

From Realistic Interactions to Collective Excitations:

Nuclear Structure in the UCOM Framework

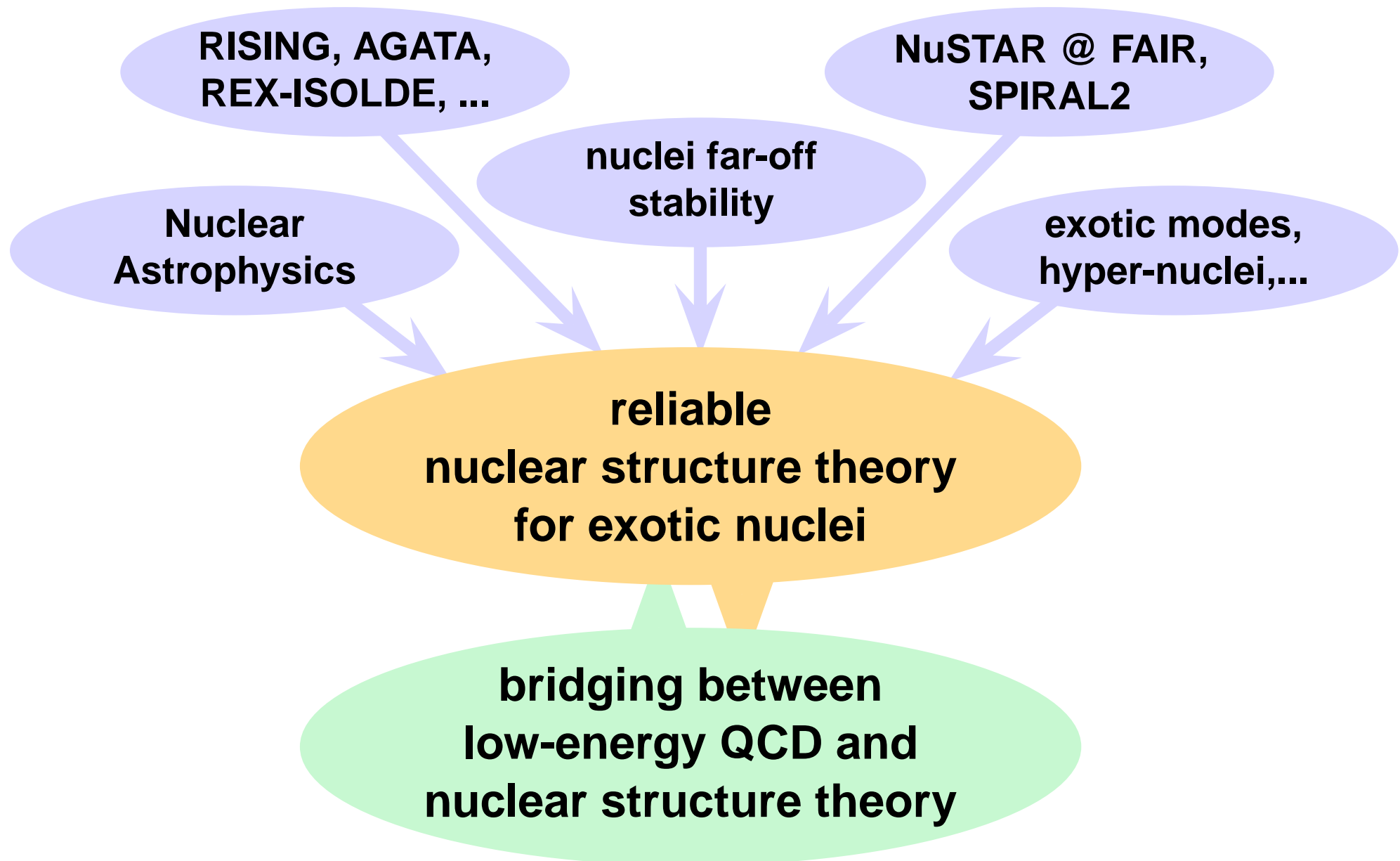


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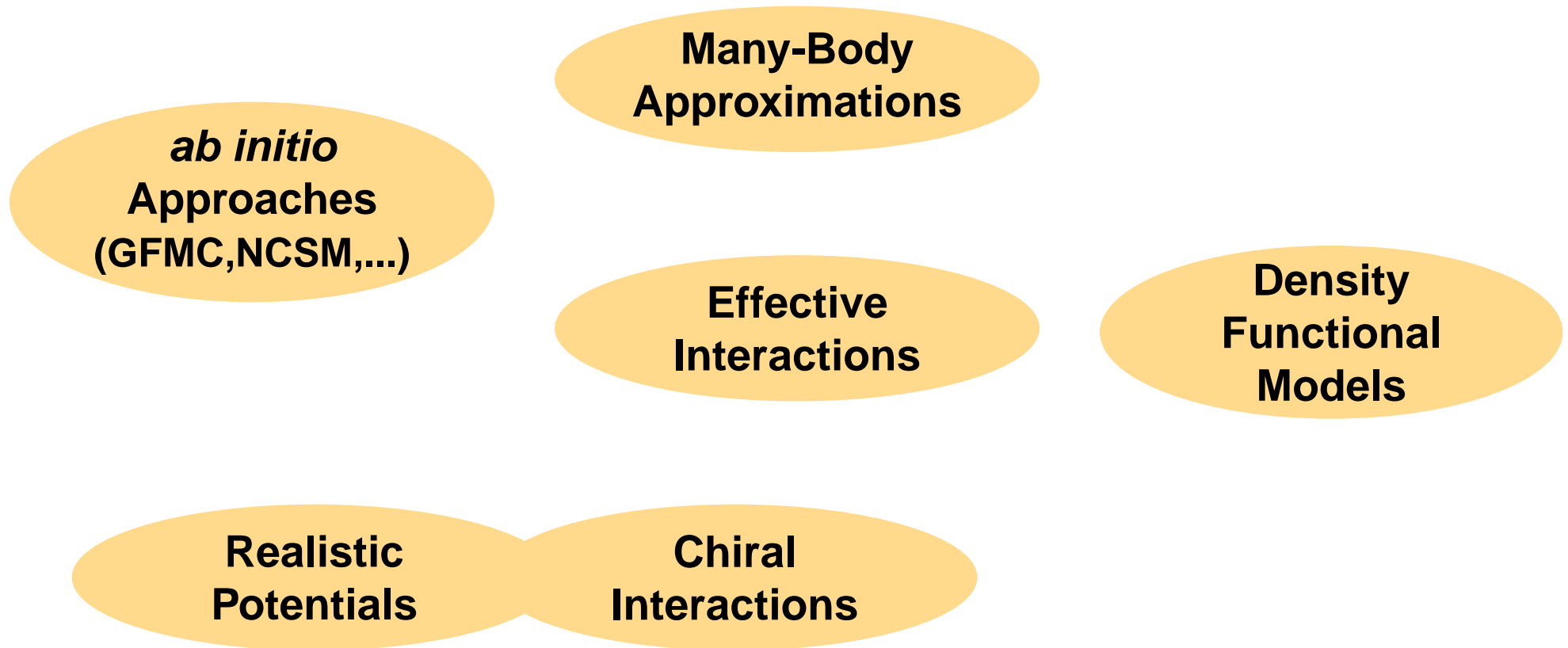
- Motivation
- Correlated Realistic NN-Potentials
 - Correlations & Unitary Correlation Operator Method
- Applications
 - No Core Shell Model
 - Hartree-Fock & Beyond
 - Random Phase Approximation & Beyond

