Nuclear Structure with Correlated Realistic Interactions

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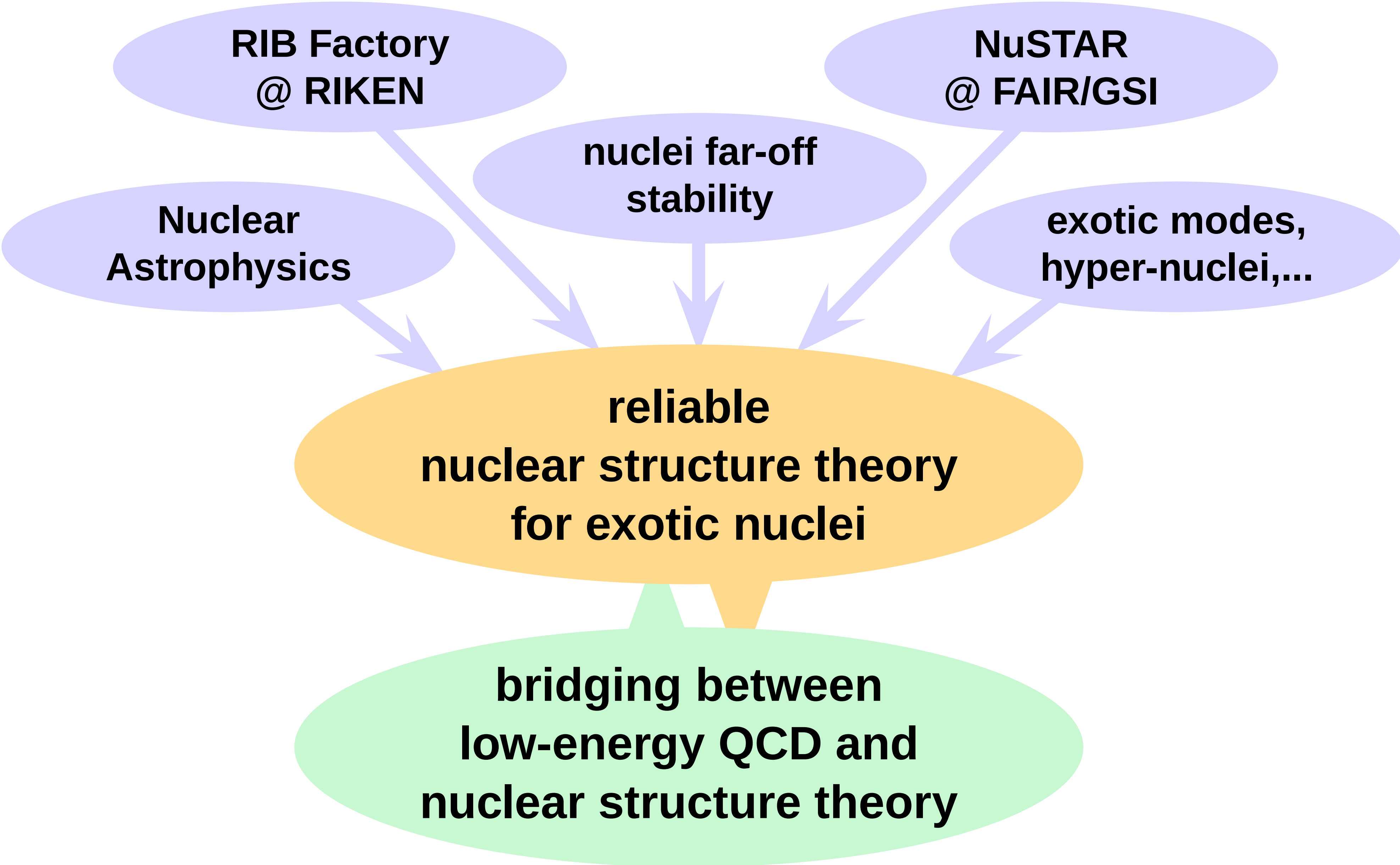
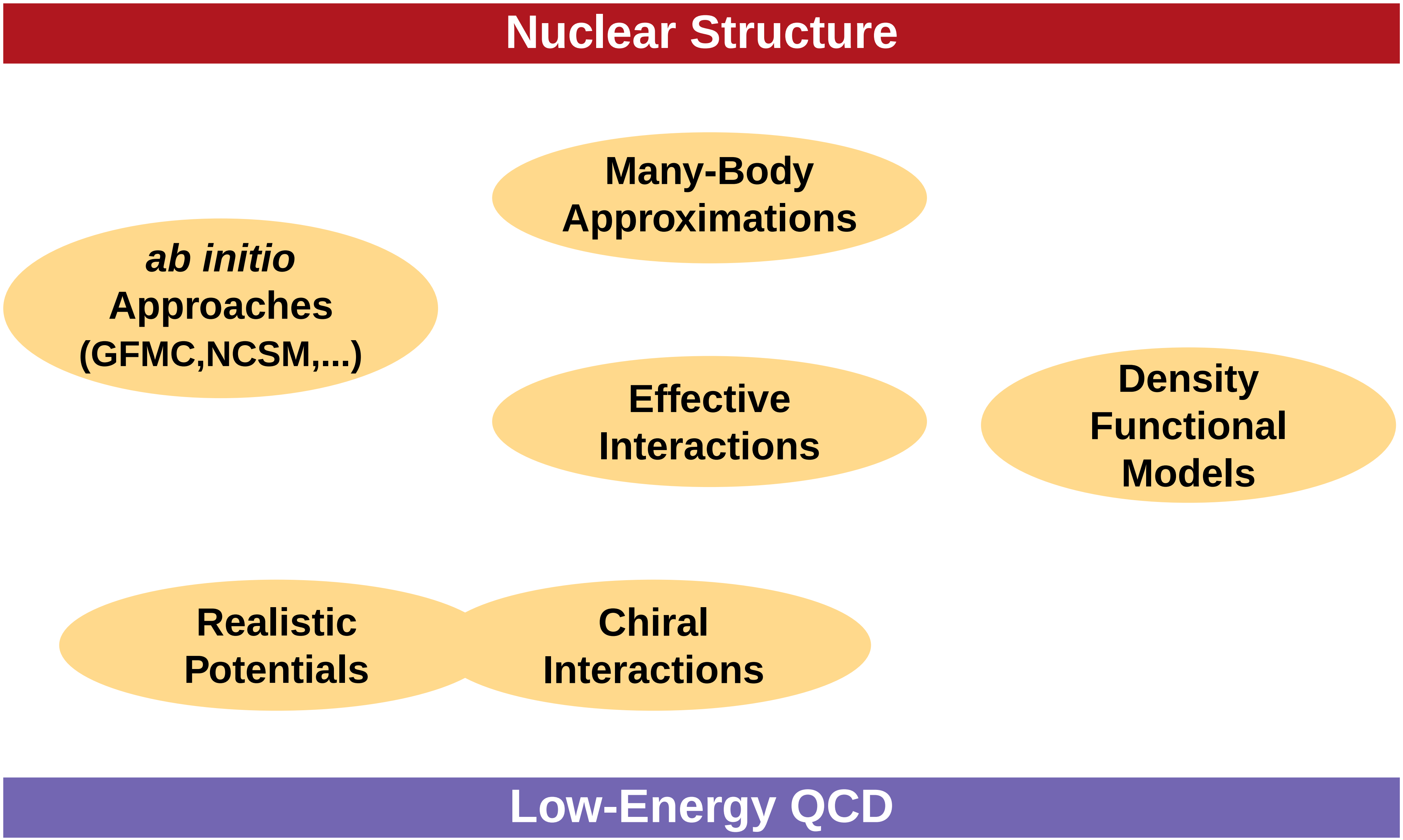
Overview

