Nuclear Structure in the UCOM Framework:

Few-Body Systems

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Overview

- The Unitary Correlation Operator Method (UCOM)
- No-Core Shell Model (NCSM) Calculations
  - Short- & Long-Range Correlations
  - Tjon-Line
  - $\boldsymbol{p}$-Shell: $^{6}\mathrm{Li}$ and $^{10}\mathrm{B}$
- Summary & Outlook

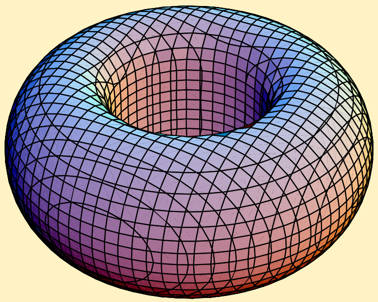
Unitary Correlation Operator Method (UCOM)

Motivation

Argonne V18 Deuteron Solution

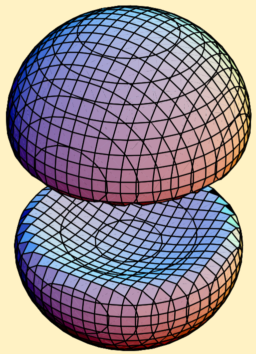
$M_S = 0$

$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$



$M_S = \pm 1$

$|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle$



$\rho^{(2)}_{1,M_S}(\vec{r})$

- **central correlations:** two-body density is suppressed at low distances
- **tensor correlations:** angular distribution depends on the relative spin alignments

use very large many-body Hilbert spaces
$\Rightarrow$ **high computational effort**

or

use numerically affordable Hilbert spaces and **treat strong correlations explicitly**

Central and Tensor Correlators

Central Correlator C_r

- radial distance-dependent shift in the relative coordinate of a nucleon pair

$$\mathrm{C}_r = \exp(-i \sum_{i,j}^{A} \mathrm{g}_{r,ij})$$

$$\mathrm{g}_r = \frac{1}{2}\big[s(\mathrm{r})\ \mathrm{q}_r + \mathrm{q}_r\ s(\mathrm{r})\big]$$

$$\mathrm{q}_r = \tfrac{1}{2}\big[\tfrac{\vec{\mathrm{r}}}{\mathrm{r}} \cdot \vec{\mathrm{q}} + \vec{\mathrm{q}} \cdot \tfrac{\vec{\mathrm{r}}}{\mathrm{r}}\big]$$

Tensor Correlator C_Ω

- angular shift, depending on the orientation of spin and relative coordinate of a nucleon pair

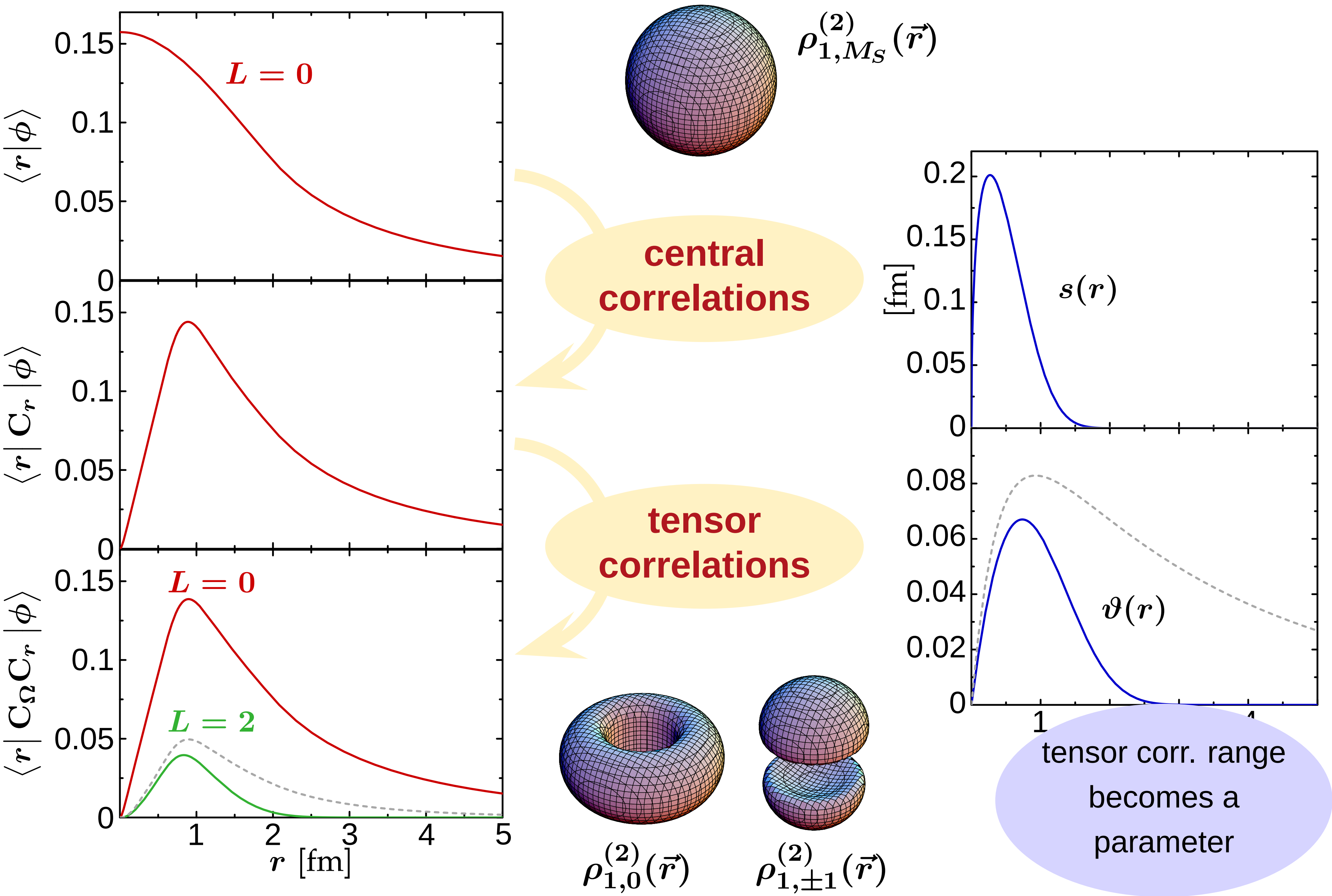
$$\mathrm{C}_\Omega = \exp(-i \sum_{i,j}^{A} \mathrm{g}_{\Omega,ij})$$

$$\mathrm{g}_\Omega = \frac{3}{2}\vartheta(\mathrm{r})\big[(\vec{\sigma}_1 \cdot \vec{\mathrm{q}}_\Omega)(\vec{\sigma}_2 \cdot \vec{\mathrm{r}}) + (\vec{\mathrm{r}} \leftrightarrow \vec{\mathrm{q}}_\Omega)\big]$$

$$\vec{\mathrm{q}}_\Omega = \vec{\mathrm{q}} - \tfrac{\vec{\mathrm{r}}}{\mathrm{r}}\ \mathrm{q}_r$$

$s(r)$ and $\vartheta(r)$
encapsulate the physics of
short-range correlations.

