Ultracold Bose Gases In Optical Superlattices

Adaptive Basis Truncation Scheme

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Summary & Motivation

- ultracold, dilute atomic gases in optical lattices provide a unique experimental tool to investigate strongly correlated quantum systems [1]
- these systems are well described by the single-band Bose-Hubbard Hamiltonian [2]
- for moderate system sizes the groundstate is obtained by exact diagonalisation of the corresponding Hamilton matrix using Lanczos algorithms
- almost every static observable throughout the phase diagram is accessible, e.g. condensate / superfluid fraction, and the interference pattern [3-5,8]
- for the exact diagonalisation we are limited in system size, we developed a physical motivated truncation scheme
- based upon the truncation scheme we are able to perform static calculations for larger systems as well as explicit time evolutions of pertubed systems [7]

Bose-Hubbard Model

- 1D optical lattice with I lattice sites and N bosonic particles
- restriction to the first energy-band, T=0, nearest neighbour hopping, and on-site two-particle interactions
- additional sinusodial two-colour superlattice potential

$$\hat{H} = -J \sum_{i=1}^{I} \left(\hat{a}^\dagger_{i+1} \hat{a}_i + h.a. \right) \quad \text{tunneling term}$$

$$+ \frac{V}{2} \sum_{i=1}^{I} \hat{n}_i \left(\hat{n}_i - 1 \right) \quad \text{interaction term}$$

$$+ \sum_{i=1}^{I} \epsilon_i \, \hat{n}_i \quad \text{superlattice potential}$$

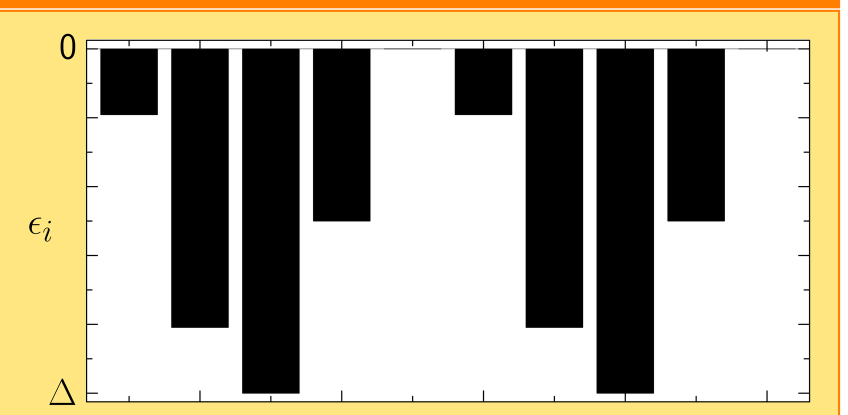
$\hat{a}^\dagger_i,\ \hat{a}_i,\ \hat{n}_i$	creation, annihilation, occupation-number operators
J	tunnelling matrix element
V	two particle interaction energy
ϵ_i	height of the superlattice potential

- states are represented in an occupation number basis with dimension D

$$|\,\psi^{(0)}\,\rangle = \sum_{\alpha=1}^{D} C^{(0)}_\alpha \,|\, \{n_1, ..., n_I\}_\alpha \,\rangle$$

Two-Colour Superlattice

- superposition of two standing wave lattices with different wavelengths [6]
- Δ/J is the energy of the deepest superlattice well



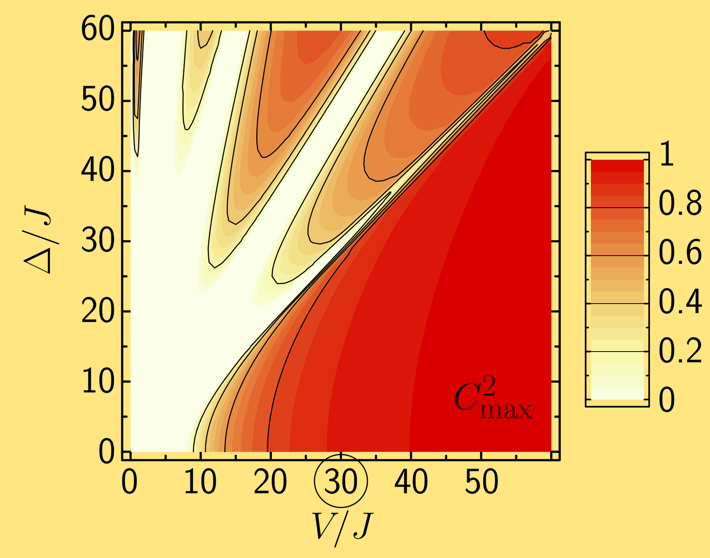
Truncation Scheme

- virtues of exact diagonalisation techniques are that one is able to compute almost every groundstate observable throughout the whole phase diagram from the Mott insulating to the superfluid phase
- drawback is that the Hilbert space grows exponentially with number of particles and lattice sites (static calculations are feasible up to $D \approx 10^7$, e.g. 12 bosons on 12 lattice sites have $D = 1352087$ number states)
- when focusing on the strongly correlated regime there are many energetically unfavourable number states that virtually do not contribute to the groundstate
- the idea is to include only states $|\,\{n_1,...,n_I\}_\alpha\,\rangle$ whose diagonal part of the Hamiltonian is below a specific truncation energy E_T