- Motivation
- Correlated Realistic NN-Potentials
  - Central and Tensor Correlations
  - Unitary Correlation Operator Method
- Applications
  - No Core Shell Model
  - Hartree-Fock & Beyond

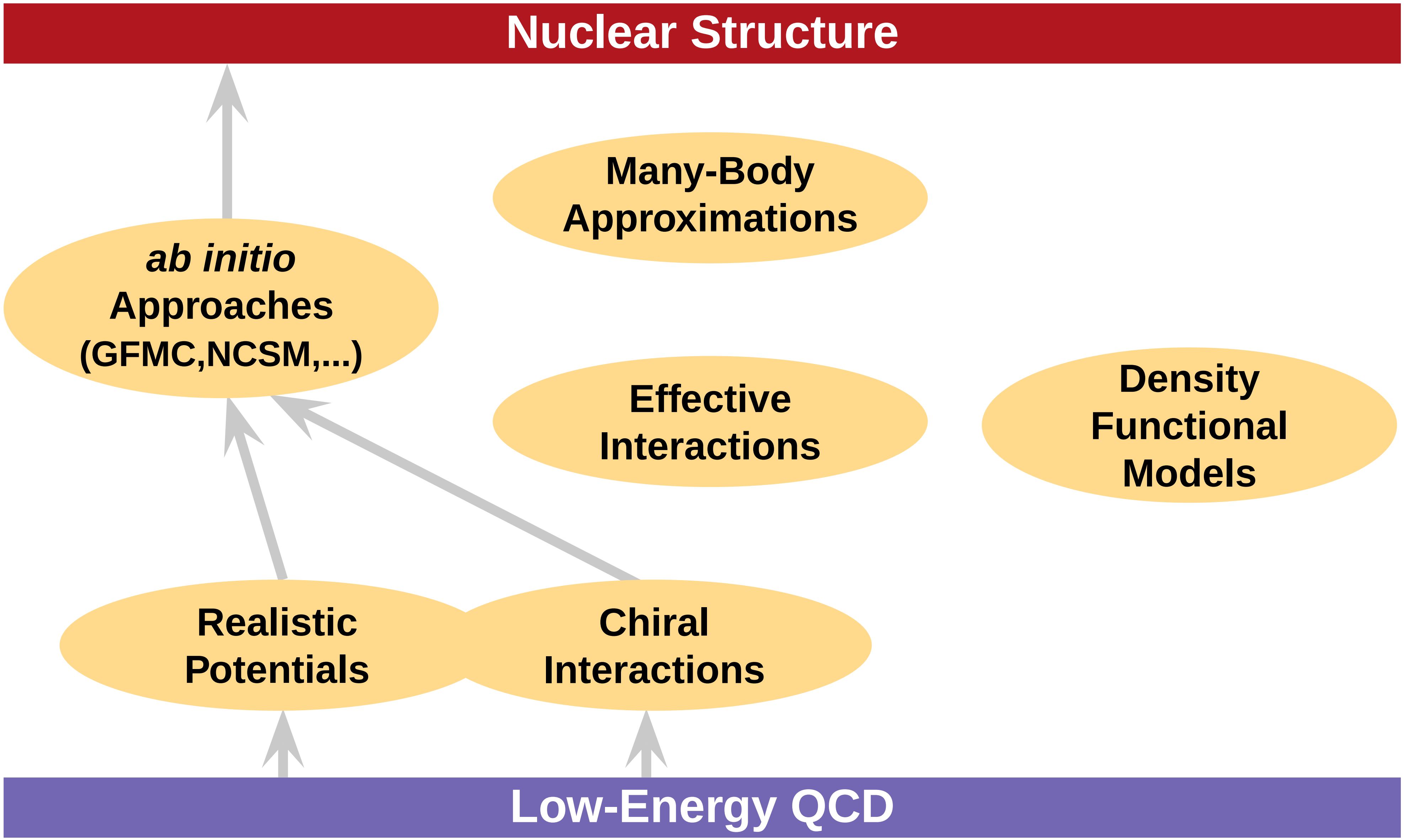
Nuclear Structure in the 21st Century



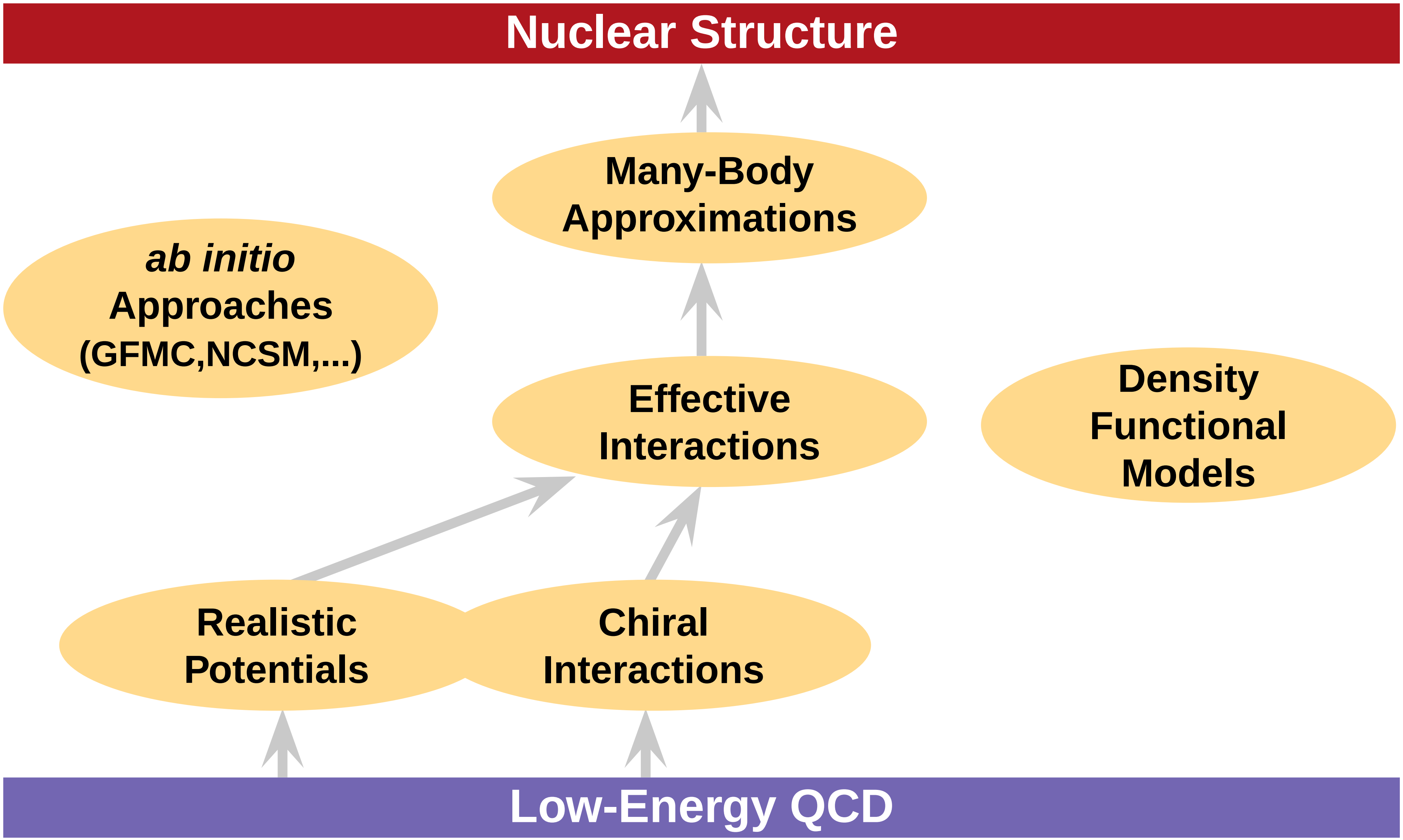
Modern Nuclear Structure Theory



Modern Nuclear Structure Theory



Modern Nuclear Structure Theory



Realistic NN-Potentials

- **QCD motivated**
  - symmetries, meson-exchange picture