Correlated States



Correlated Interaction

Correlated Hamiltonian

$$\widetilde{\mathrm{H}} = \mathrm{T}^{[1]} + \mathrm{V}_{\mathrm{UCOM}} + \mathrm{V}_{\mathrm{UCOM}}^{[3]} + \dots$$

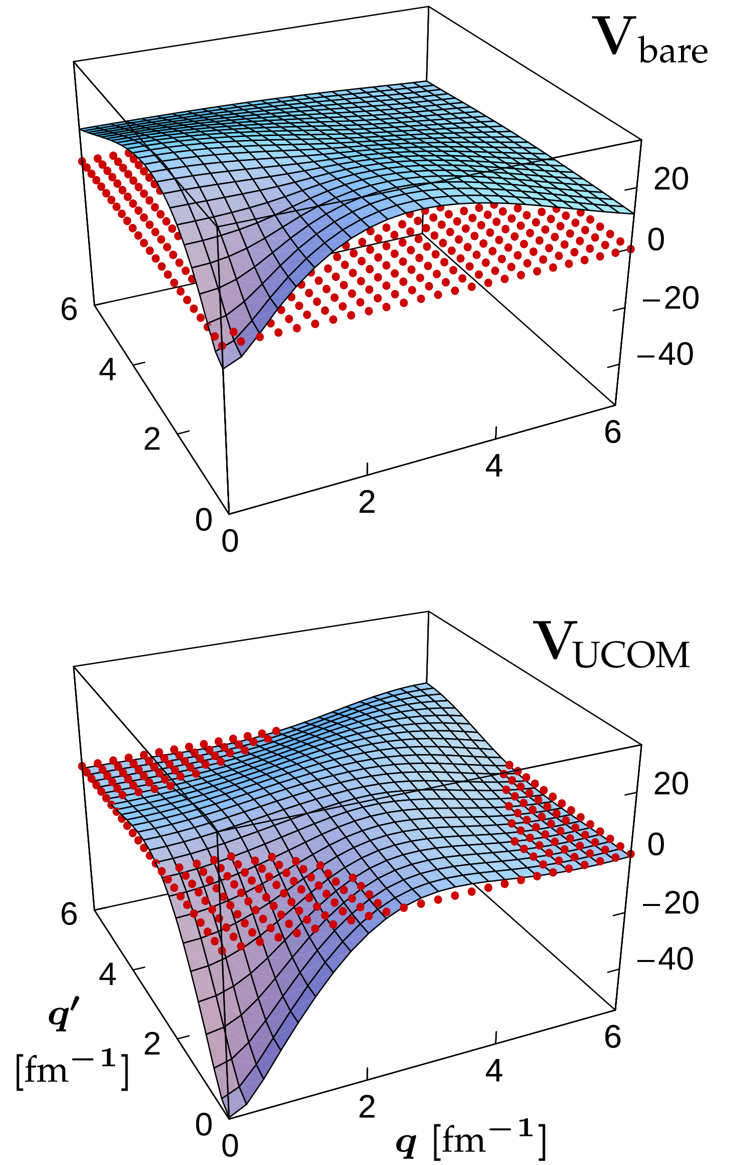
- **closed operator representation** of $\mathrm{V}_{\mathrm{UCOM}}$ in two-body approximation

  ⇒ usable with **arbitrary many-body basis**

- $\mathrm{V}_{\mathrm{UCOM}}$ is **phase-shift equivalent** to the underlying bare nucleon-nucleon interaction

- $\mathrm{V}_{\mathrm{UCOM}}$ is pre-diagonalized in momentum space, i. e. **high-momentum components are decoupled** (similar to $\mathrm{V}_{\text{low-k}}$)