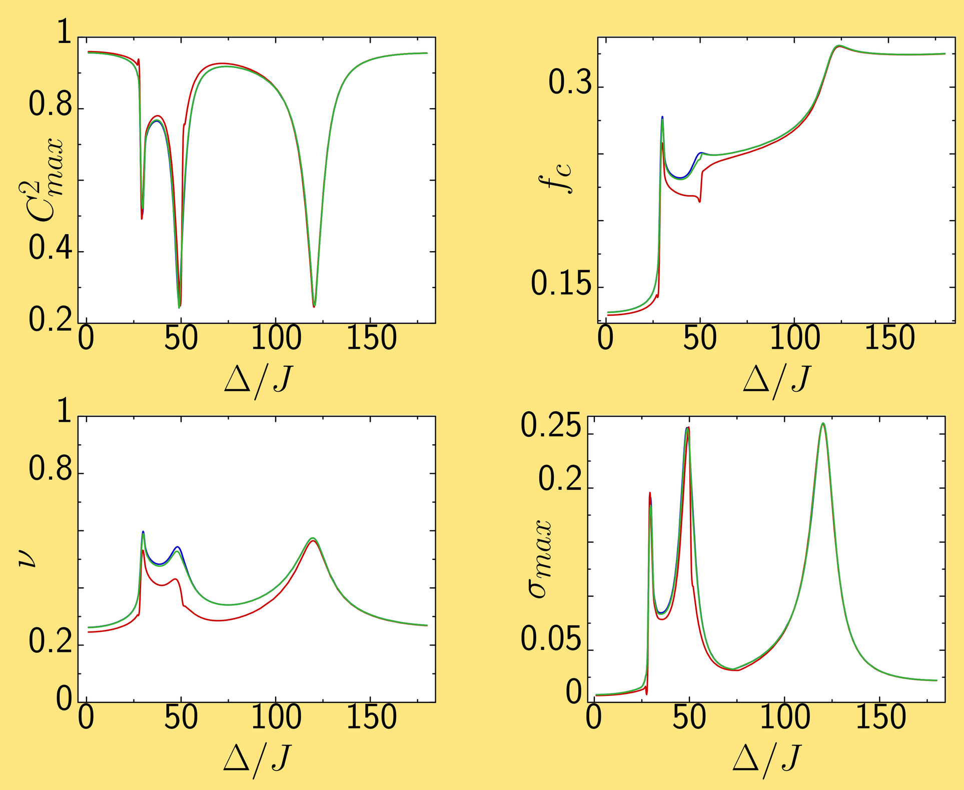
$$\langle\, {}_\alpha\{n_1,...,n_I\} \,|\, \frac{V}{2}\sum_{i=1}^{I} \hat{n}_i(\hat{n}_i - 1) + \sum_{i=1}^{I} \epsilon_i \hat{n}_i \;|\, \{n_1,...,n_I\}_\alpha \,\rangle \le E_T$$

- depending on the lattice topology and the position in the phase-diagram one can reduce the basis dimension by some orders of magnitude without significant consequences

Test of Truncation Scheme ($I = 10,\ N = 10$)



- phase diagram in the V-Δ-plane shows the localised phase ($V/J \le 1$), the Mott-insulator phase ($V < \Delta$) and the quasi Bose-glass phase ($V \ge \Delta$) [3]
- we fix the interaction strength at $V/J = 30$ and increase the value of Δ/J