Nuclear Structure in the 21st Century



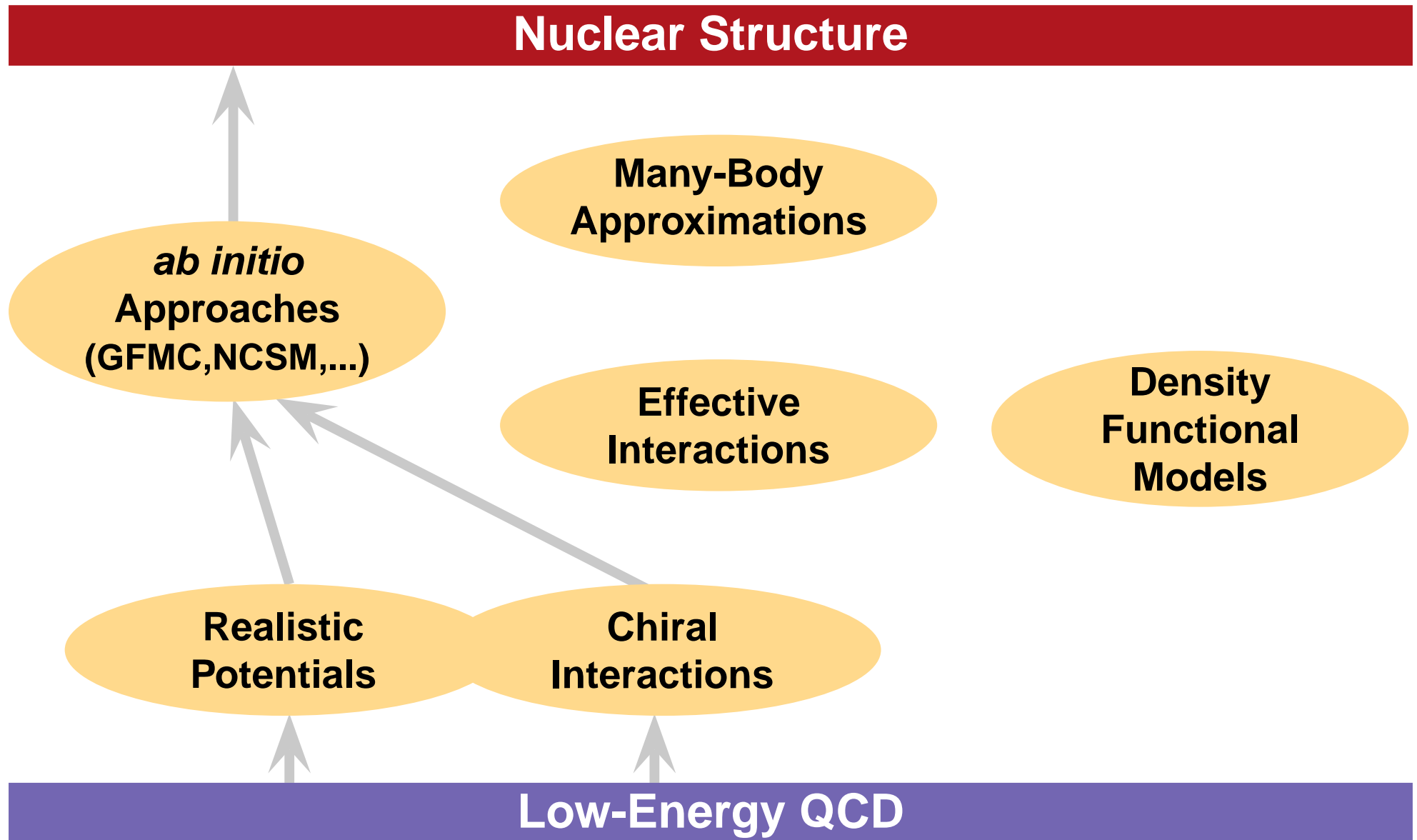
Modern Nuclear Structure Theory

Nuclear Structure

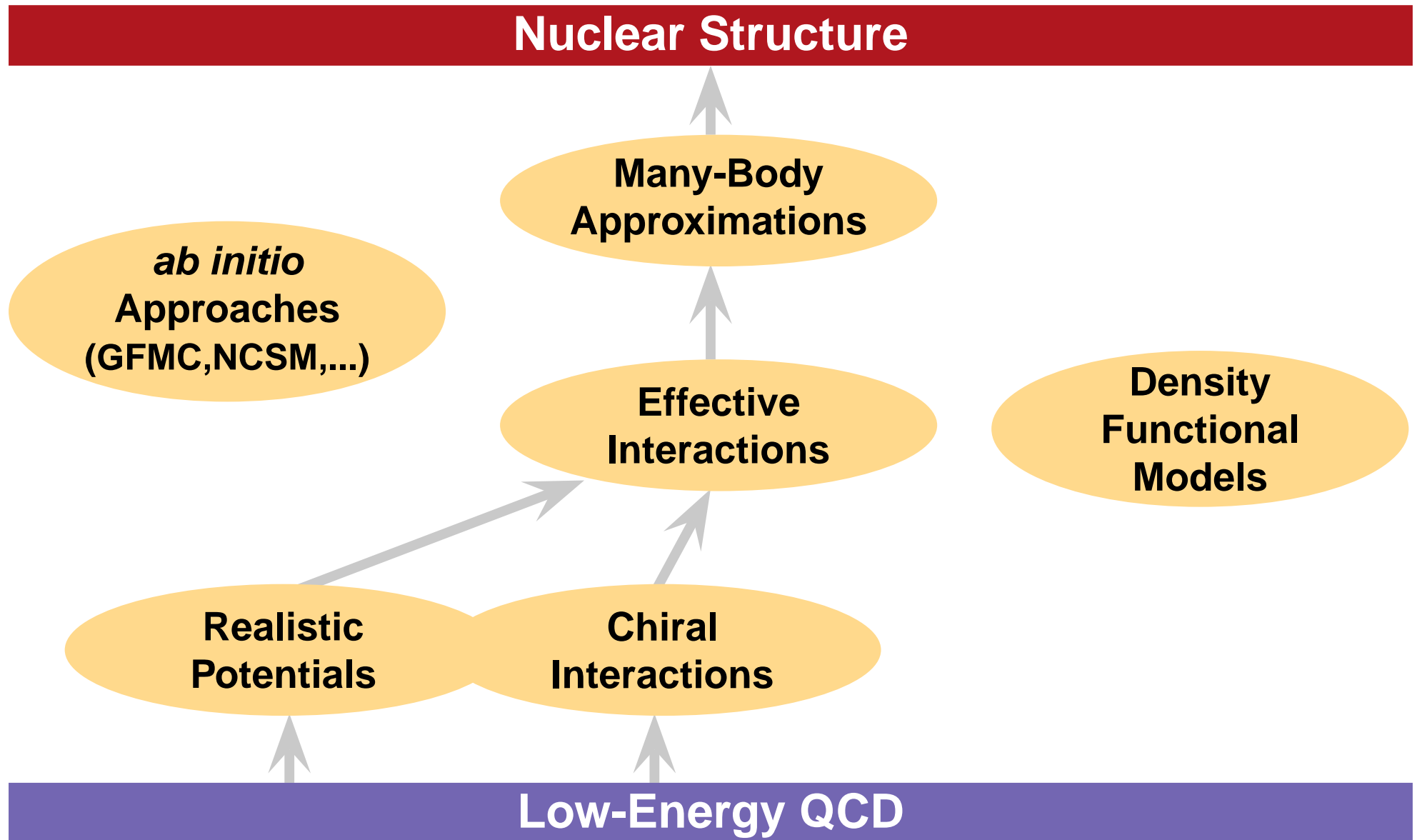


Low-Energy QCD

Modern Nuclear Structure Theory



Modern Nuclear Structure Theory



Realistic NN-Potentials

■ QCD motivated

- symmetries, meson-exchange picture
- chiral effective field theory

■ short-range phenomenology

- short-range parametrization or contact terms

■ experimental two-body data

- scattering phase-shifts & deuteron properties reproduced with high precision

■ supplementary three-nucleon force

- adjusted to spectra of light nuclei

Argonne V18

CD Bonn

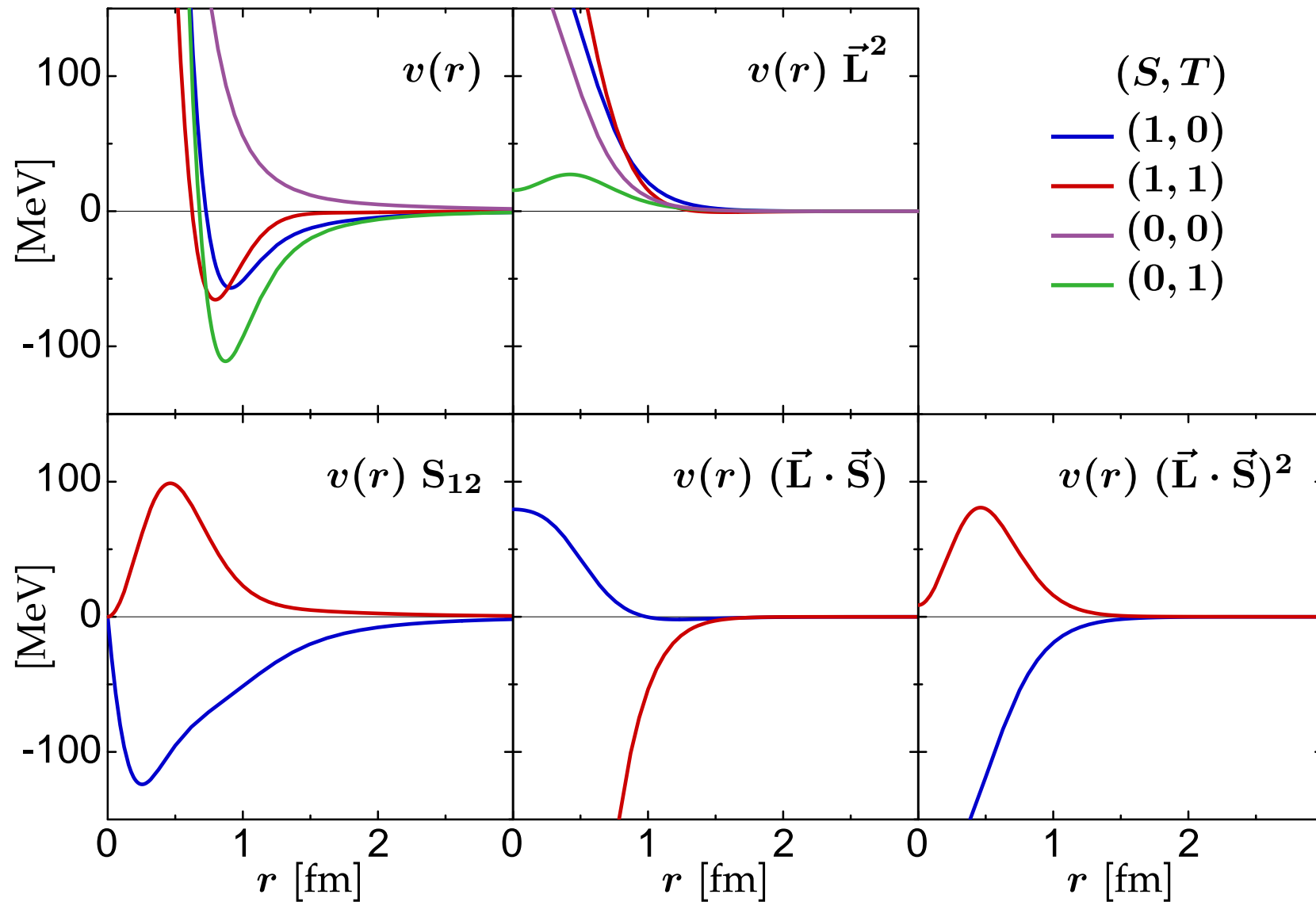
Nijmegen I/II

Chiral N3LO

Argonne V18 +
Illinois 2

Chiral N3LO +
N2LO

Argonne V18 Potential



Why Effective Interactions?

Realistic Potentials

- generate strong correlations in many-body states
- short-range central & tensor correlations most important

Many-Body Approximations

- rely on truncated many-nucleon Hilbert spaces for larger A
- not capable of describing short-range correlations
- extreme: Hartree-Fock based on single Slater determinant

Modern Effective Interactions

- adapt realistic potential to the available model spaces
- conserve experimentally constrained properties (phase shifts)

Unitary Correlation Operator Method (UCOM)

Unitary Correlation Operator Method

Correlation Operator

introduce short-range correlations by means of a unitary transformation with respect to the relative coordinates of all pairs

$$\mathbf{C} = \exp[-i \mathbf{G}] = \exp\left[-i \sum_{i < j} g_{ij}\right]$$

$$\begin{aligned} \mathbf{G}^\dagger &= \mathbf{G} \\ \mathbf{C}^\dagger \mathbf{C} &= \mathbf{1} \end{aligned}$$

Correlated States

$$|\tilde{\psi}\rangle = \mathbf{C} |\psi\rangle$$

Correlated Operators

$$\tilde{\mathbf{O}} = \mathbf{C}^\dagger \mathbf{O} \mathbf{C}$$

$$\langle \tilde{\psi} | \mathbf{O} | \tilde{\psi}' \rangle = \langle \psi | \mathbf{C}^\dagger \mathbf{O} \mathbf{C} | \psi' \rangle = \langle \psi | \tilde{\mathbf{O}} | \psi' \rangle$$

Central and Tensor Correlators

$$C = C_{\Omega} C_r$$

Central Correlator C_r

- radial distance-dependent shift in the relative coordinate of a nucleon pair

$$g_r = \frac{1}{2} [s(r) q_r + q_r s(r)]$$

$$q_r = \frac{1}{2} \left[\frac{\vec{r}}{r} \cdot \vec{q} + \vec{q} \cdot \frac{\vec{r}}{r} \right]$$