  - chiral effective field theory
- **short-range phenomenology**
  - short-range parametrization or contact terms
- **experimental two-body data**
  - scattering phase-shifts & deuteron properties reproduced with high precision
- **supplementary three-nucleon force**
  - adjusted to spectra of light nuclei

Argonne V18

CD Bonn

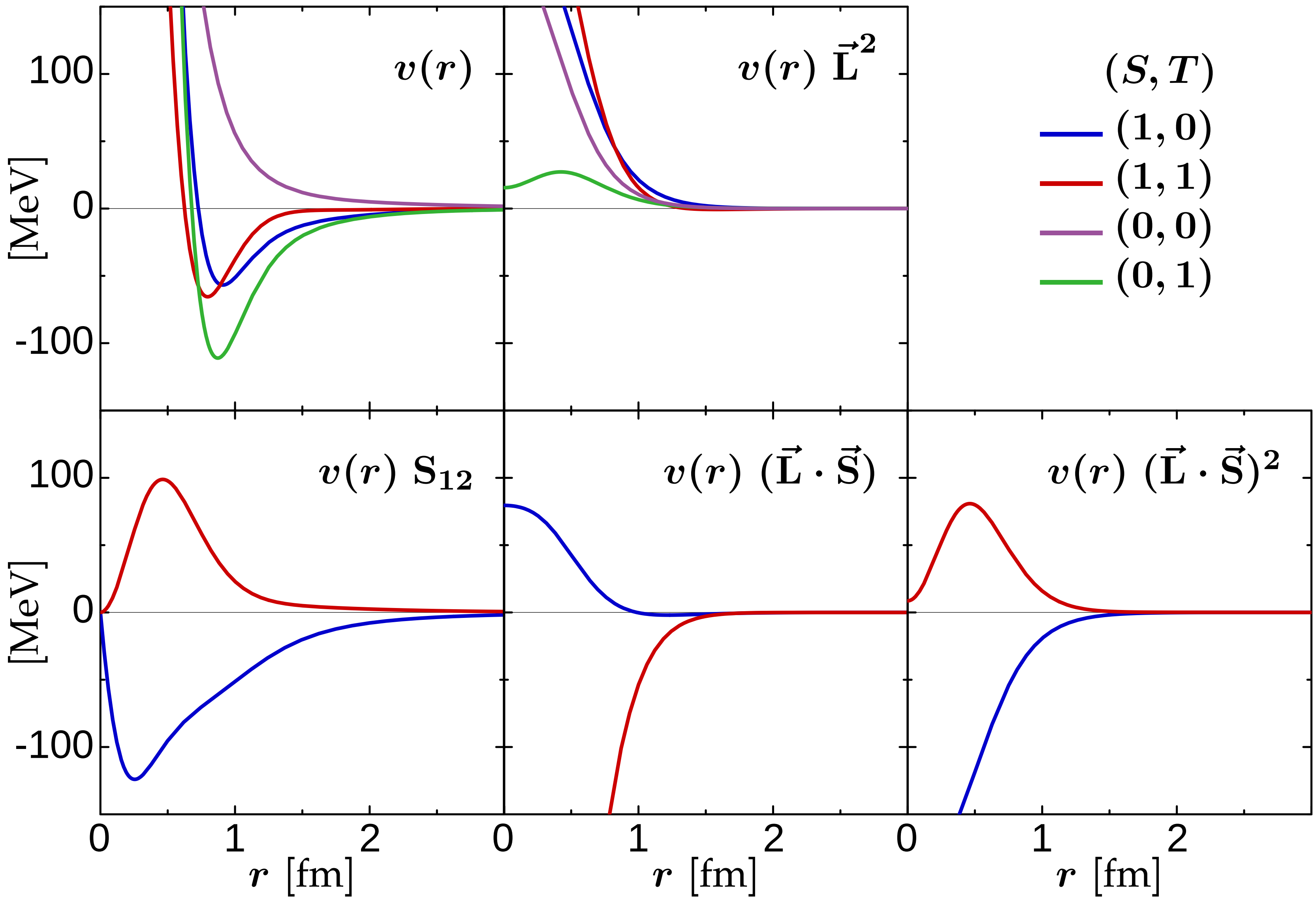
Nijmegen I/II

Chiral N3LO

Argonne V18 + **Illinois 2**

Chiral N3LO + **N2LO**

Argonne V18 Potential



Why Effective Interactions?