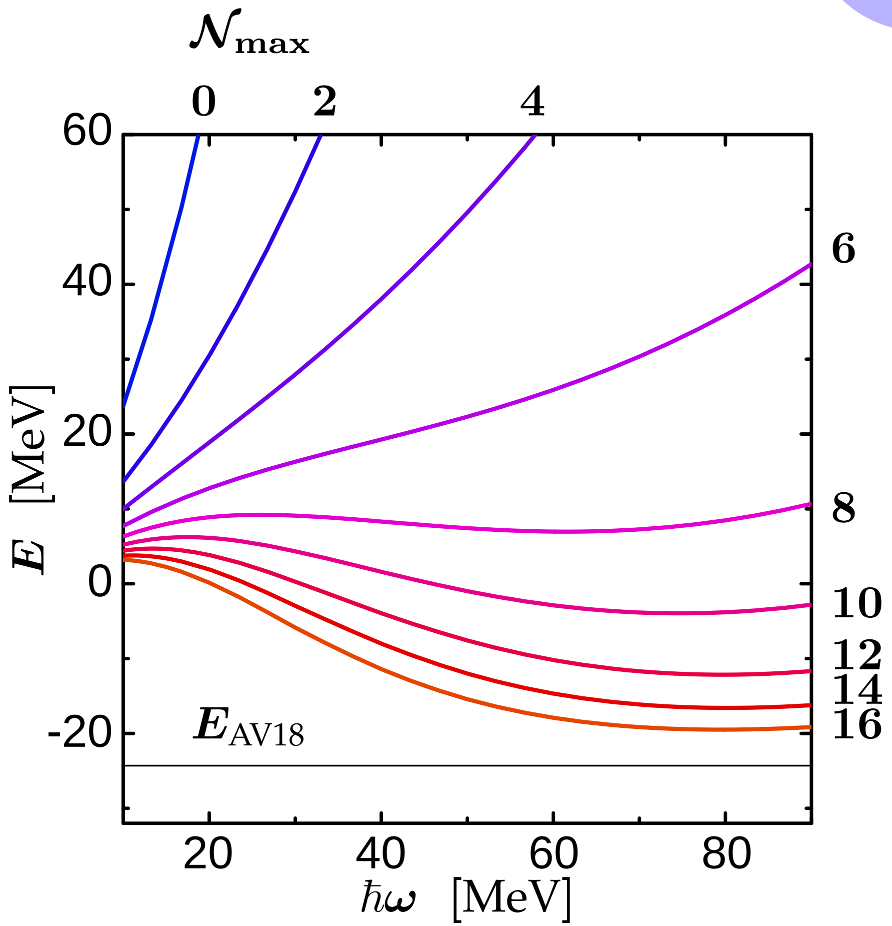
AV18 $^3S_1$

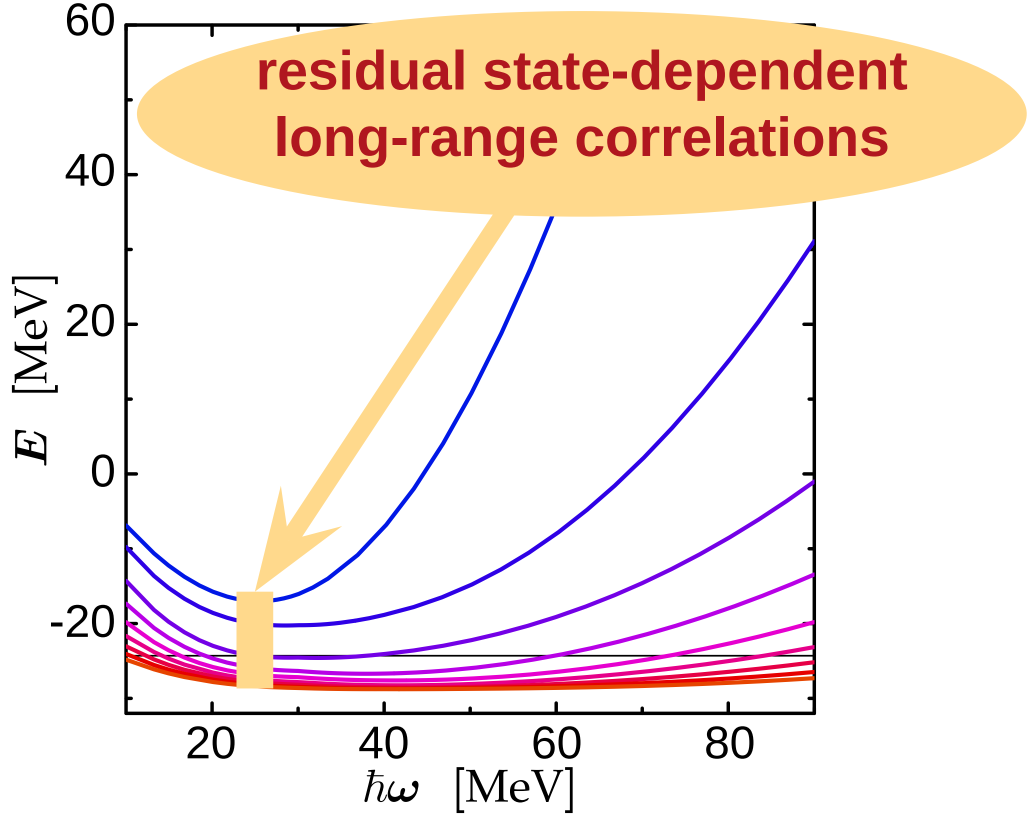


No-Core Shell Model (NCSM) Calculations

^{4}He: Convergence

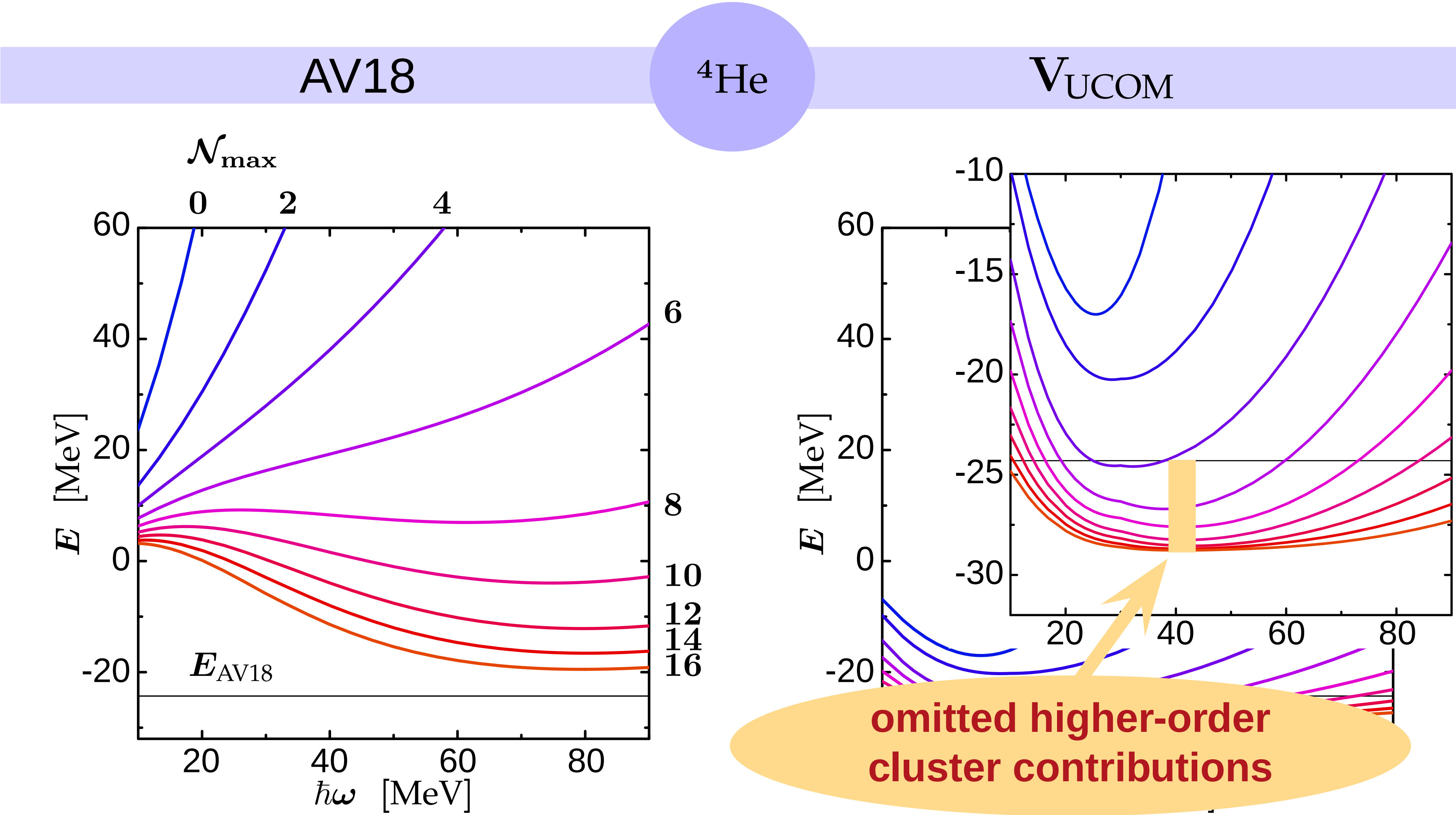
AV18 | ^{4}He | V_{UCOM}





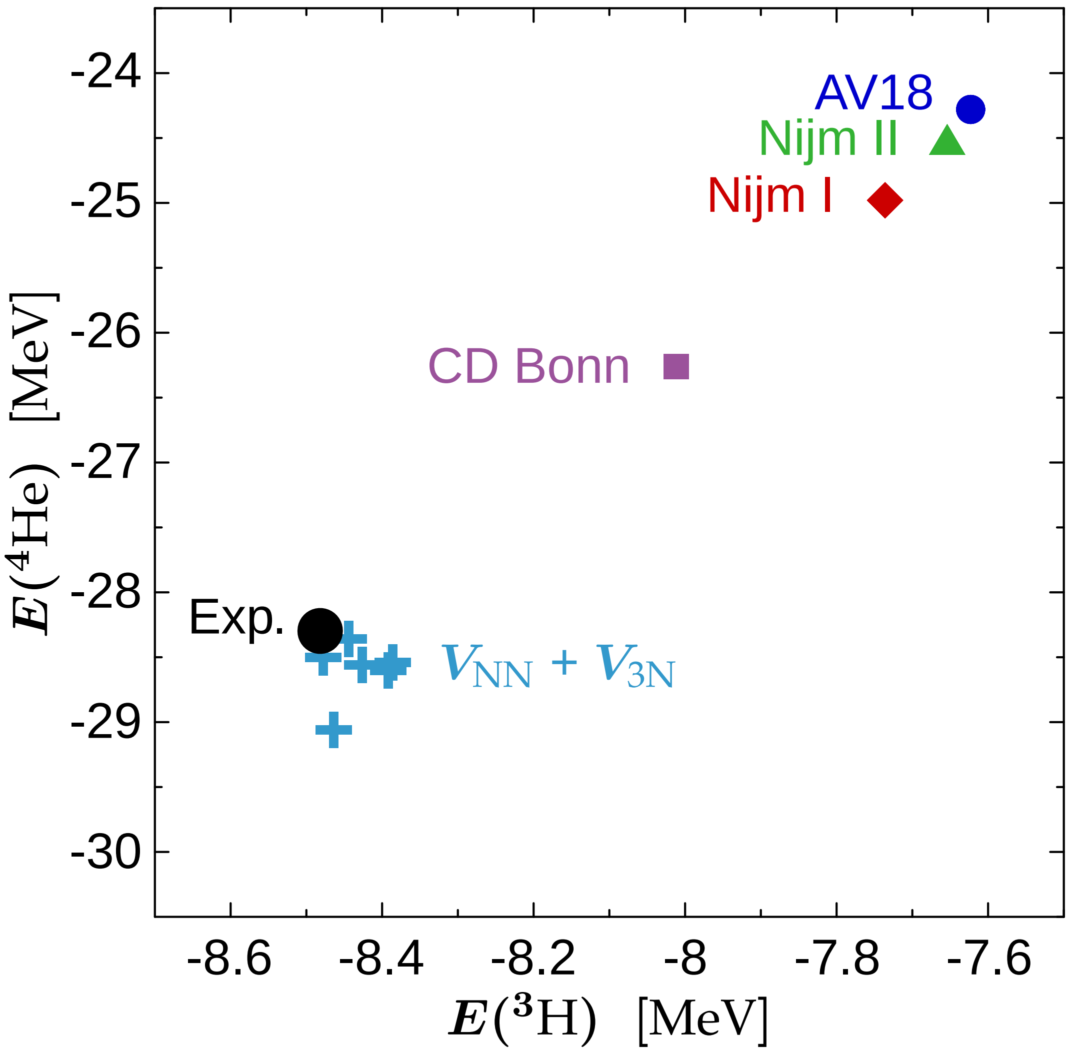
NCSM code by P. Navrátil [PRC 61, 044001 (2000)]