Tensor Correlator C_{Ω}

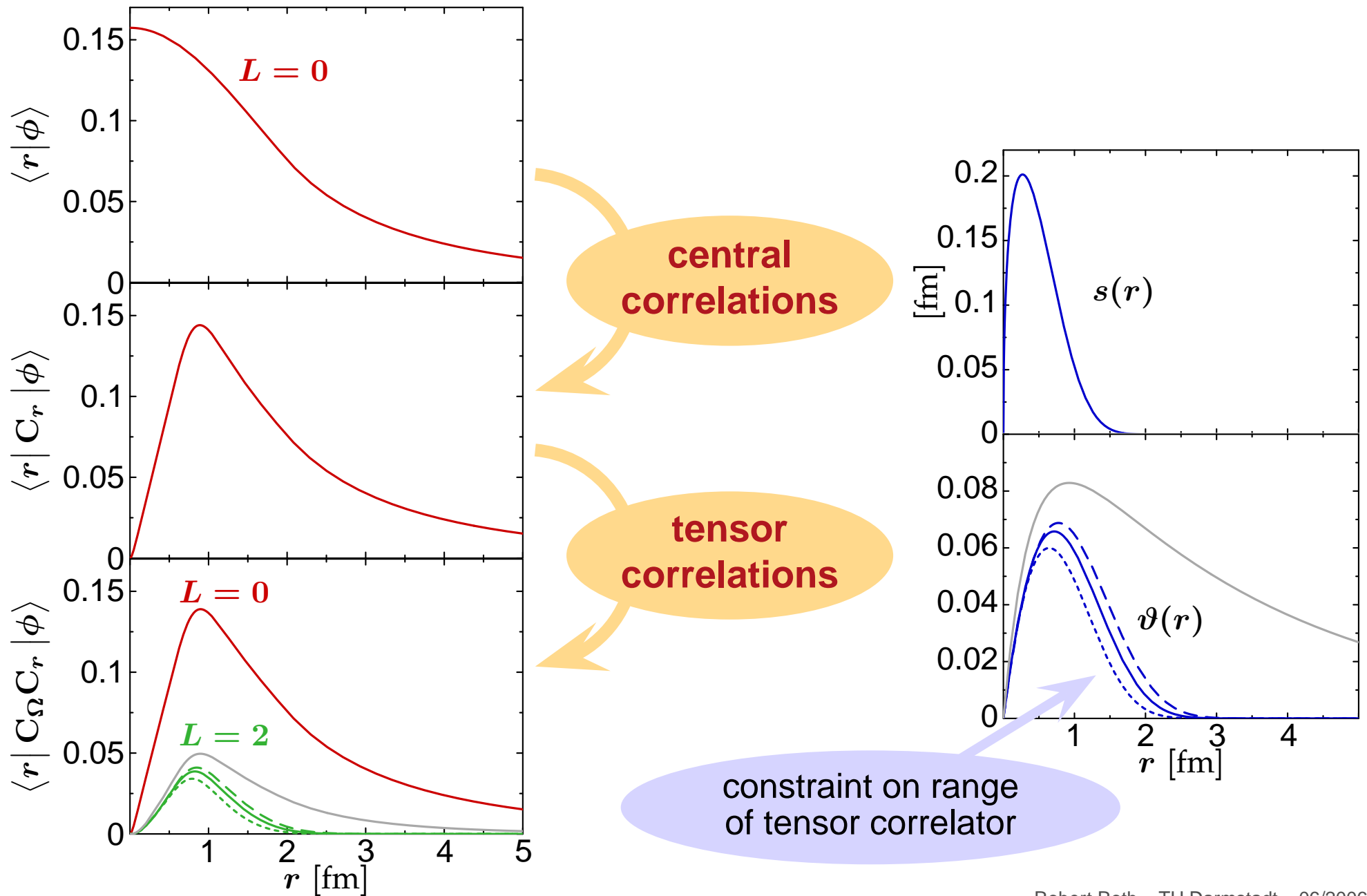
- angular shift depending on the orientation of spin and relative coordinate of a nucleon pair

$$g_{\Omega} = \frac{3}{2} \vartheta(r) [(\vec{\sigma}_1 \cdot \vec{q}_{\Omega})(\vec{\sigma}_2 \cdot \vec{r}) + (\vec{r} \leftrightarrow \vec{q}_{\Omega})]$$

$$\vec{q}_{\Omega} = \vec{q} - \frac{\vec{r}}{r} q_r$$

$s(r)$ and $\vartheta(r)$
for given potential determined
in the two-body system

Correlated States: The Deuteron



Correlated Interaction — V_{UCOM}

$$\tilde{H} = T + V_{\text{UCOM}} + V_{\text{UCOM}}^{[3]} + \dots$$

- **closed operator expression** for the correlated interaction V_{UCOM} in two-body approximation
- correlated interaction and original NN-potential are **phase shift equivalent** by construction
- unitary transformation results in a **pre-diagonalization** of Hamiltonian
- momentum-space matrix elements of correlated interaction are **similar to** $V_{\text{low-}k}$

Application I

No-Core Shell Model

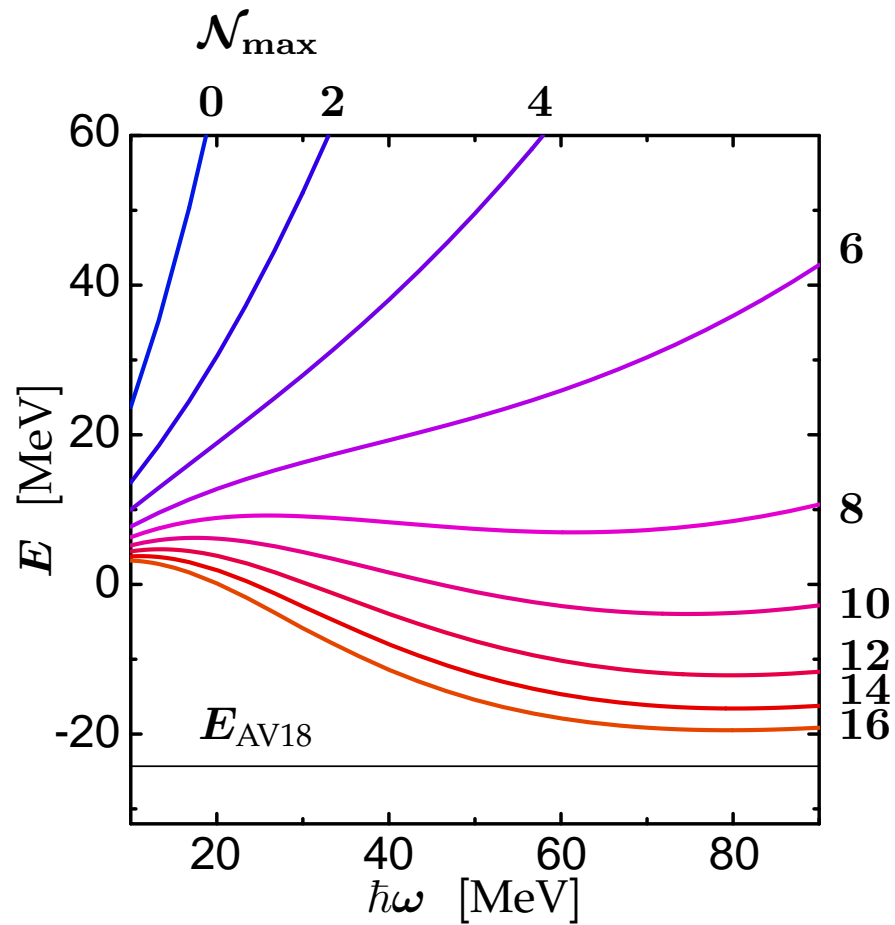
No-Core Shell Model
+
**Matrix Elements of Correlated
Realistic NN-Interaction V_{UCOM}**

- many-body state is **expanded in Slater determinants** of harmonic oscillator single-particle states
- **large scale diagonalization** of Hamiltonian within a truncated model space ($\mathcal{N}^{\hbar\omega}$ truncation)
- assessment of **short and long-range correlations**

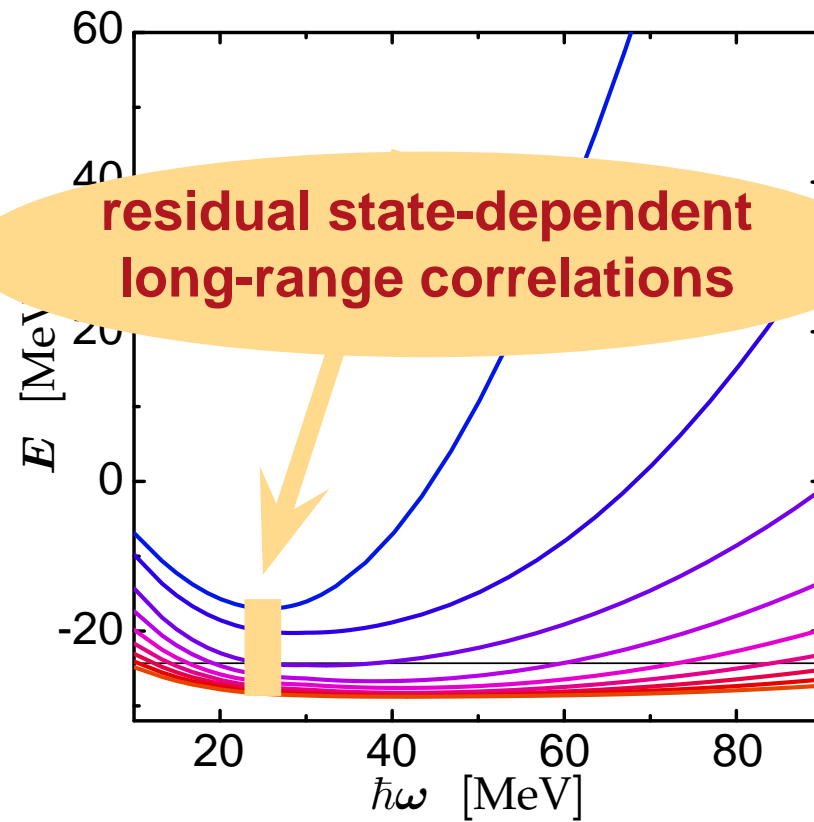
NCSM code by Petr Navrátil [PRC 61, 044001 (2000)]