Realistic Potentials

- generate strong correlations in many-body states
- short-range central & tensor correlations most important

Many-Body Approximations

- rely on truncated many-nucleon Hilbert spaces for larger A
- not capable of describing short-range correlations
- extreme: Hartree-Fock based on single Slater determinant

Modern Effective Interactions

- adapt realistic potential to the available model spaces
- conserve experimentally constrained properties (phase shifts)

Unitary Correlation Operator Method (UCOM)

Unitary Correlation Operator Method

Correlation Operator

introduce short-range correlations by means of a unitary transformation with respect to the relative coordinates of all pairs

$$\mathrm{C} = \exp[-\mathrm{i}\,\mathrm{G}] = \exp\Big[-\,\mathrm{i}\sum_{i<j} \mathrm{g}_{ij}\Big]$$

$$\mathrm{G}^\dagger = \mathrm{G}$$
$$\mathrm{C}^\dagger \mathrm{C} = 1$$

Correlated States

$$|\widetilde{\psi}\rangle = \mathrm{C}\;|\psi\rangle$$

Correlated Operators

$$\widetilde{\mathrm{O}} = \mathrm{C}^\dagger\;\mathrm{O}\;\mathrm{C}$$

$$\langle\widetilde{\psi}|\,\mathrm{O}\,|\widetilde{\psi}'\rangle = \langle\psi|\,\mathrm{C}^\dagger\;\mathrm{O}\;\mathrm{C}\,|\psi'\rangle = \langle\psi|\,\widetilde{\mathrm{O}}\,|\psi'\rangle$$

Central and Tensor Correlators

$$\mathbf{C} = \mathbf{C}_{\Omega}\mathbf{C}_{r}$$

Central Correlator C_r

- radial distance-dependent shift in the relative coordinate of a nucleon pair

$$\mathrm{g}_r = \frac{1}{2}\big[s(\mathrm{r})\ \mathrm{q}_r + \mathrm{q}_r\ s(\mathrm{r})\big]$$

$$\mathrm{q}_r = \tfrac{1}{2}\big[\tfrac{\vec{\mathrm{r}}}{\mathrm{r}} \cdot \vec{\mathrm{q}} + \vec{\mathrm{q}} \cdot \tfrac{\vec{\mathrm{r}}}{\mathrm{r}}\big]$$

Tensor Correlator C_Ω

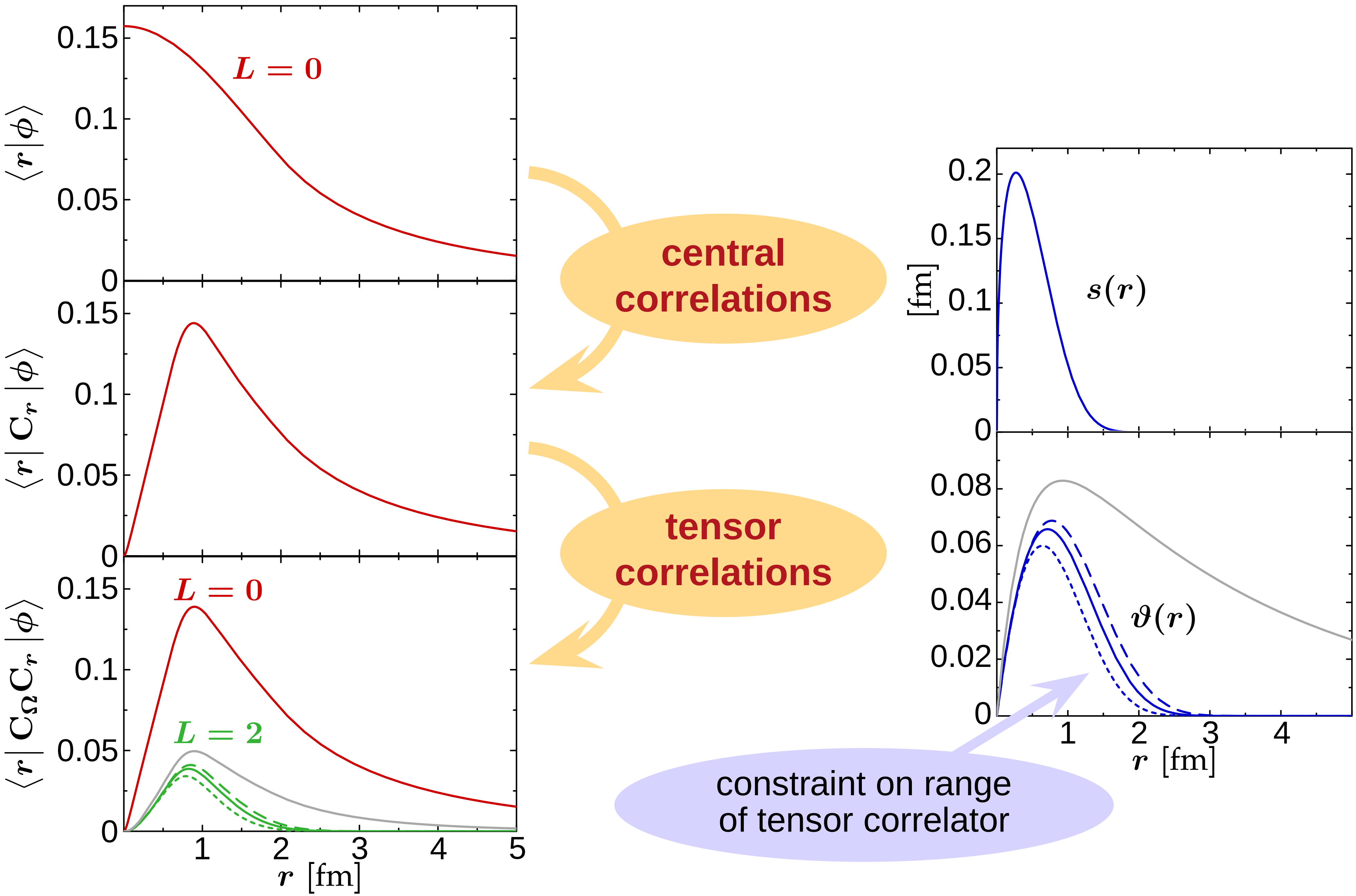
- angular shift depending on the orientation of spin and relative coordinate of a nucleon pair