^{4}He: Convergence



NCSM code by P. Navrátil [PRC 61, 044001 (2000)]

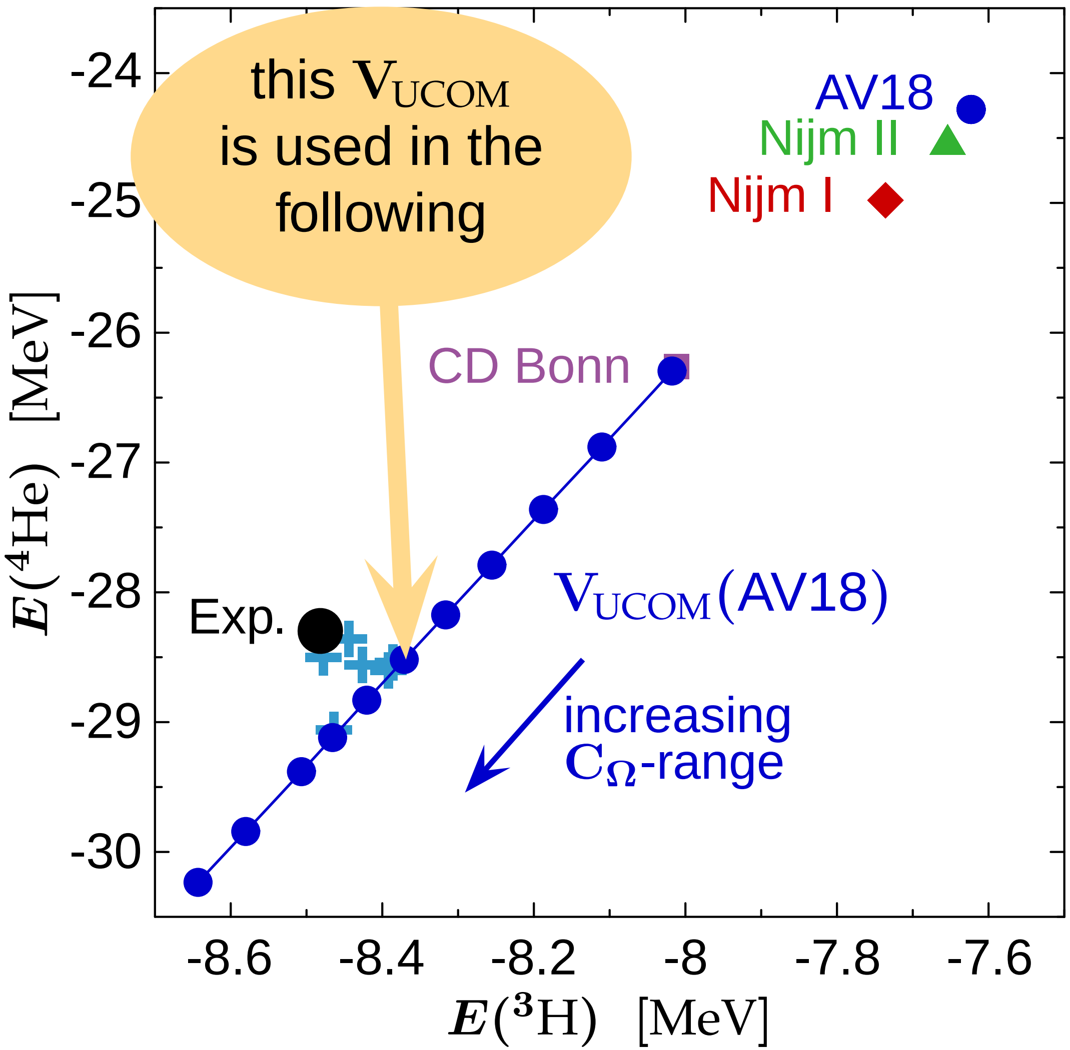
Tjon-Line and Correlator Range



- **Tjon-line**: $E(^4\text{He})$ vs. $E(^3\text{H})$ for phase-shift equivalent NN-interactions

Data points: A. Nogga et al., Phys. Rev. Lett. **85**, 944 (2000)

Tjon-Line and Correlator Range



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- change of C_Ω-correlator range results in shift along Tjon-line

minimize net three-body force by choosing correlator with energies close to experimental value

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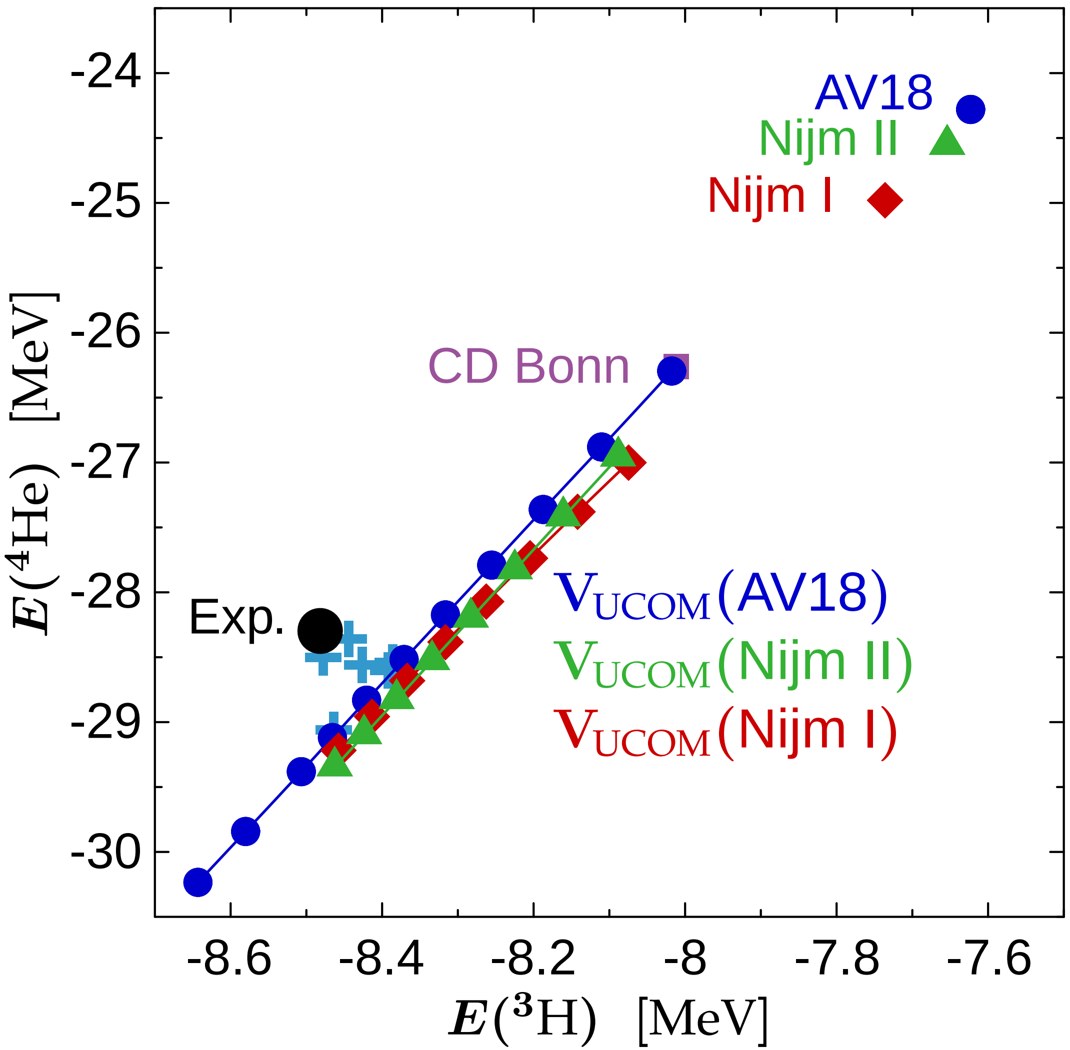