^4He : Convergence

V_{AV18}

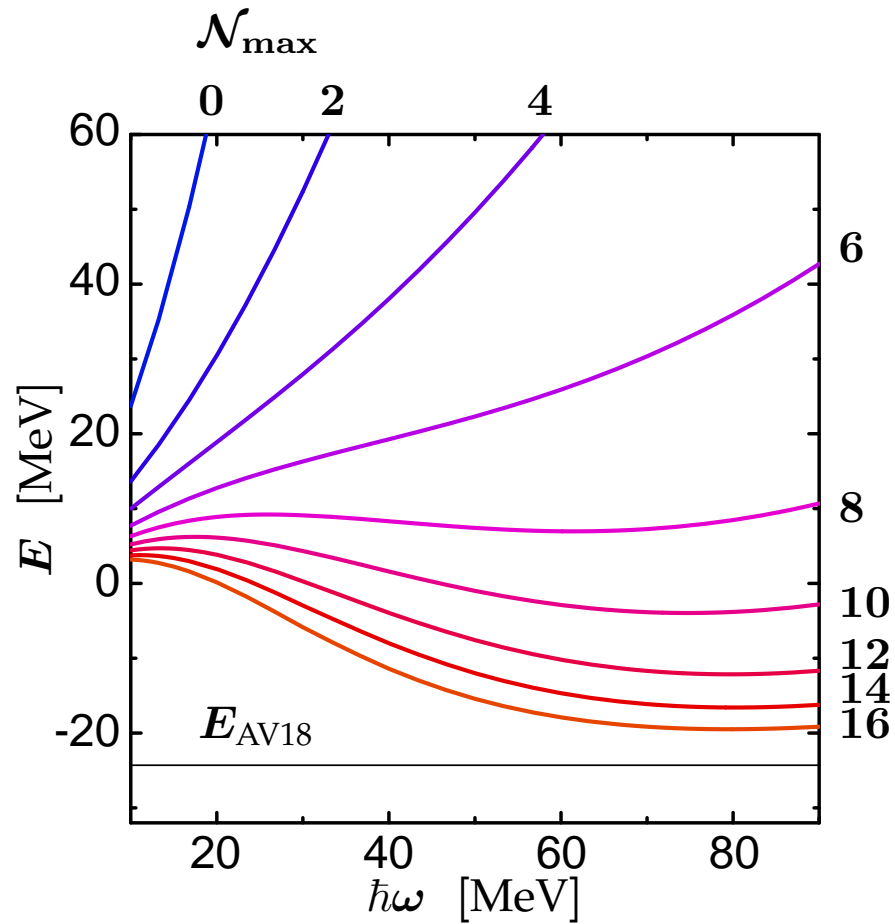


V_{UCOM}

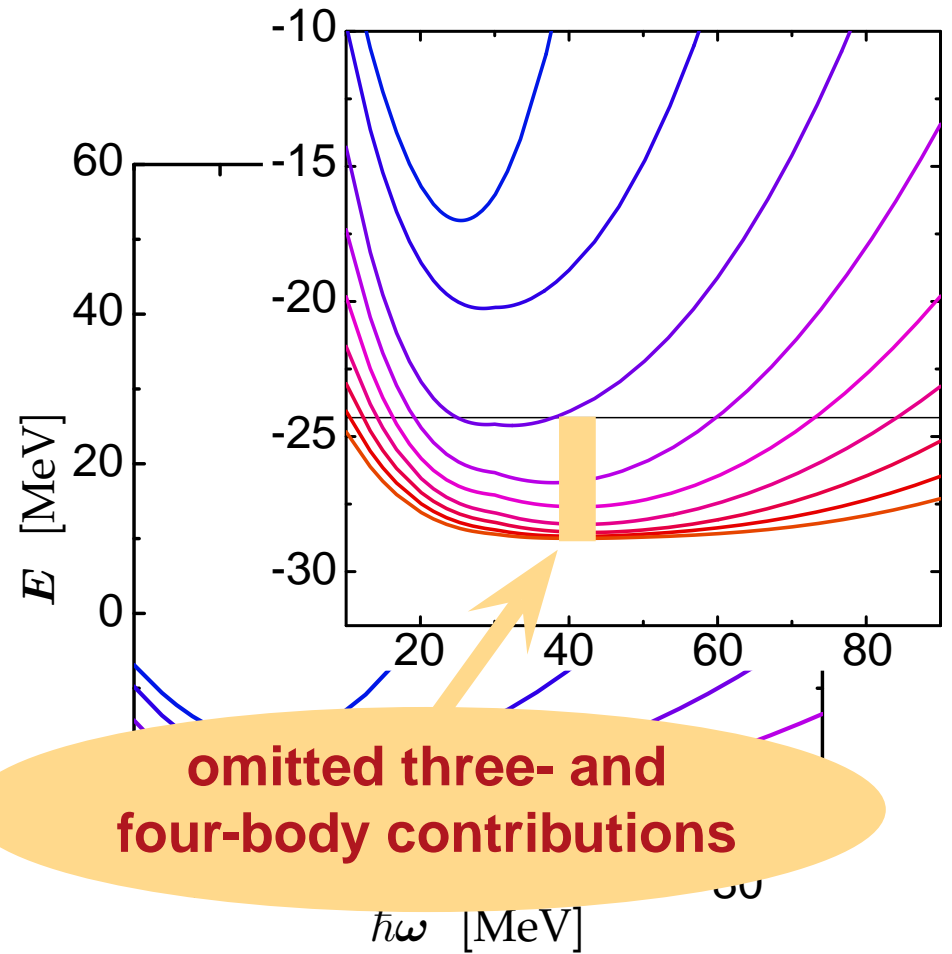


^4He : Convergence

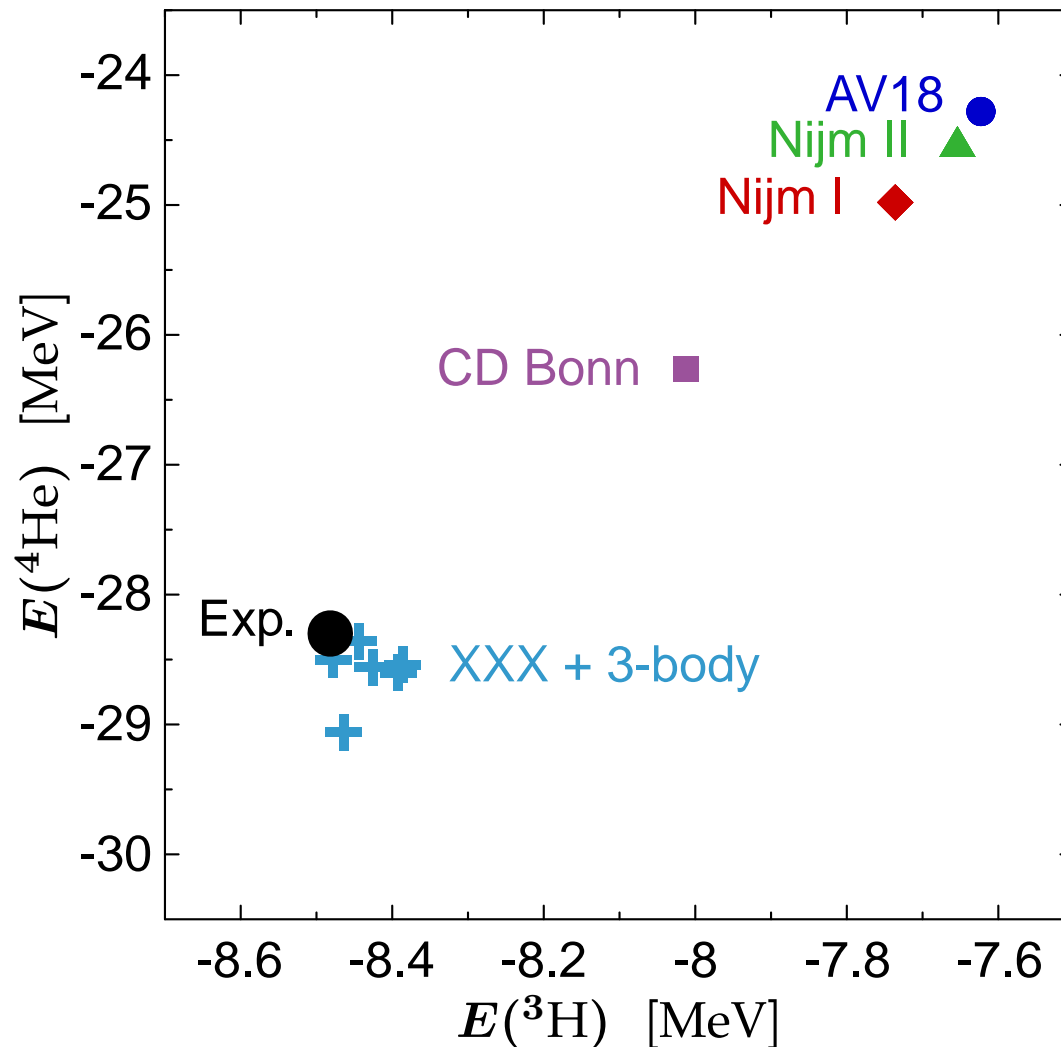
V_{AV18}



V_{UCOM}

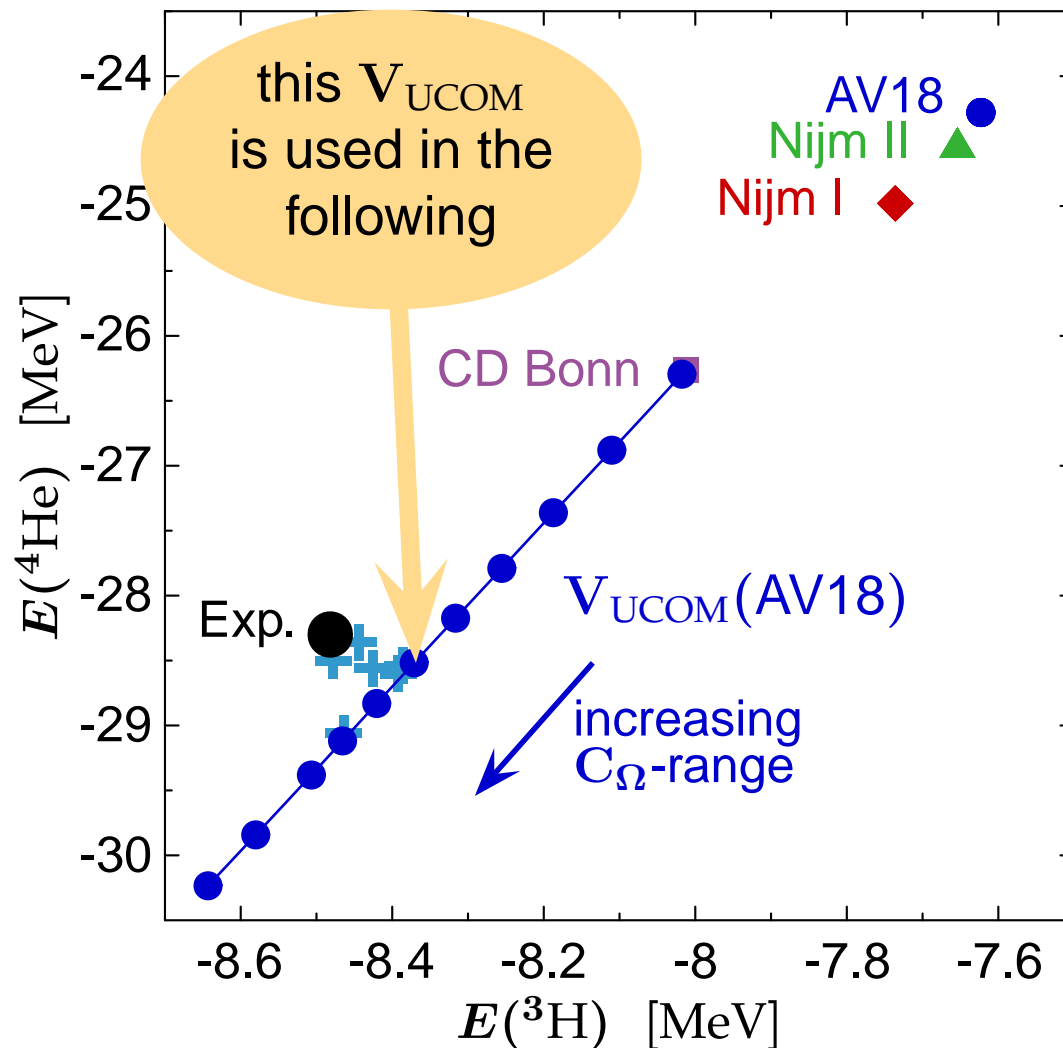


Tjon-Line and Correlator Range



- **Tjon-line:** $E({}^4\text{He})$ vs. $E({}^3\text{H})$ for phase-shift equivalent NN-interactions