$$\mathrm{g}_\Omega = \frac{3}{2}\vartheta(\mathrm{r})\big[(\vec{\sigma}_1 \cdot \vec{\mathrm{q}}_\Omega)(\vec{\sigma}_2 \cdot \vec{\mathrm{r}}) + (\vec{\mathrm{r}} \leftrightarrow \vec{\mathrm{q}}_\Omega)\big]$$

$$\vec{\mathrm{q}}_\Omega = \vec{\mathrm{q}} - \tfrac{\vec{\mathrm{r}}}{\mathrm{r}}\ \mathrm{q}_r$$

$s(r)$ and $\vartheta(r)$
for given potential determined
in the two-body system

Correlated States: The Deuteron

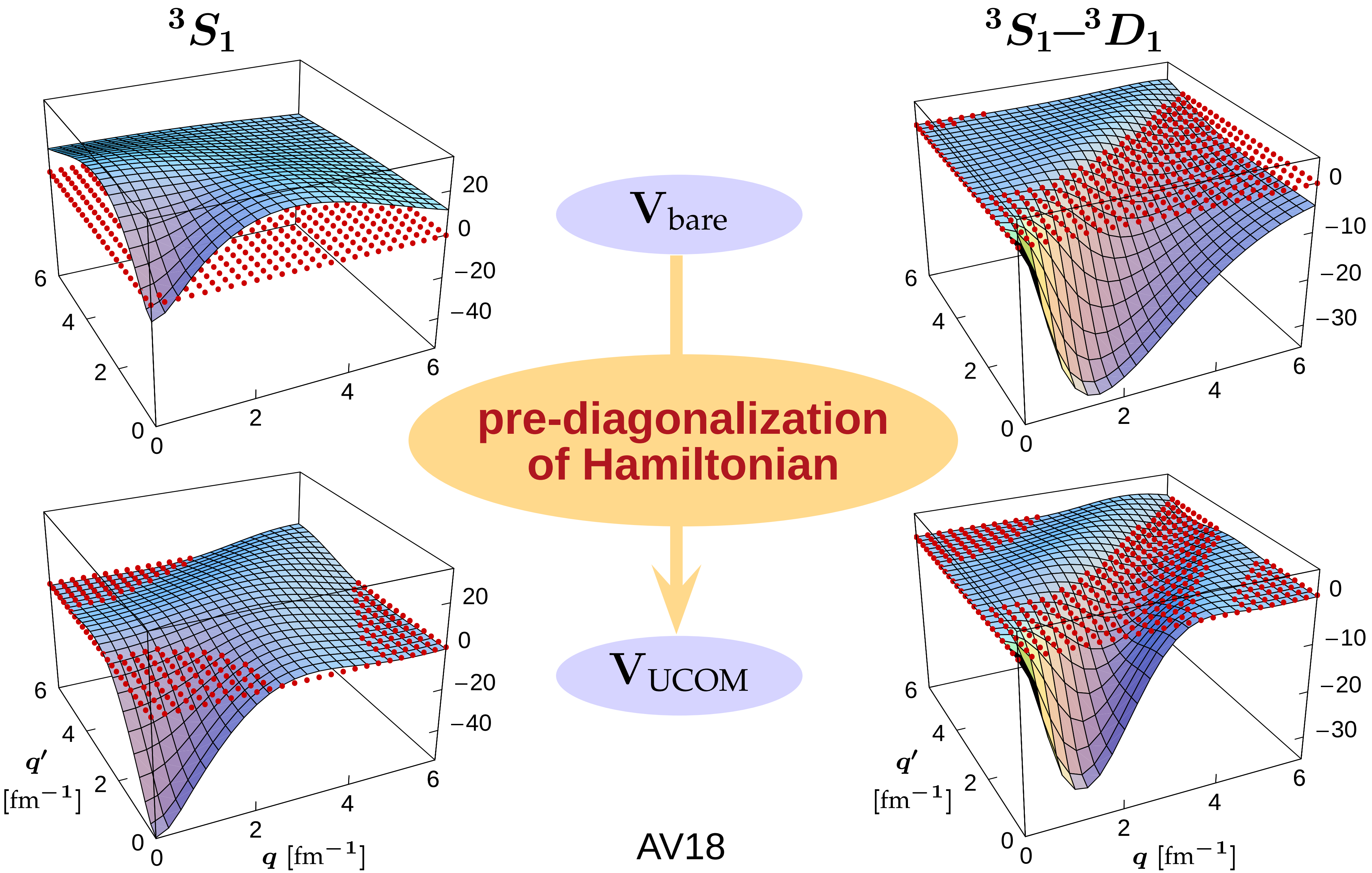


Correlated Interaction — V_{UCOM}

$$\widetilde{\mathrm{H}} = \mathrm{T} + \mathrm{V}_{\mathrm{UCOM}} + \mathrm{V}_{\mathrm{UCOM}}^{[3]} + \cdots$$

- **closed operator expression** for the correlated interaction $\mathrm{V}_{\mathrm{UCOM}}$ in two-body approximation
