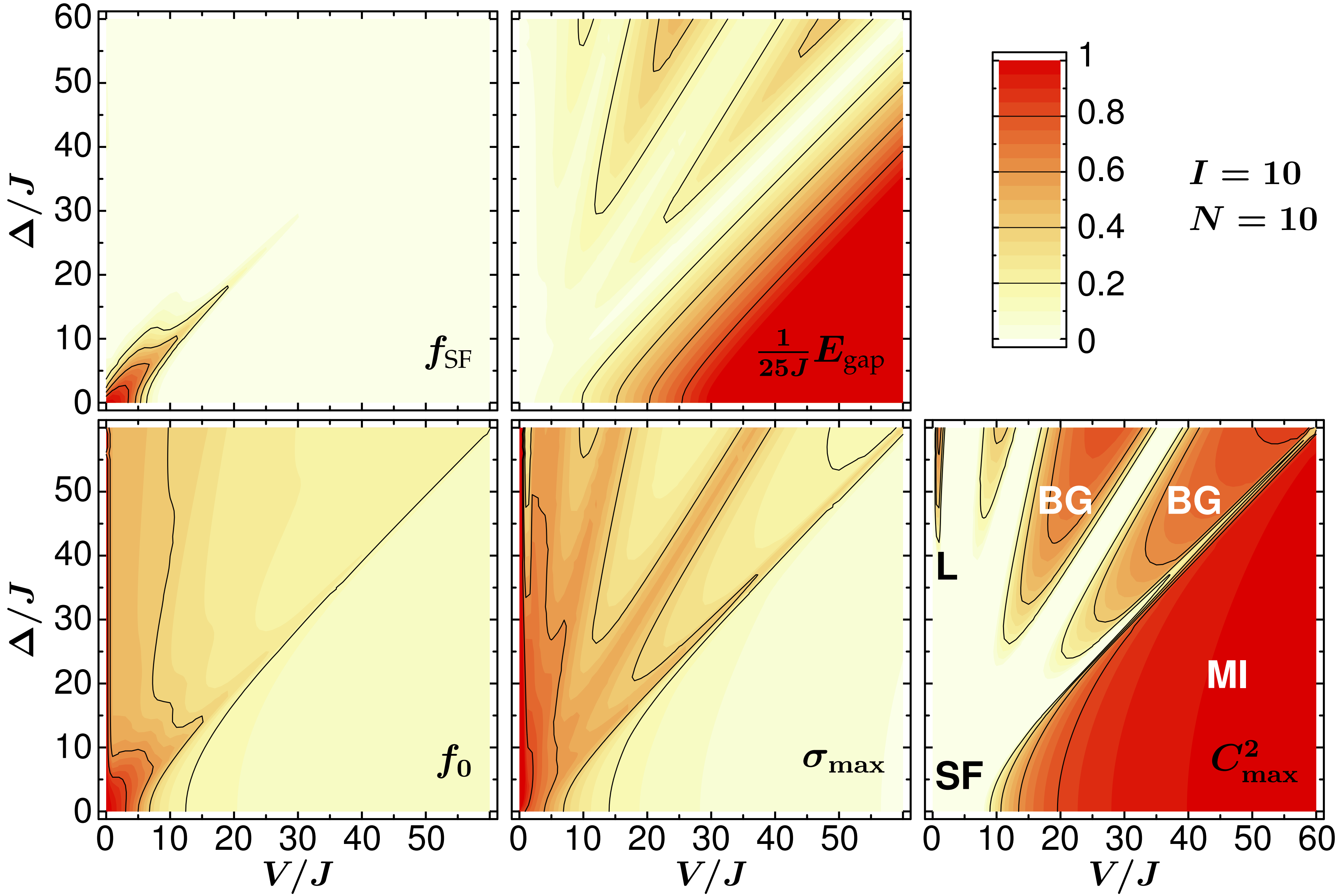
- Bose-Hubbard model: varying on-site energies $\epsilon_i \in [0, -\Delta]$

► **controlled lattice irregularities** open novel possibilities to study “disorder” related effects; more complex topologies easily possible

Interaction -vs- Lattice Irregularity



V-Δ Phase Diagrams



Boson-Fermion Mixtures in Lattices

Bose-Fermi-Hubbard Hamiltonian

- second quantized Hamiltonian containing boson (B) and fermion (F) operators [Albus et al. (2003)]