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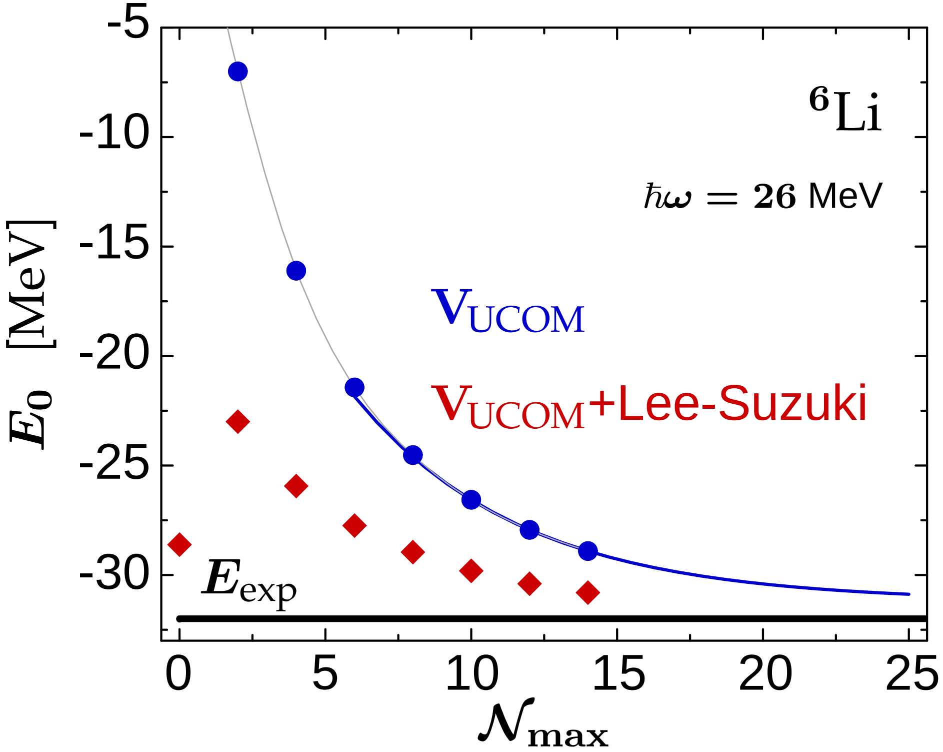
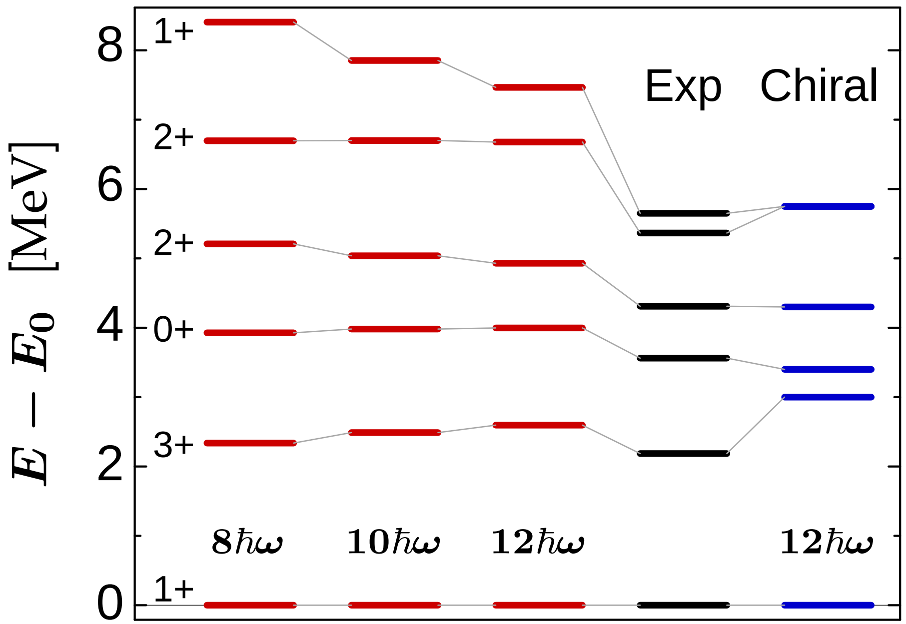
^{6}Li: NCSM + UCOM for p-Shell Nuclei