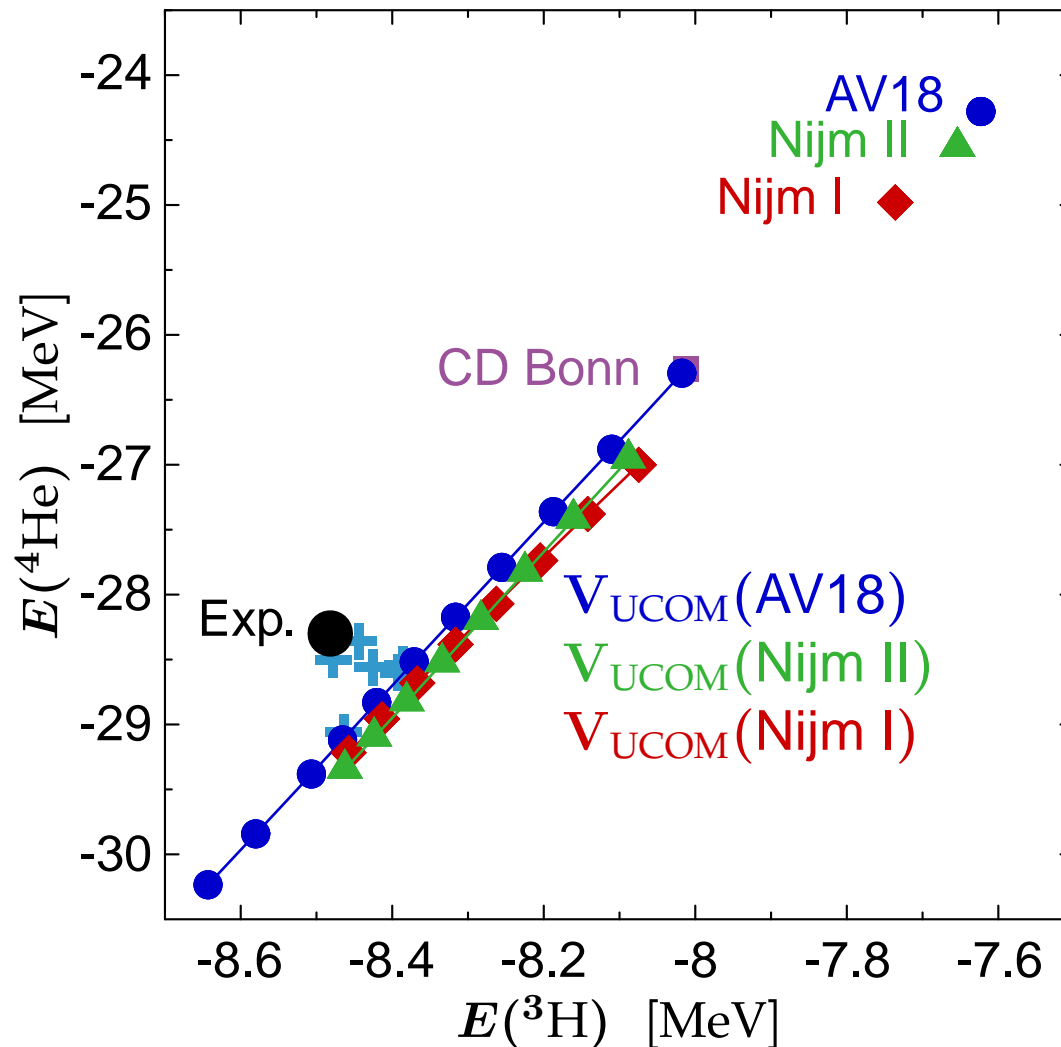
Tjon-Line and Correlator Range



- **Tjon-line:** $E(^4\text{He})$ vs. $E(^3\text{H})$ for phase-shift equivalent NN-interactions
- change of C_{Ω} -correlator range results in shift along Tjon-line

minimise net three-body force by choosing correlator with energies close to experimental value