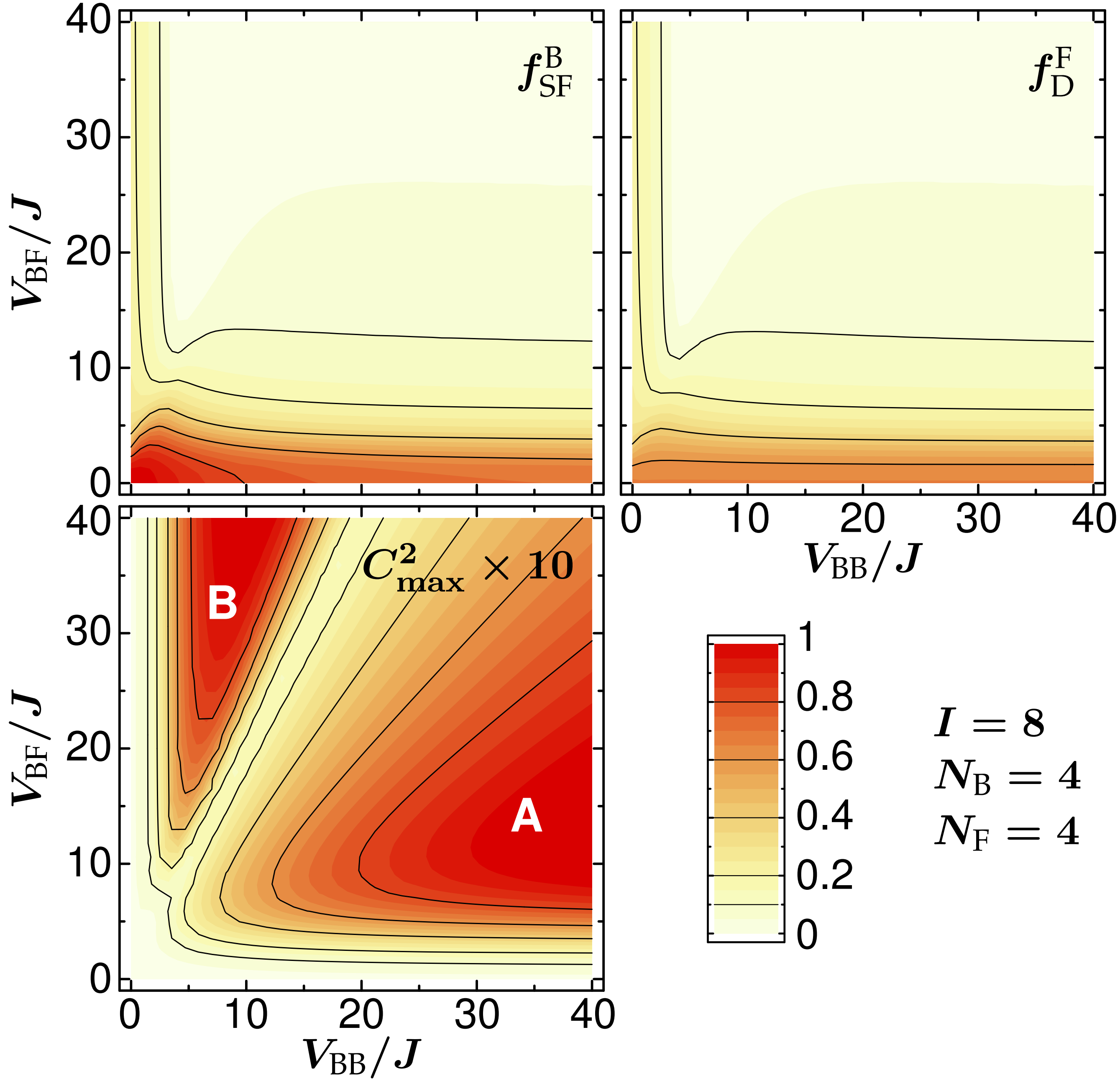
$$\mathrm{H}_0 = -J_{\mathrm{B}} \sum_{i=1}^{I} (\mathrm{a}_{i+1}^{\mathrm{B}\dagger} \mathrm{a}_i^{\mathrm{B}} + \mathrm{h.a.}) + \frac{V_{\mathrm{BB}}}{2} \sum_{i=1}^{I} \mathrm{n}_i^{\mathrm{B}} (\mathrm{n}_i^{\mathrm{B}} - 1)$$

$$- J_{\mathrm{F}} \sum_{i=1}^{I} (\mathrm{a}_{i+1}^{\mathrm{F}\dagger} \mathrm{a}_i^{\mathrm{F}} + \mathrm{h.a.}) + V_{\mathrm{BF}} \sum_{i=1}^{I} \mathrm{n}_i^{\mathrm{B}}\, \mathrm{n}_i^{\mathrm{F}}$$

- exact solution of eigenvalue problem in combined Fock-state representation

- in addition to ground state observables we employ two stiffnesses to characterize the various phases
  - **bosonic phase stiffness** ➔ boson superfluid fraction
  - **fermionic phase stiffness** ➔ Drude weight, fermionic conductivity