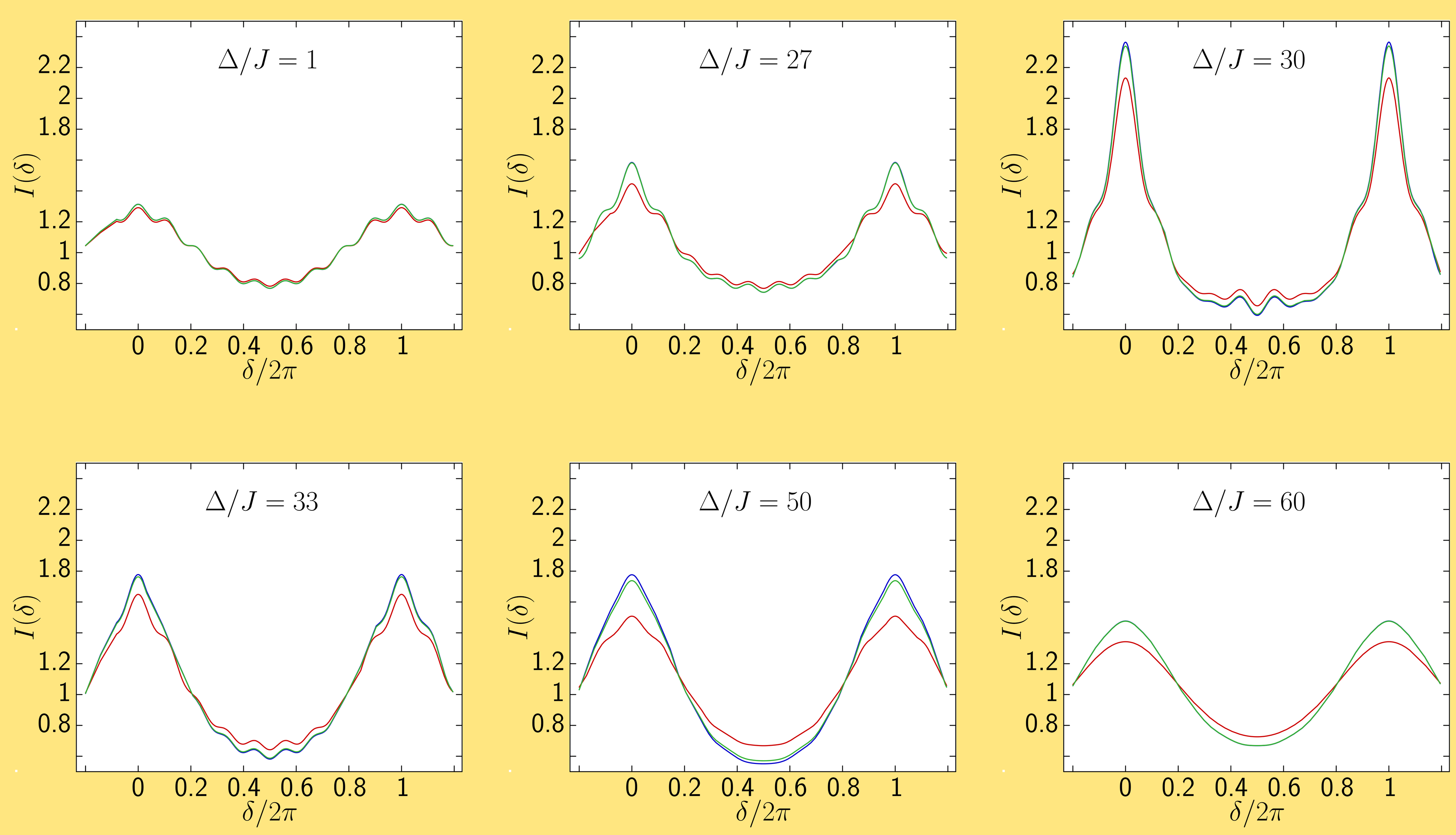


C^2_{max}	:	$max(C^{(0)}_\alpha)$
f_c	:	condensate fraction of particles
ν	:	visibility of the interference fringes
σ_{max}	:	max fluctuation of particles

- less than 1% of the states are able to reproduce all observables qualitatively
- about 5% of the states fit the complete cacualation almost perfectly

range	complete	Δ/J 1-28	Δ/J 29-50	Δ/J 51-180	Δ/J 1-29	Δ/J 30-50	Δ/J 51-180
dimension	92378	91	288	199	3743	1866	2737

Interference Patterns



- matter-wave interference patterns of the atom cloud after release from the lattice and a certain time of flight are a genuine experimental observable
- again the truncated bases seem to include all physically relevant information
- if δ is a multiple of $1/I = 0.1$ the intensities are the quasi-momentum occupation numbers
- for small superlattice amplitudes the systems is incoherent and therefore the interference is almost completely suppressed, this an inherent feature of the Mott-insulating state
- entering the quasi Bose-glass phase ($\Delta \approx V$) the quasi-momentum zero peaks and a characteristic interference pattern appears [5]

[1] Immanuel Bloch, Physics World (2004)
[2] D. Jacksch et al., Phys. Rev. Lett. 81, 31083111 (1998)
[3] R. Roth and K. Burnett, Phys. Rev. A 67 031692(R) (2003)
[4] R. Roth and K. Burnett, Phys. Rev. A 68 023604 (2003)
[5] R. Roth and K. Burnett, J. Opt. B 5 S50 (2003)
[6] J.E. Lye et al. Phys. Rev. Lett. 95 070401 (2005)
[7] Markus Hild, Felix Schmitt, Robert Roth Q 5.3
[8] Felix Schmitt, Diploma Thesis (2005)