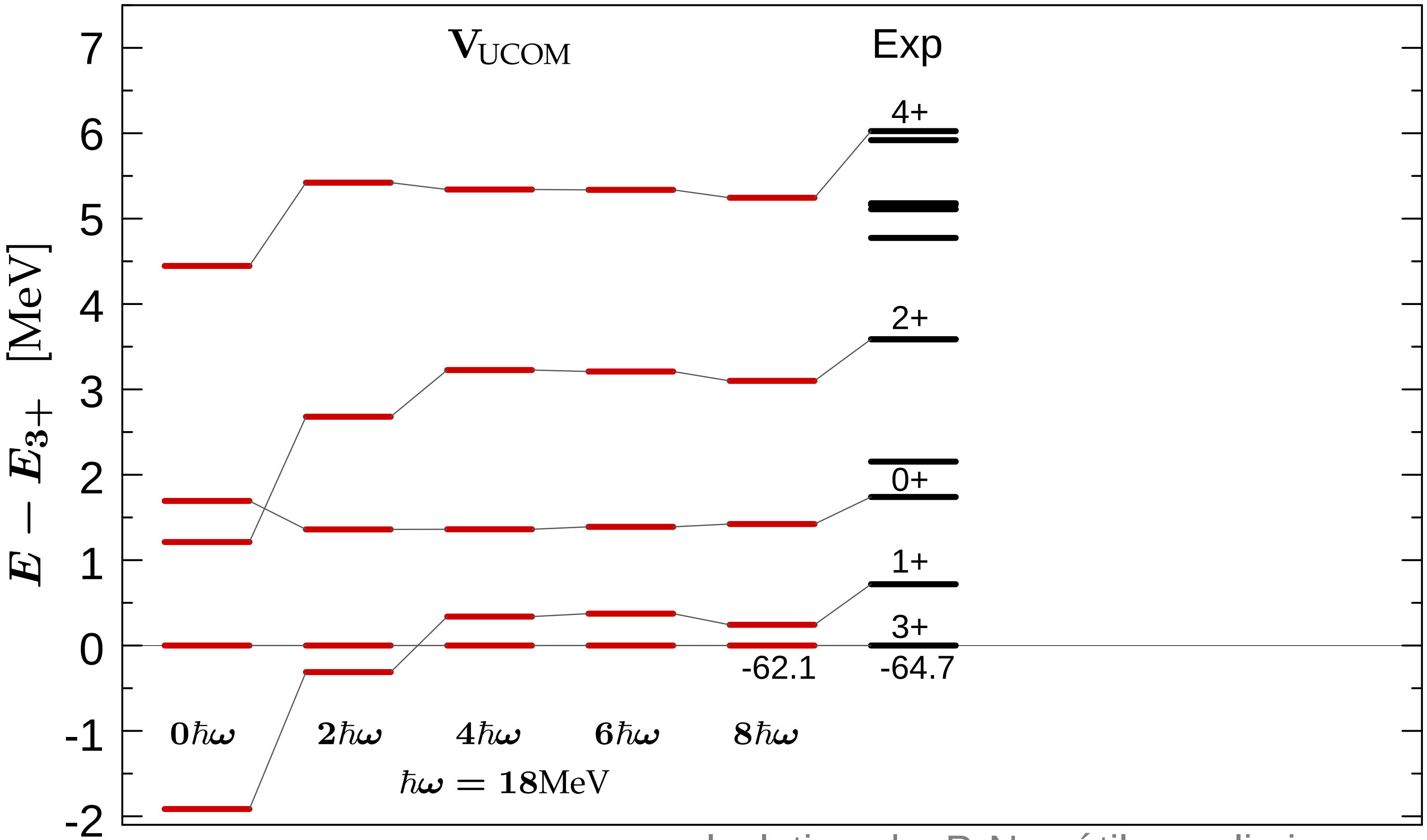


E_0 [MeV] (extrapolation)	-31.23
E_0 [MeV] (experiment)	-32.00

NCSM calculations by P. Navrátil (LLNL)

Without $3N$-Interaction!

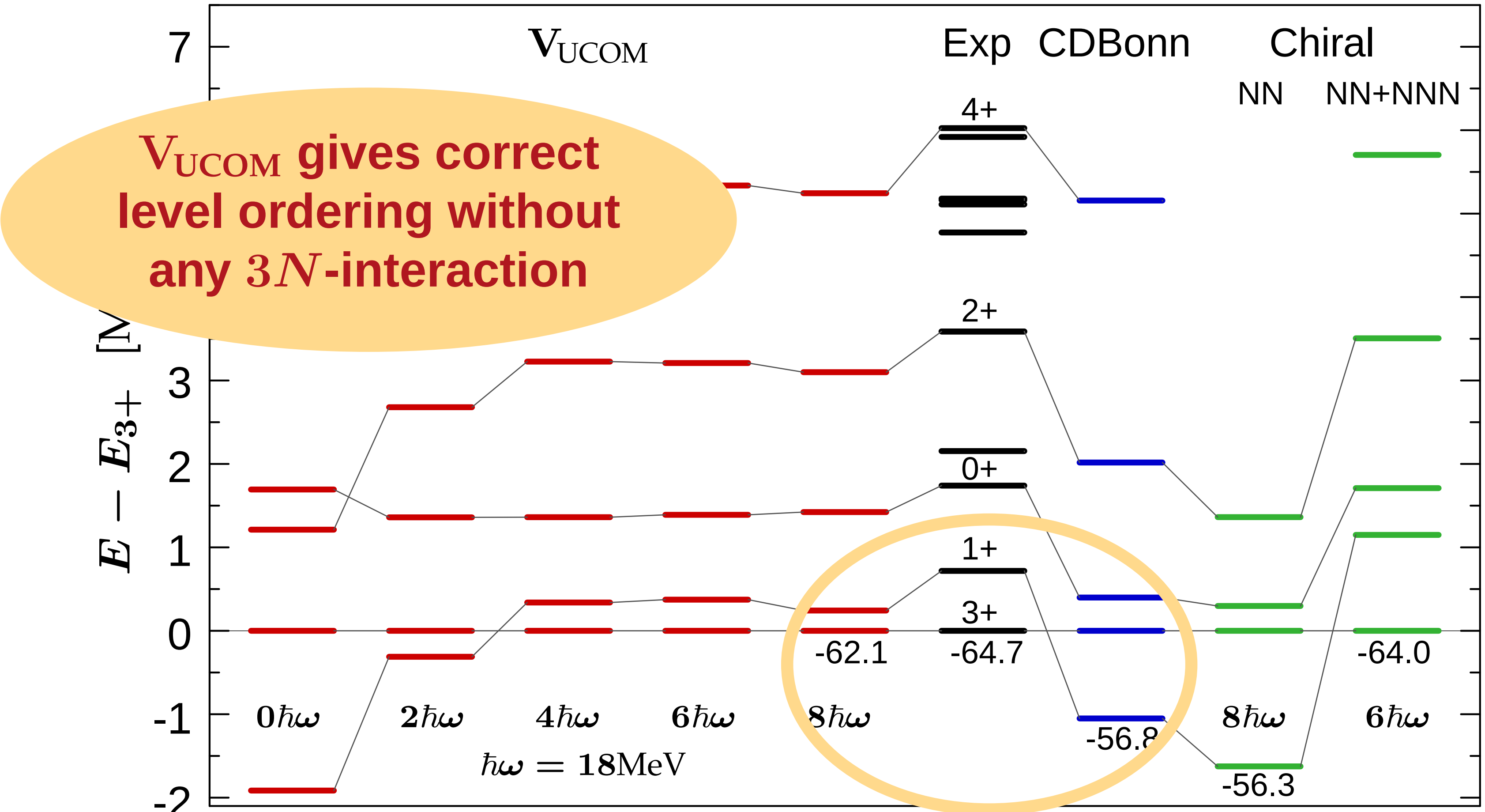
^{10}B: Benchmarking V_{UCOM}



calculations by P. Navrátil – preliminary

- stable spectrum from $\sim 4\hbar\omega$
- E_0 in good agreement with experiment

^{10}B: Benchmarking V_{UCOM}



calculations by P. Navrátil – preliminary

- stable spectrum from $\sim 4\hbar\omega$
- E_0 in good agreement with experiment

Summary & Outlook

Summary

- UCOM treats **short-range central** and **tensor correlations explicitly**

  ⇒ **improved convergence** through pre-diagonalization

  ⇒ **minimize net three-body forces**

- **few-body calculations** (NCSM) can be used to **constrain and optimize** $\mathbf{V}_{\mathrm{UCOM}}$

- $\mathbf{V}_{\mathrm{UCOM}}$ is suited for a wide range of many-body methods: NCSM, FMD, HF, MBPT, RPA, ...

  ⇒ **“Universal” phase-shift equivalent** $\boldsymbol{NN}$-interaction!

Outlook

- systematic NCSM study of p-shell Nuclei
- construction & inclusion of simple **three-nucleon forces** (encouraging early results)
- **reactions & resonances** in Fermionic Molecular Dynamics (UCOM-FMD)
- further & extended applications: **pairing** (HFB), etc.

(In)Famous Last Words...