Tjon-Line and Correlator Range

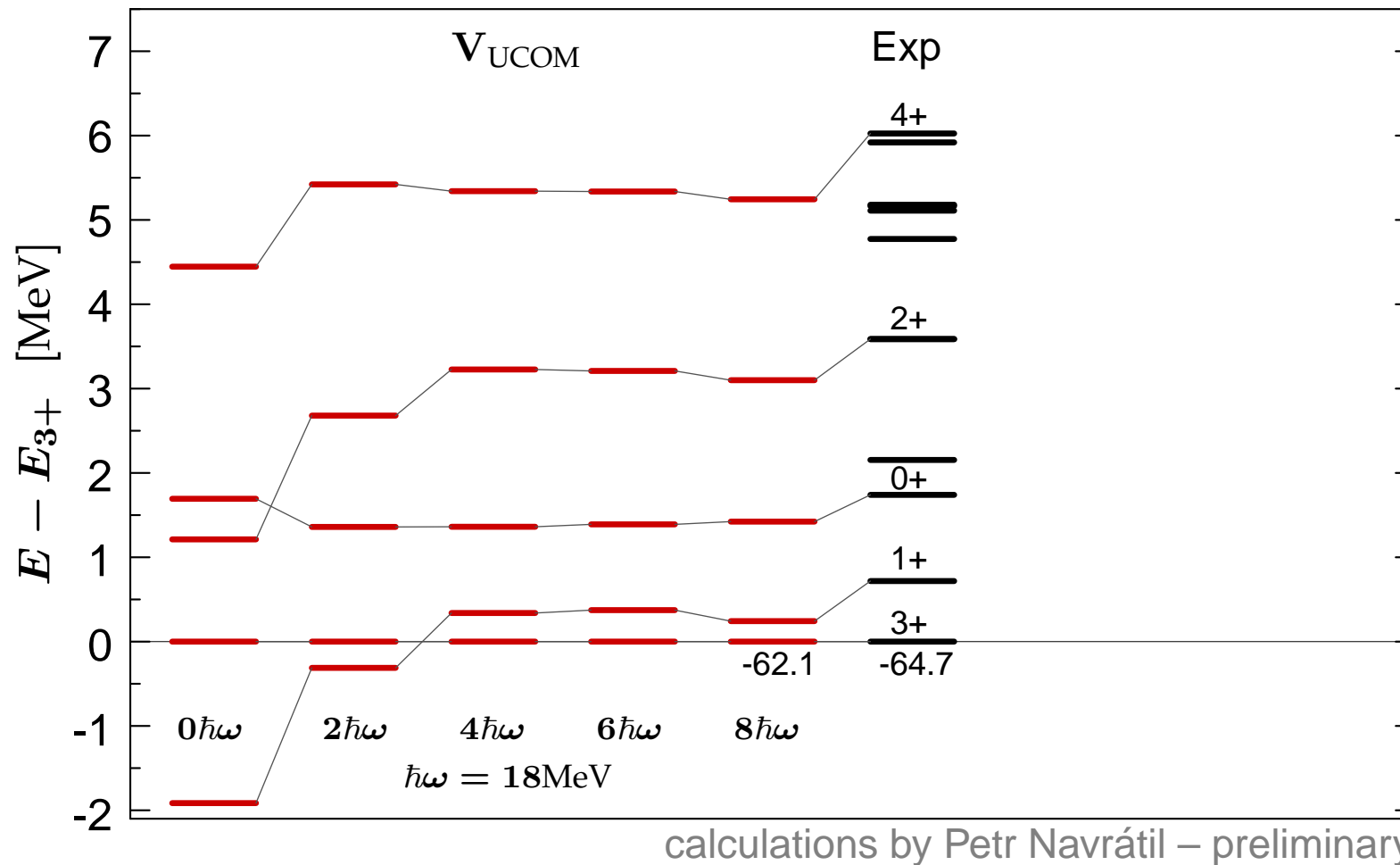


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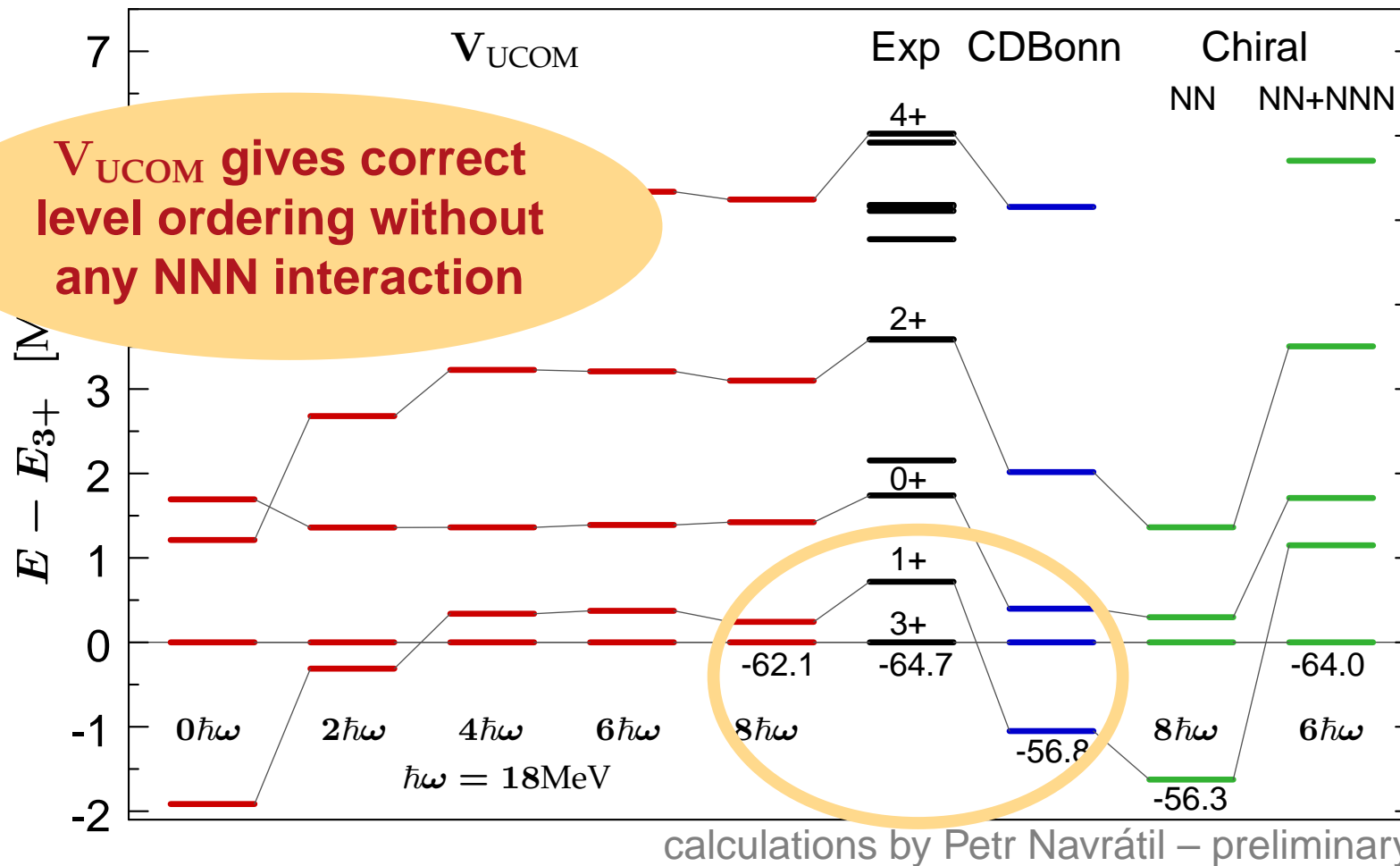
^{10}B : Benchmarking V_{UCOM}

- large-scale NCSM calculations throughout the p-shell in progress (with and w/o Lee-Suzuki transformation)



¹⁰B: Benchmarking V_{UCOM}

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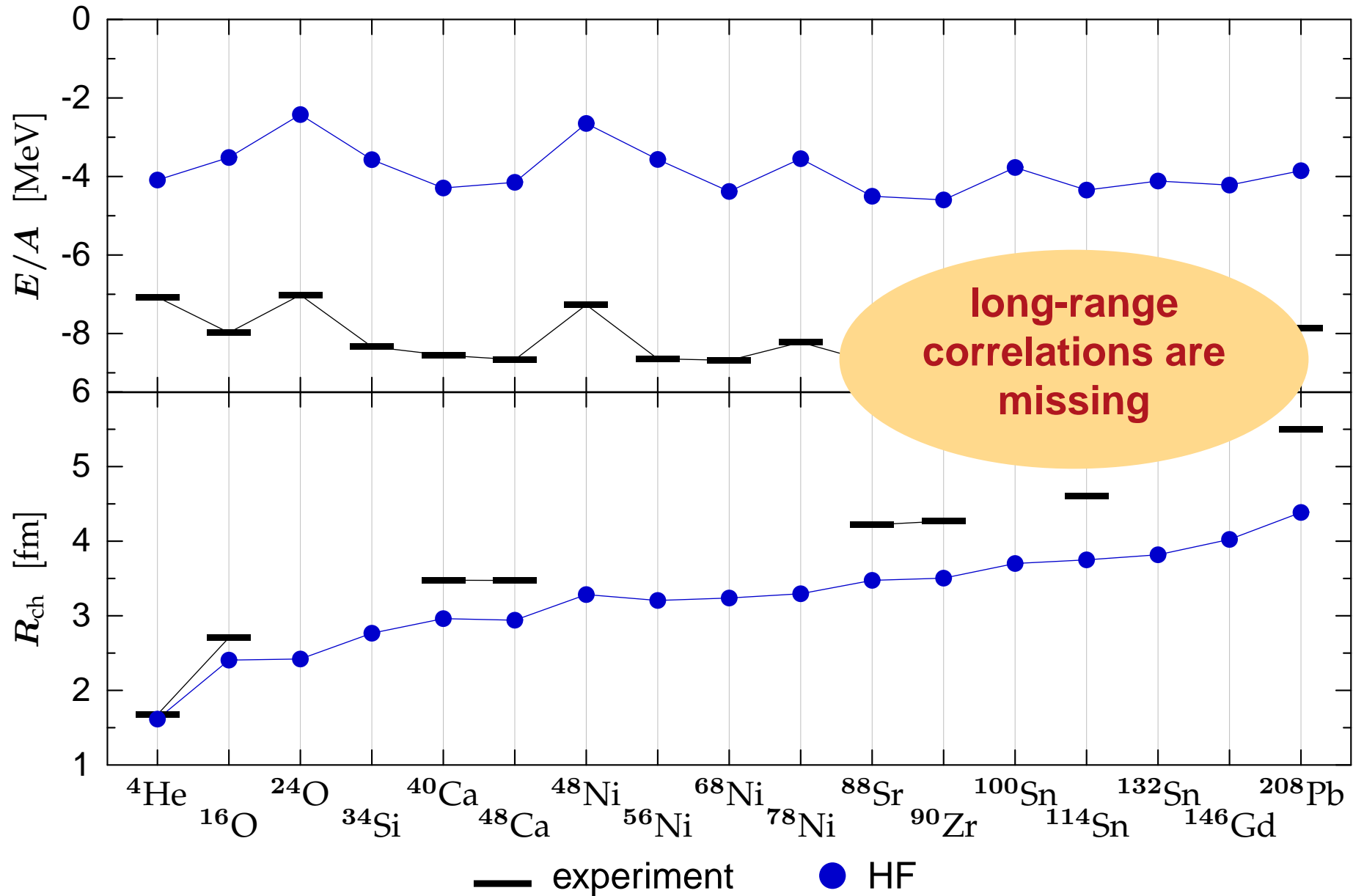
Application II:

Hartree-Fock & Beyond

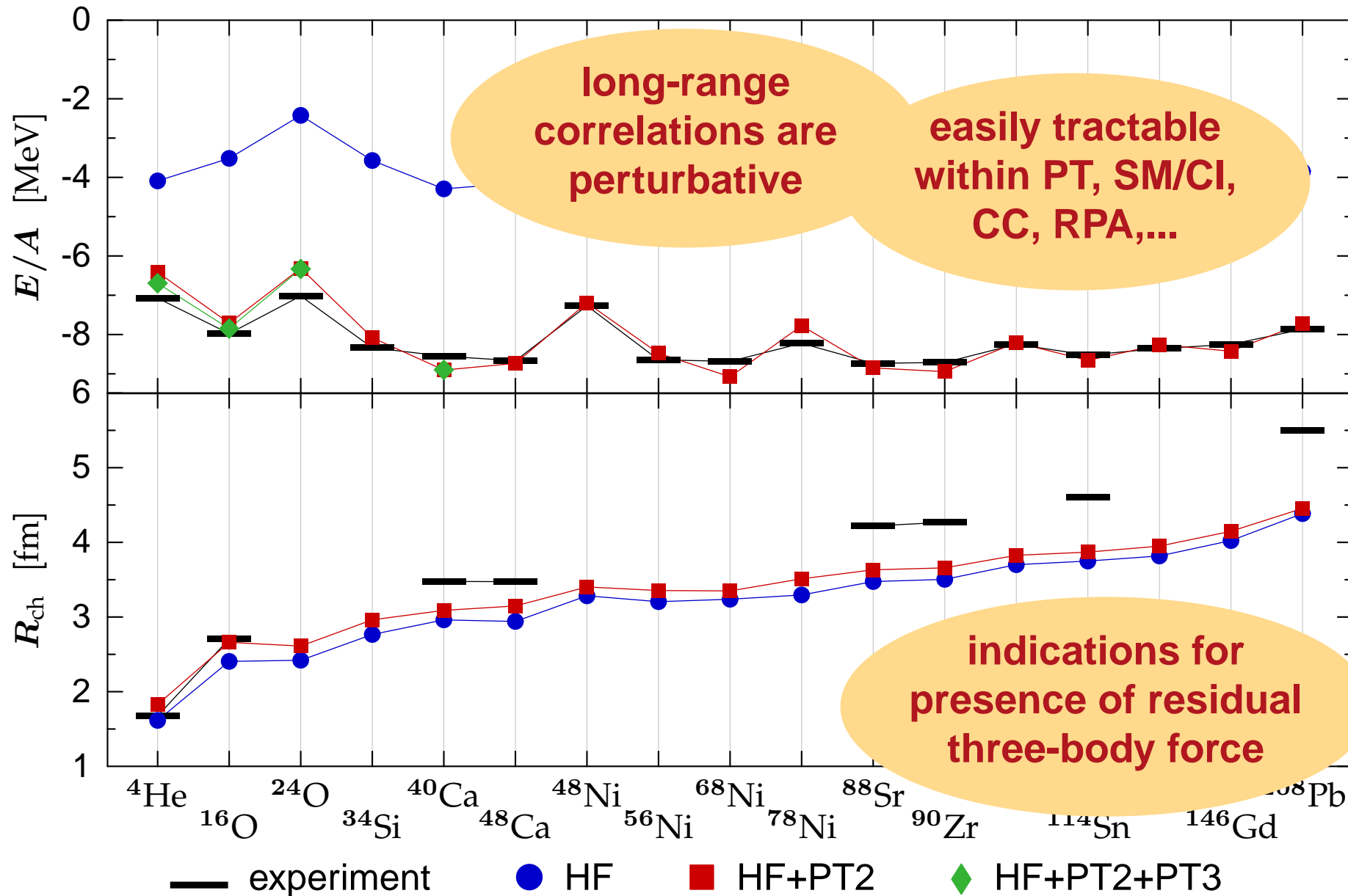
Standard Hartree-Fock
+
**Matrix Elements of Correlated
Realistic NN-Interaction V_{UCOM}**

- many-body state is a **Slater determinant** of single-particle states expanded in oscillator basis (typically 13 major shells)
- **correlations cannot be described** by Hartree-Fock states
- starting point for **improved many-body calculations**: MBPT, RPA, SM/CI, CC,...