V_{BB}-V_{BF} Phase Diagrams



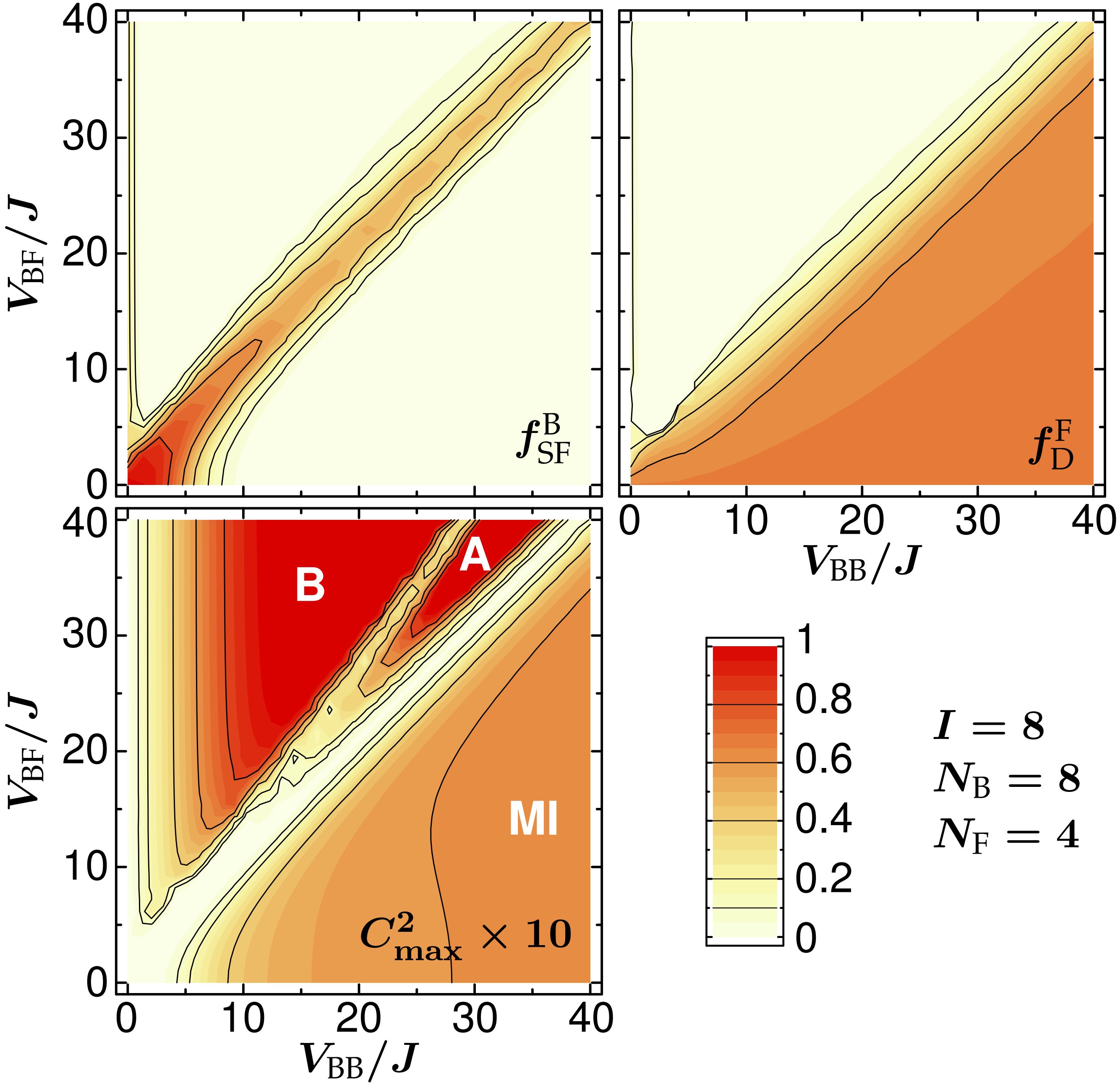
A: alternating boson-fermion occupation

➔ crystalline diagonal long-range order

B: continuous boson and fermion blocks

➔ component separation

V_{BB}-V_{BF} Phase Diagrams



A: alternating boson-fermion occupation

➔ crystalline diagonal long-range order

B: continuous boson and fermion blocks

➔ component separation

MI: bosonic Mott insulator

➔ fermions not affected

Summary

- **superfluidity**
  - stiffness under phase twists; depends crucially on the excitation spectrum
- **condensate & coherence**
  - property of the one-body density matrix of the ground state
- **two-color superlattices**
  - rich phase diagram with several insulating phases: localized, quasi Bose-glass, Mott-insulator
- **boson-fermion mixtures in lattices**
  - novel class of lattice systems with largely unexplored phase diagram

Epilogue

- unique degree of **experimental control** makes ultracold atomic gases in optical lattices...
  - ideal model systems to study strong correlation effects (quantum phase transitions) and other solid-state questions
  - promising “hardware” for quantum information processing

- many **fascinating questions** still open...
  - fermions in lattices, attractive interactions & Cooper pairing
  - Bragg spectroscopy & dynamic structure factor
  - simulation of time-dependent optical lattices

Epilogue

- **thanks to my collaborators**
  - M. Hild, F. Schmitt
    Technical University Darmstadt
  - K. Burnett, J. Dunningham, K. Braun-Munzinger
    University of Oxford

- **references**

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