My Collaborators

- R. Roth, N. Paar, P. Papakonstantinou, A. Zapp

  Institut für Kernphysik, TU Darmstadt

- T. Neff

  NSCL, Michigan State University

- H. Feldmeier

  Gesellschaft für Schwerionenforschung (GSI)

Recent References

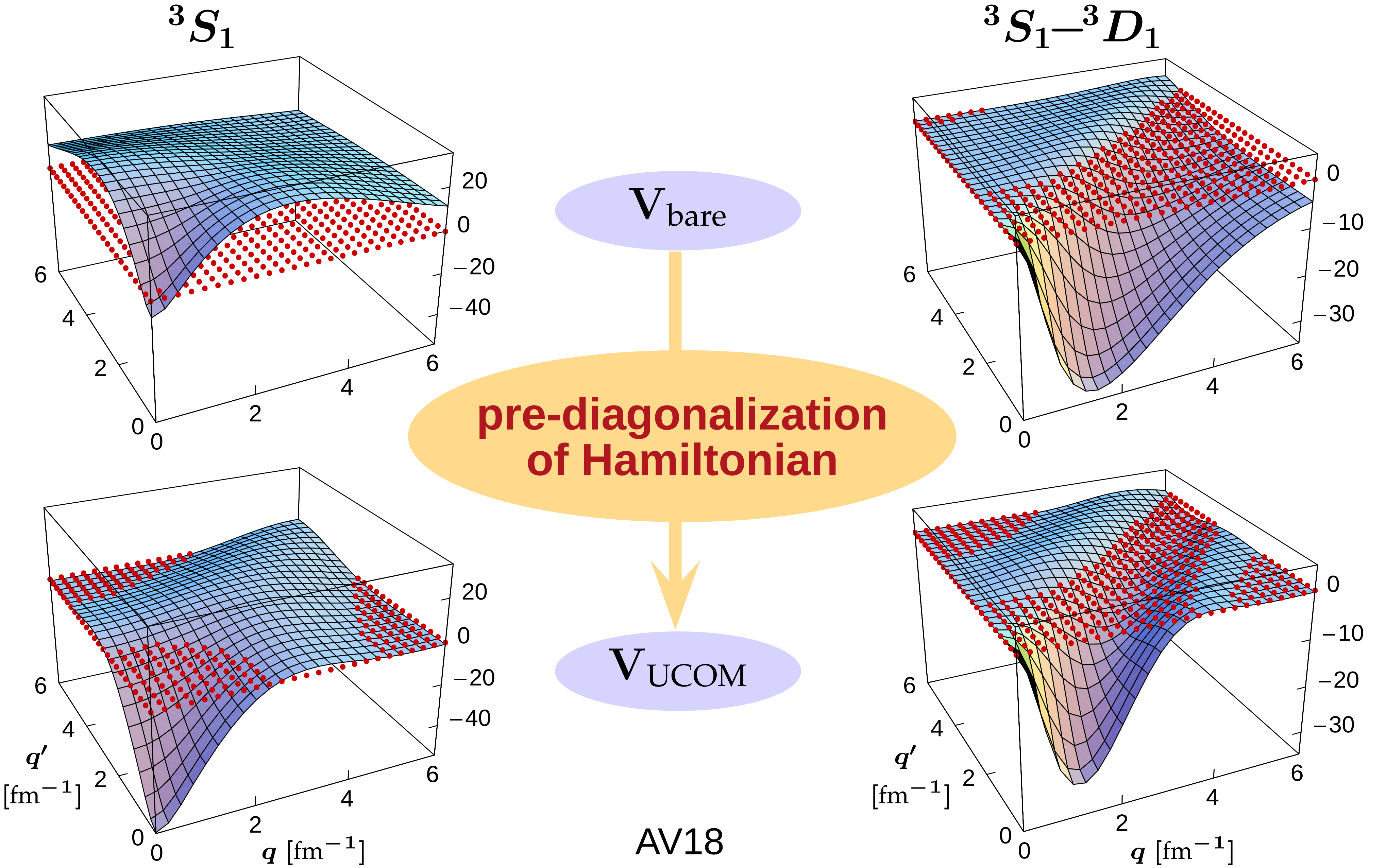
- R. Roth, H. Hergert, P. Papakonstantinou, T. Neff, and H. Feldmeier, Phys. Rev. C**72**, 034002 (2005)

- R. Roth, P. Papakonstantinou, N.Paar, H. Hergert, T. Neff, and H. Feldmeier, nucl-th/0510036, accepted for publication in Phys. Rev. C

- http://crunch.ikp.physik.tu-darmstadt.de/tnp/

Optional / Appendices

Momentum-Space Matrix Elements



UCOM-FMD Approach

Gaussian Single-Particle States

$$\left|q\right\rangle = \sum_{\nu=1}^{n} c_\nu \left|a_\nu, \vec{b}_\nu\right\rangle \otimes \left|\chi_\nu\right\rangle \otimes \left|m_t\right\rangle$$

$$\left\langle \vec{x} \middle| a_\nu, \vec{b}_\nu \right\rangle = \exp\left[- \frac{(\vec{x} - \vec{b}_\nu)^2}{2\, a_\nu} \right]$$

a_ν : complex width

χ_ν : spin orientation

$\vec{b}_\nu$: mean position & momentum

Slater Determinant

$$\left|Q\right\rangle = \mathcal{A} \left(\left|q_1\right\rangle \otimes \left|q_2\right\rangle \otimes \cdots \otimes \left|q_A\right\rangle \right)$$

Correlated Hamiltonian

$$\widetilde{\mathrm{H}} = \mathrm{T} + \mathrm{V}_{\mathrm{UCOM}} + \delta \mathrm{V}_{c+p+ls}$$

Variation

$$\frac{\left\langle Q \right| \widetilde{\mathrm{H}} - \mathrm{T}_{\mathrm{cm}} \left| Q \right\rangle}{\left\langle Q \middle| Q \right\rangle} \rightarrow \min$$