Hartree-Fock with V_{UCOM}



Perturbation Theory with V_{UCOM}



Application III

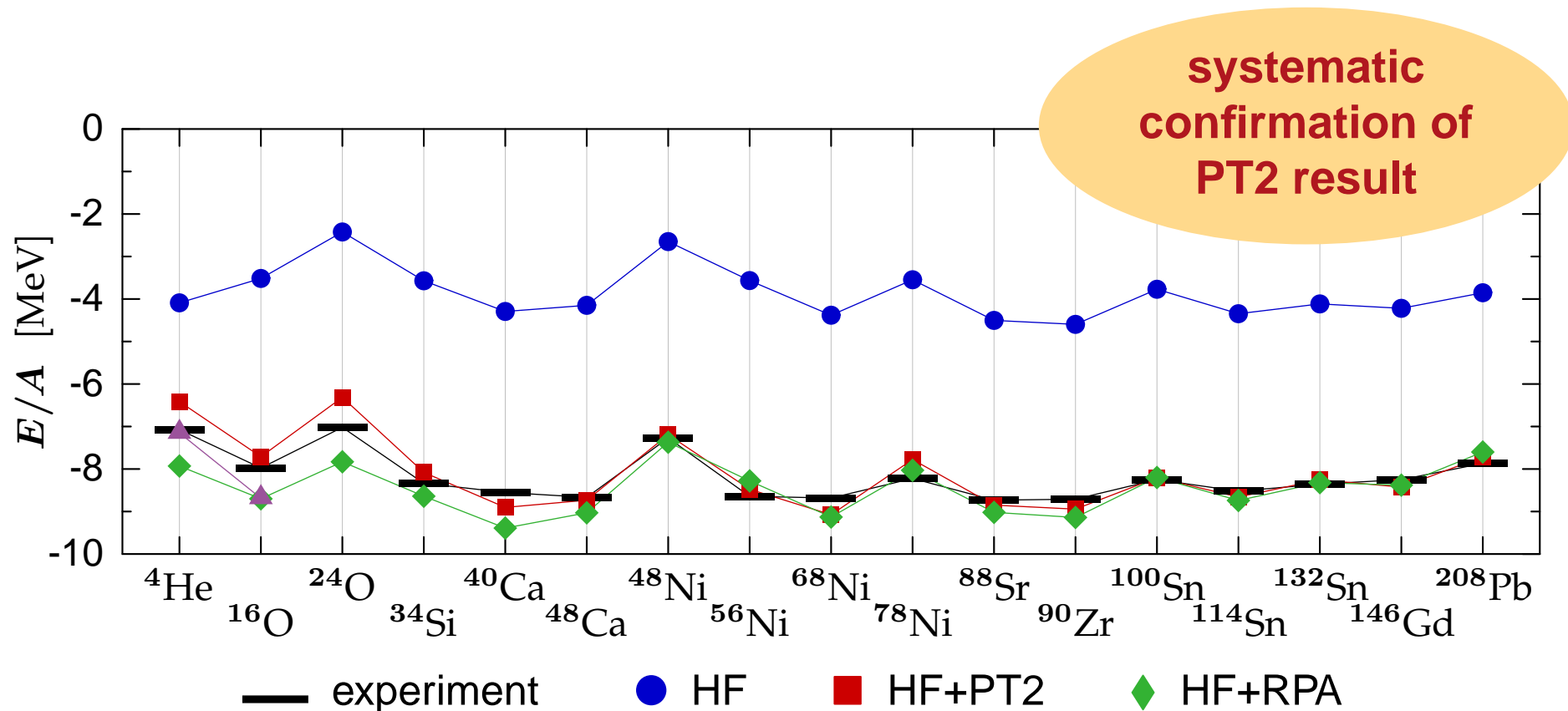
RPA & Beyond

RPA, ERPA & SRPA
+
**Matrix Elements of Correlated
Realistic NN-Interaction V_{UCOM}**

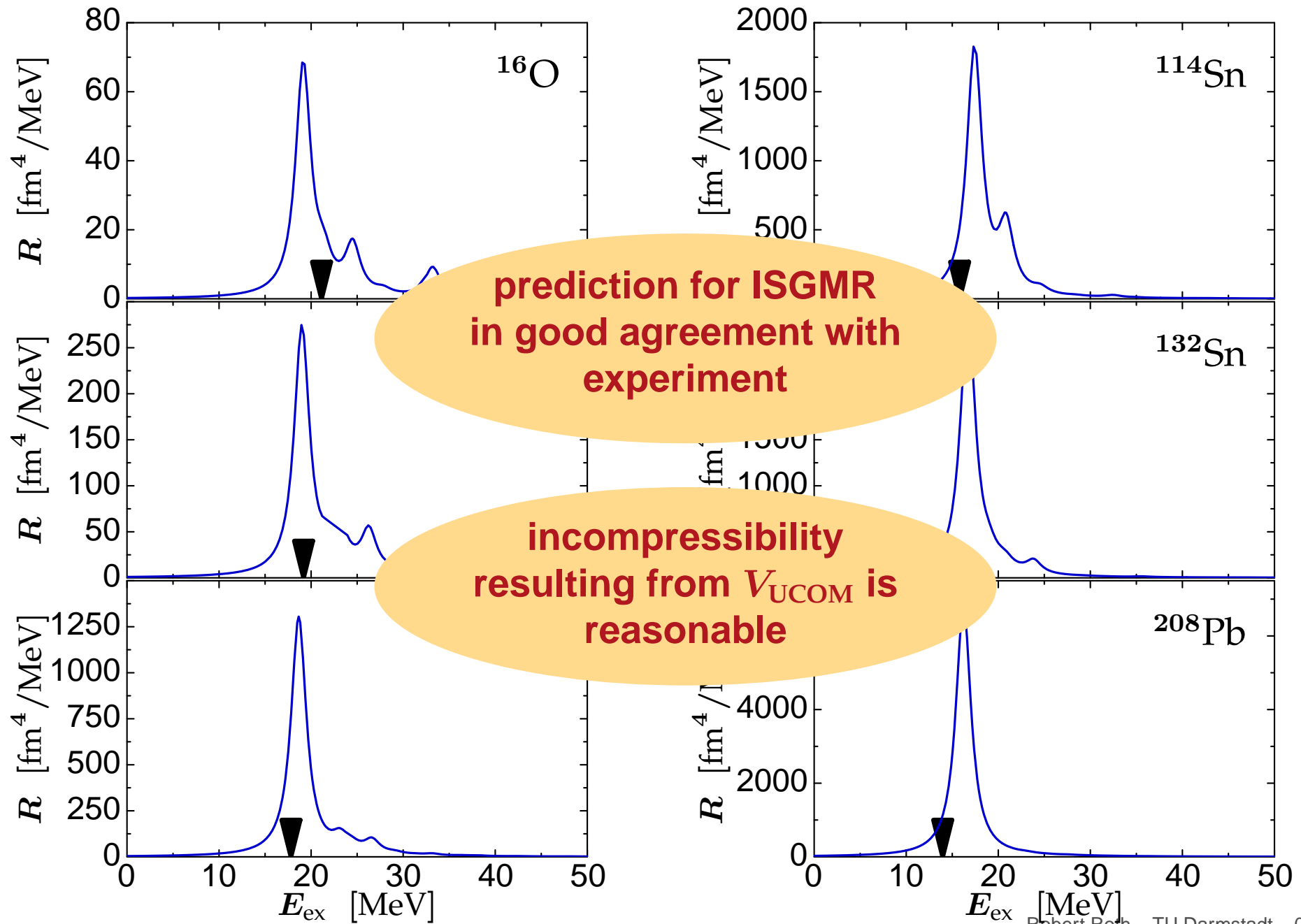
- **fully self-consistent RPA** based on the Hartree-Fock orbits using the same V_{UCOM}
- recovering sum rules with high precision, spurious center-of-mass mode fully decoupled at ~ 10 keV
- **Extended-RPA and Second-RPA** to include effects of ground state correlations and complex configurations

RPA Ground State Correlations

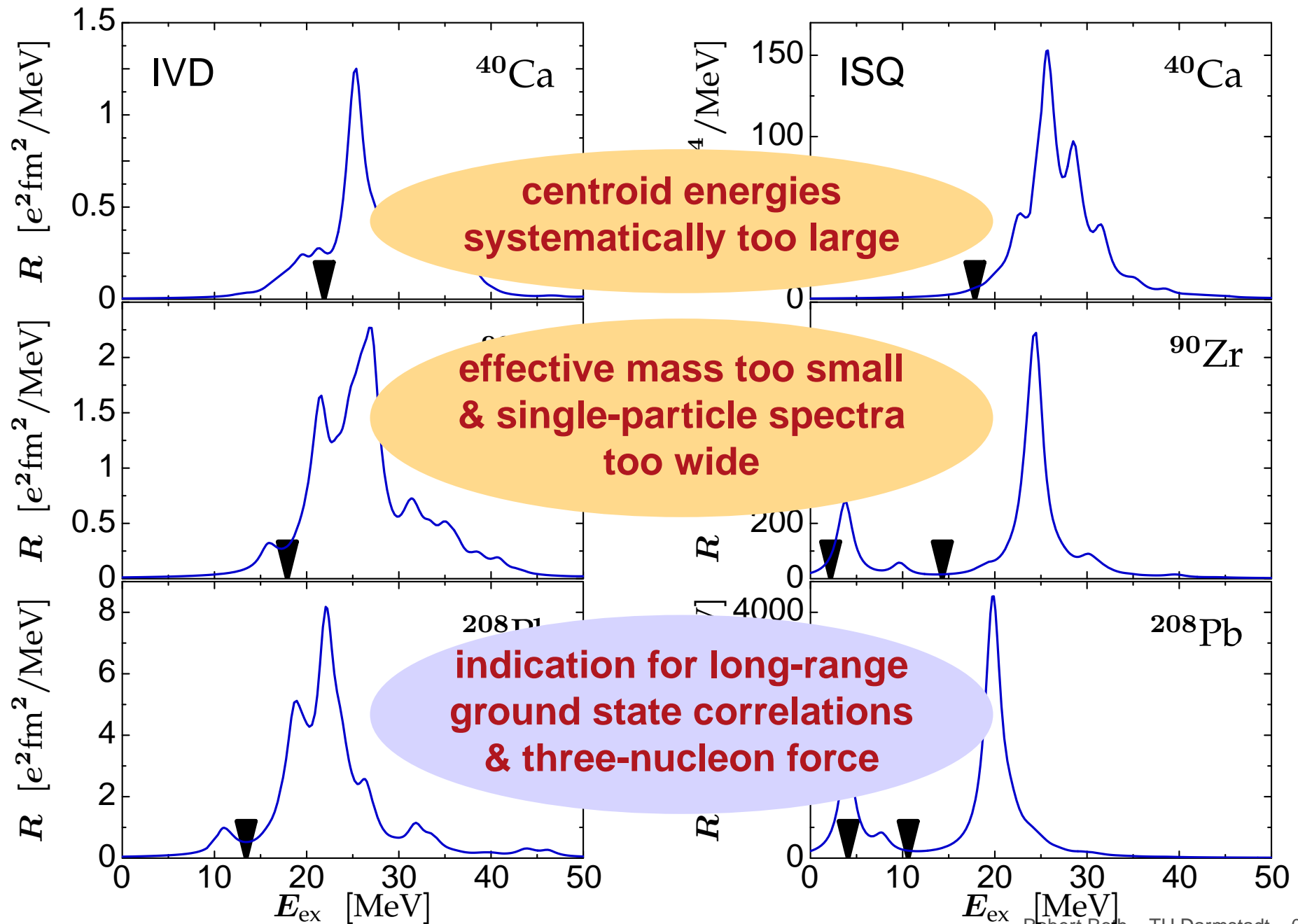
- evaluate correlation energy beyond Hartree-Fock via **ring summation** using RPA amplitudes
- include all parities and charge exchange and correct for double-counting of 2nd order term