- correlated interaction and original NN-potential are **phase shift equivalent** by construction
- unitary transformation results in a **pre-diagonalization** of Hamiltonian
- momentum-space matrix elements of correlated interaction are **similar to $V_{\mathrm{low}-k}$**

Momentum-Space Matrix Elements



Application I

No-Core Shell Model

UCOM + NCSM

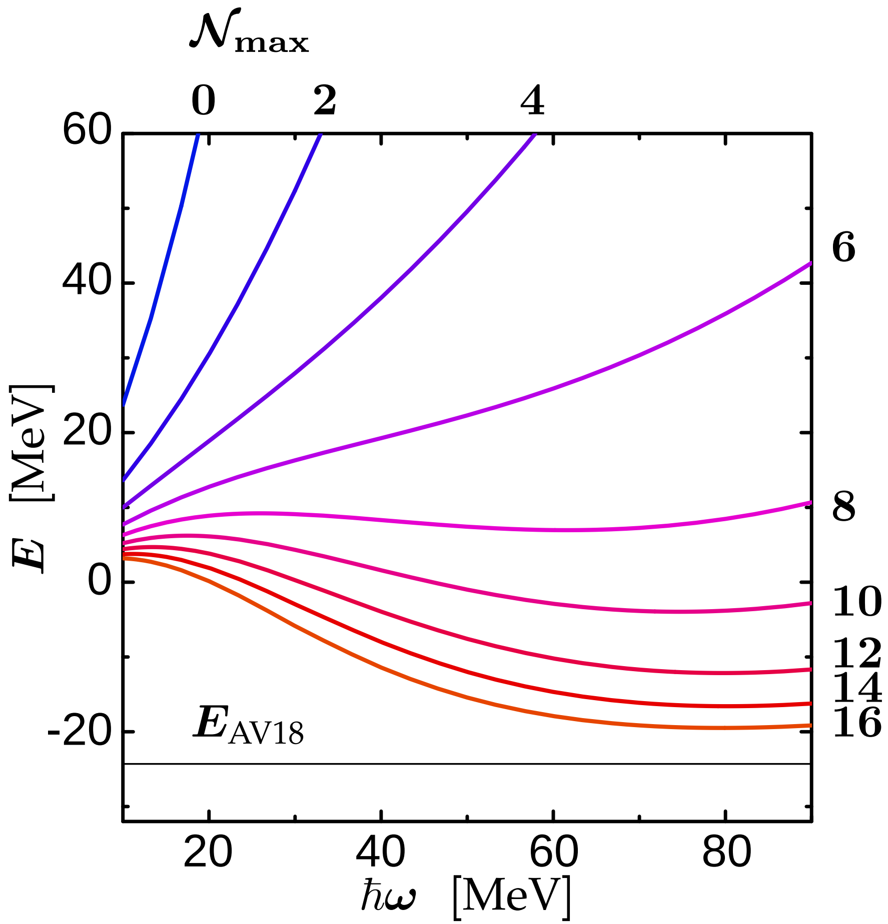
No-Core Shell Model
+
Matrix Elements of Correlated Realistic NN-Interaction V_{UCOM}

- many-body state is **expanded in Slater determinants** of harmonic oscillator single-particle states
- **large scale diagonalization** of Hamiltonian within a truncated model space ($\mathcal{N}\hbar\omega$ truncation)
- assessment of **short and long-range correlations**