Projection

restoration of parity and rotational symmetry

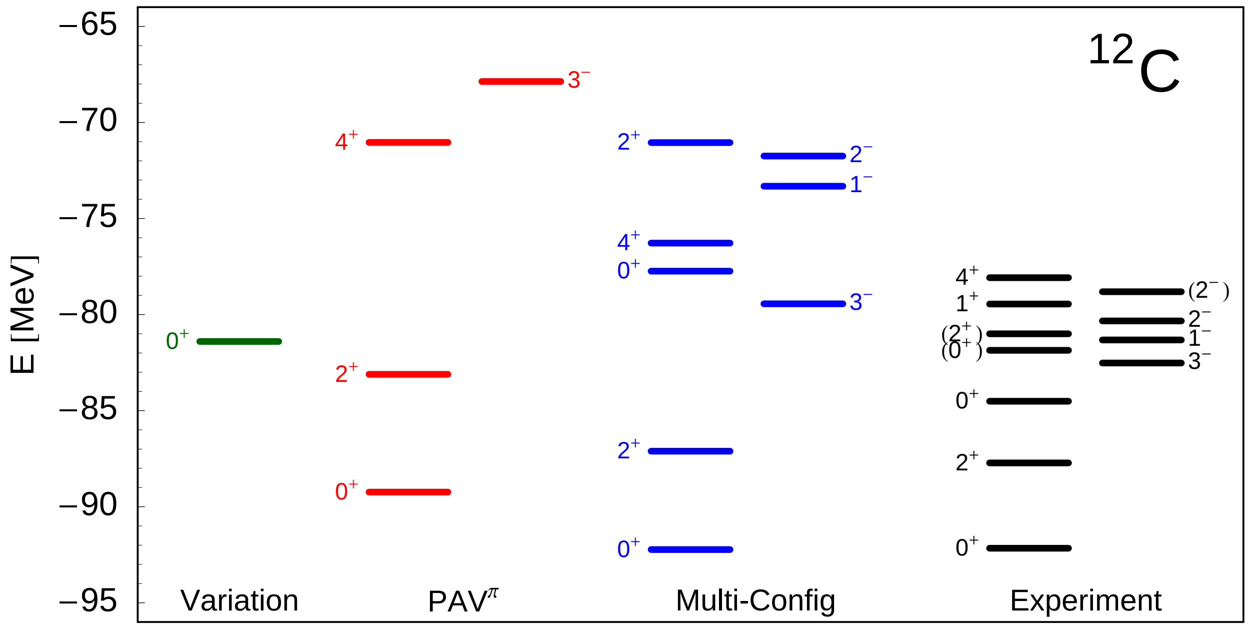
PAV / VAP

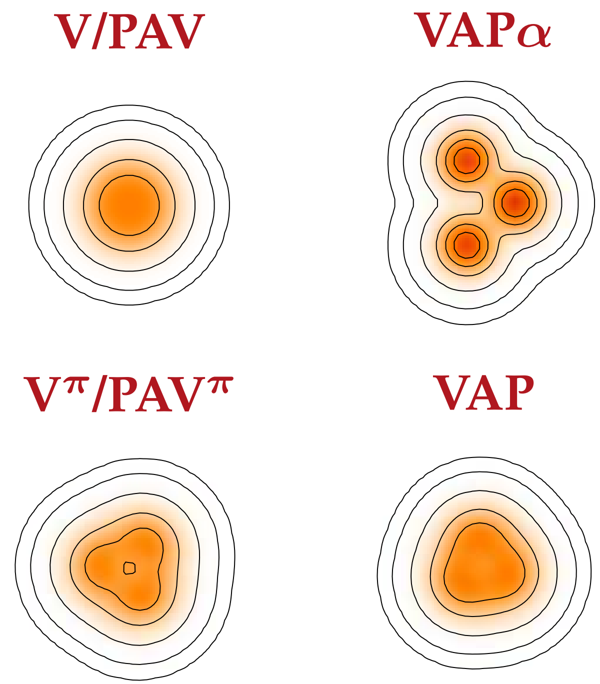
Multi-Configuration

mixing of several intrinsic configurations

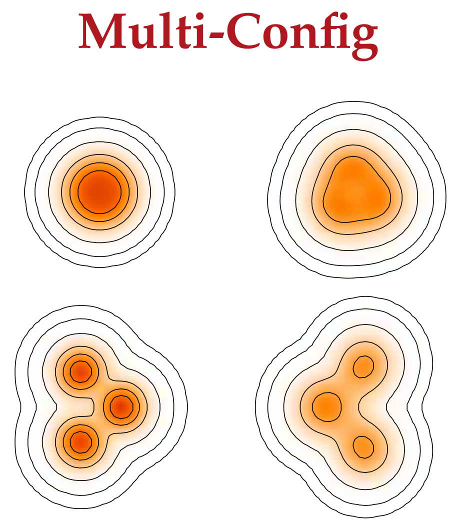
GCM

Structure of ^{12}C

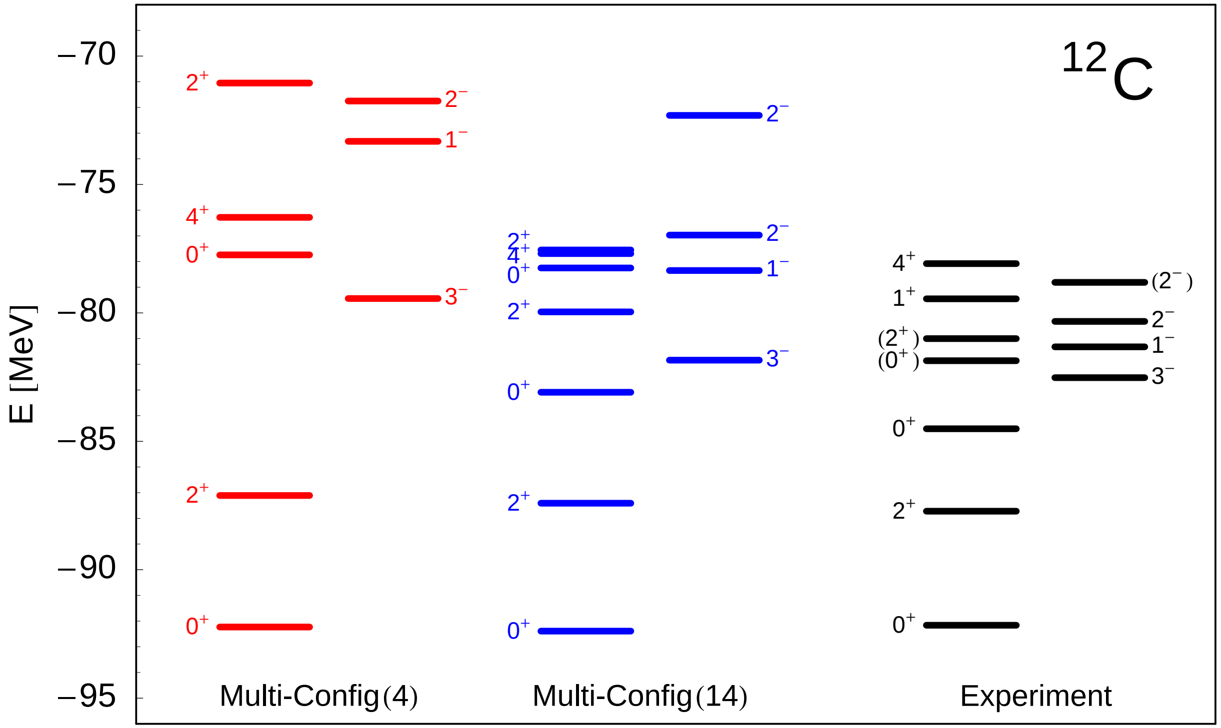


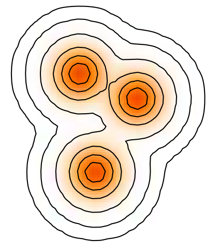


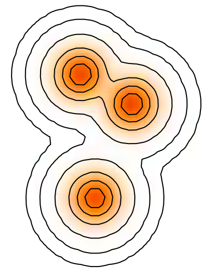
	E [MeV]	R_{ch} [fm]	$B(E2)$ [e^2 fm^4]
V/PAV	81.4	2.36	-
VAP α-cluster	79.1	2.70	76.9
PAV$^\pi$	88.5	2.51	36.3
VAP	89.2	2.42	26.8
Multi-Config	92.2	2.52	42.8
Experiment	92.2	2.47	39.7 ± 3.3

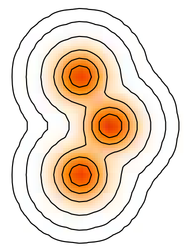


Structure of ^{12}C — Hoyle State



$\left|\langle$  $|0_2^+\rangle\right| = 0.76$

$\left|\langle$  $|0_2^+\rangle\right| = 0.71$

$\left|\langle$  $|0_2^+\rangle\right| = 0.50$

	Multi-Config	Experiment
E [MeV]	92.4	92.2
R_{ch} [fm]	2.52	2.47
$B(E2, 0_1^+ \to 2_1^+)$ [e^2 fm^4]	42.9	39.7 ± 3.3
$M(E0, 0_1^+ \to 0_2^+)$ [fm^2]	5.67	5.5 ± 0.2

More References

- H. Feldmeier, T. Neff, R. Roth, and J. Schnack, Nucl. Phys. **A632**, 61 (1998)
- T. Neff, and H. Feldmeier, Nucl. Phys. **A713**, 311 (2003)
- R. Roth, T. Neff, H. Hergert, and H. Feldmeier, Nucl. Phys. **A745**, 3 (2004)
- R. Roth, H. Hergert, P. Papakonstantinou, T. Neff, and H. Feldmeier, Phys. Rev. C**72**, 034002 (2005)
- R. Roth, P. Papakonstantinou, N.Paar, H. Hergert, T. Neff, and H. Feldmeier, nucl-th/0510036, accepted for publication in Phys. Rev. C
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