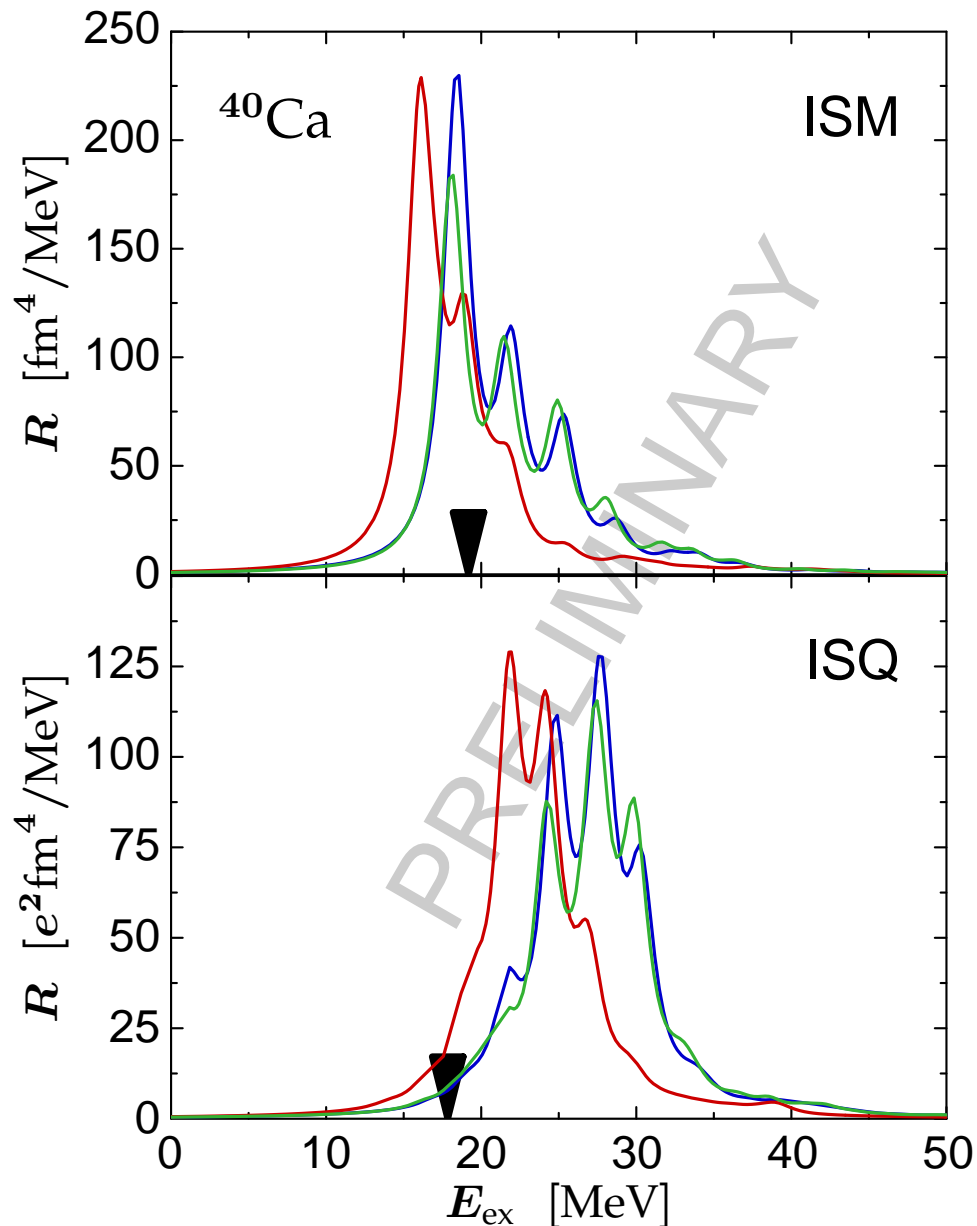
Isoscalar Giant Monopole



Isvector Dipole & Isoscalar Quadrupole



Outlook: Extended & Second RPA



- **ERPA**: iteratively build on the true RPA ground state (not the HF state)

→ small effect on response

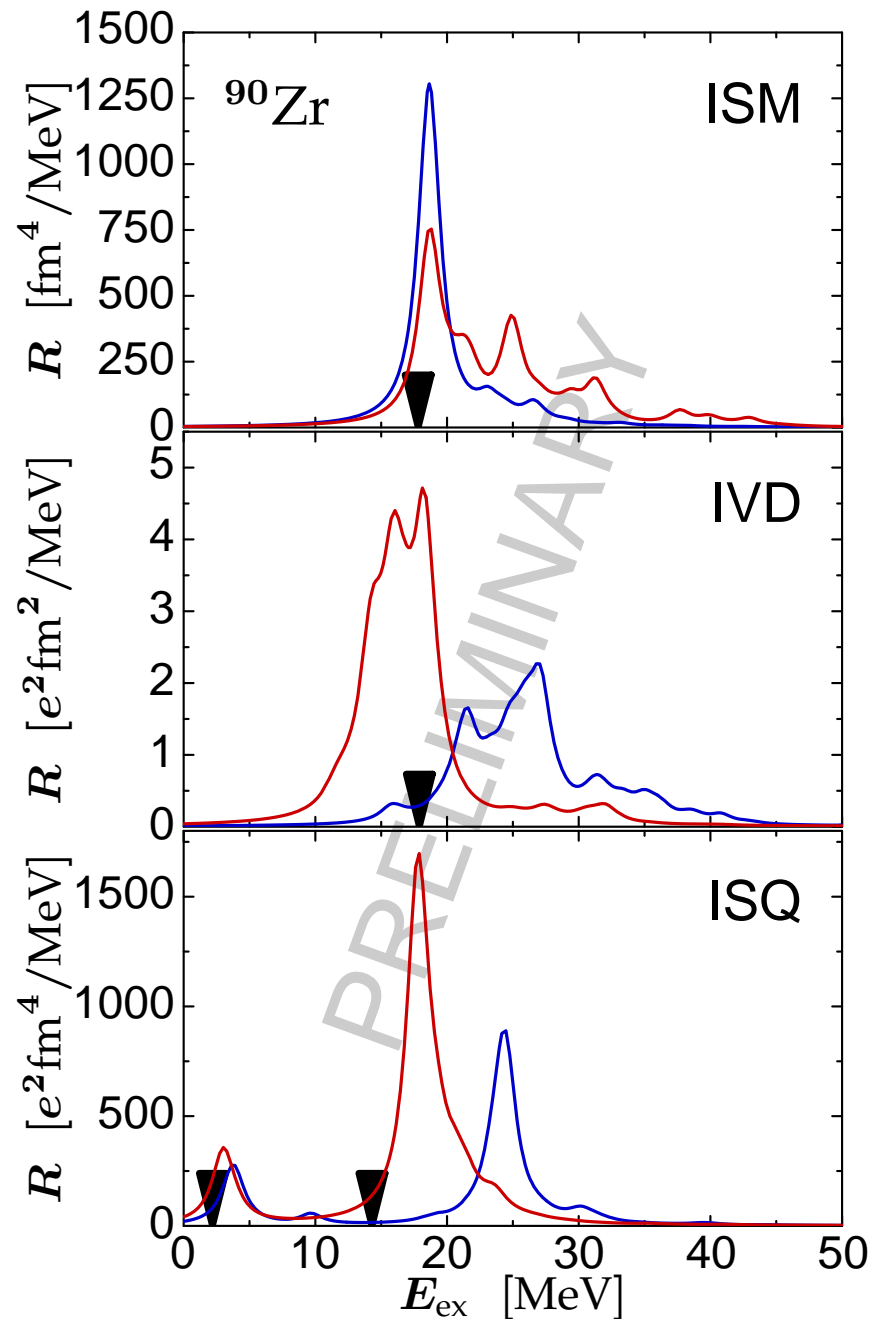
- **SRPA**: inclusion of 2p2h configurations

→ sizable shift of resonances

see poster by
P. Papakonstantinou

- RPA
- ERPA
- SRPA ($E_{2p2h} \leq 100\text{MeV}$)

Outlook: RPA with Three-Body Forces



- long-range tensor correlator & **repulsive three-body contact interaction**

- systematic improvement of

- rms-radii

- single-particle spectra

- strength distributions

— standard V_{UCOM}

— V_{UCOM} with long-range tensor & three-body contact interaction

Conclusions

■ Unitary Correlation Operator Method (UCOM)

- explicit description of short-range central and tensor correlations
- universal phase-shift equivalent correlated interaction V_{UCOM}

■ Innovative Many-Body Methods

- No-Core Shell Model
- Hartree-Fock, MBPT, SM/CI, CC,...
- RPA, ERPA, SRPA, GFRPA,...
- Fermionic Molecular Dynamics

**unified description of nuclear
structure across the whole
nuclear chart is within reach**

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