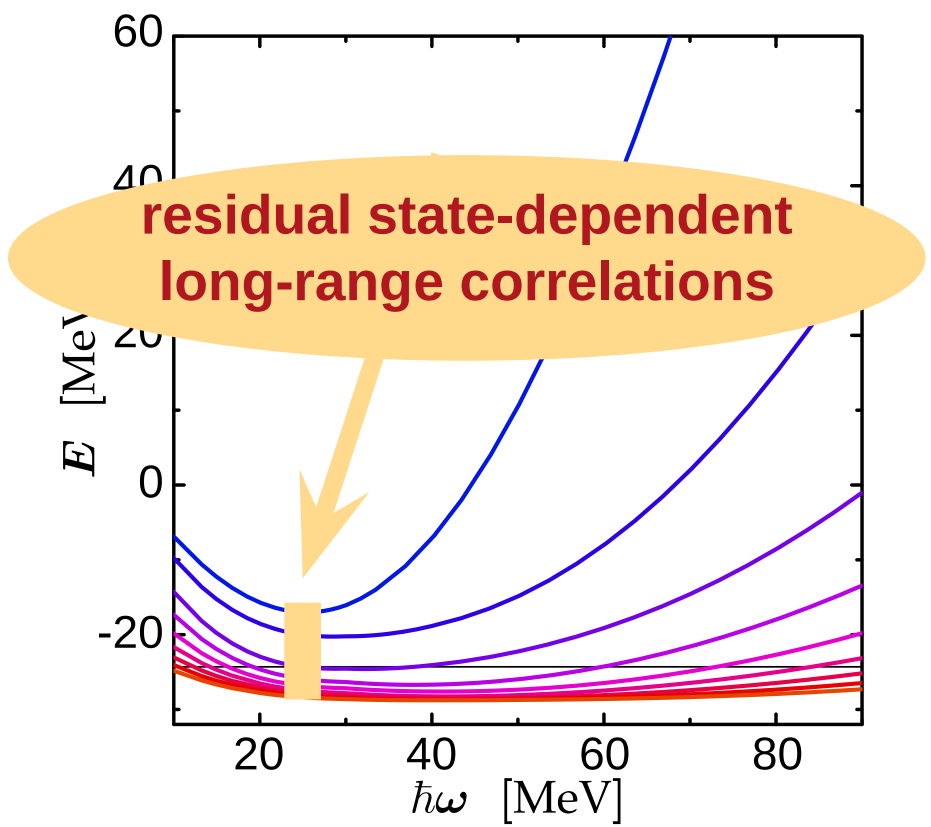
NCSM code by Petr Navrátil [PRC 61, 044001 (2000)]

^{4}He: Convergence

V_{AV18}



V_{UCOM}



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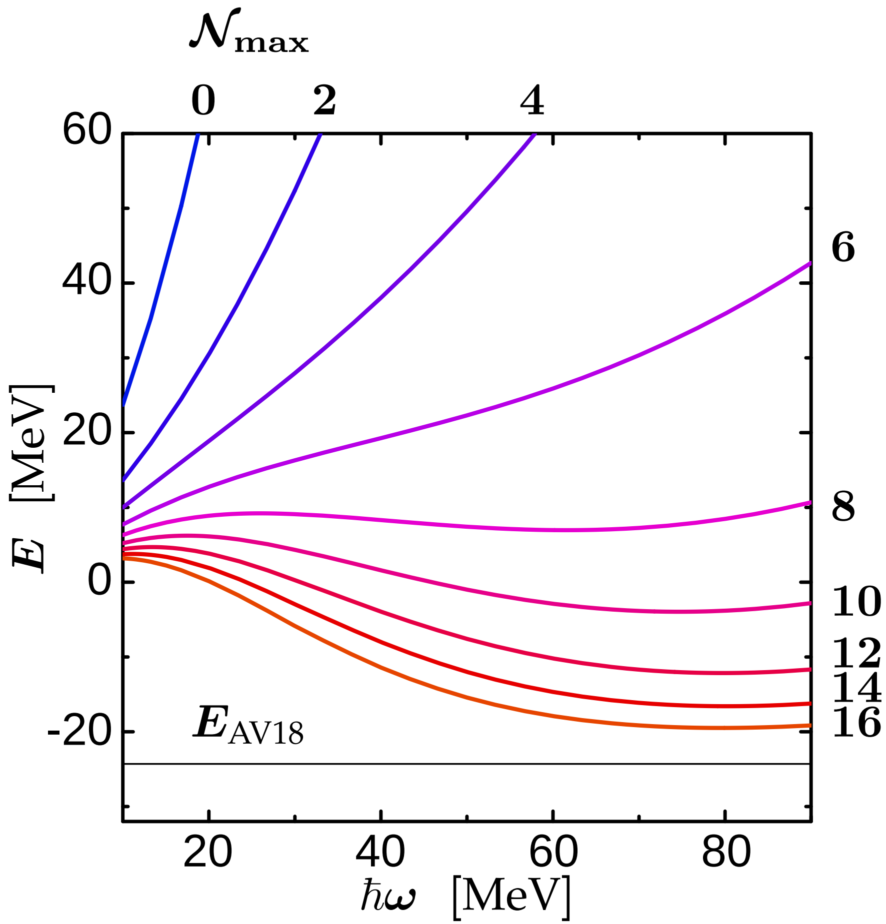
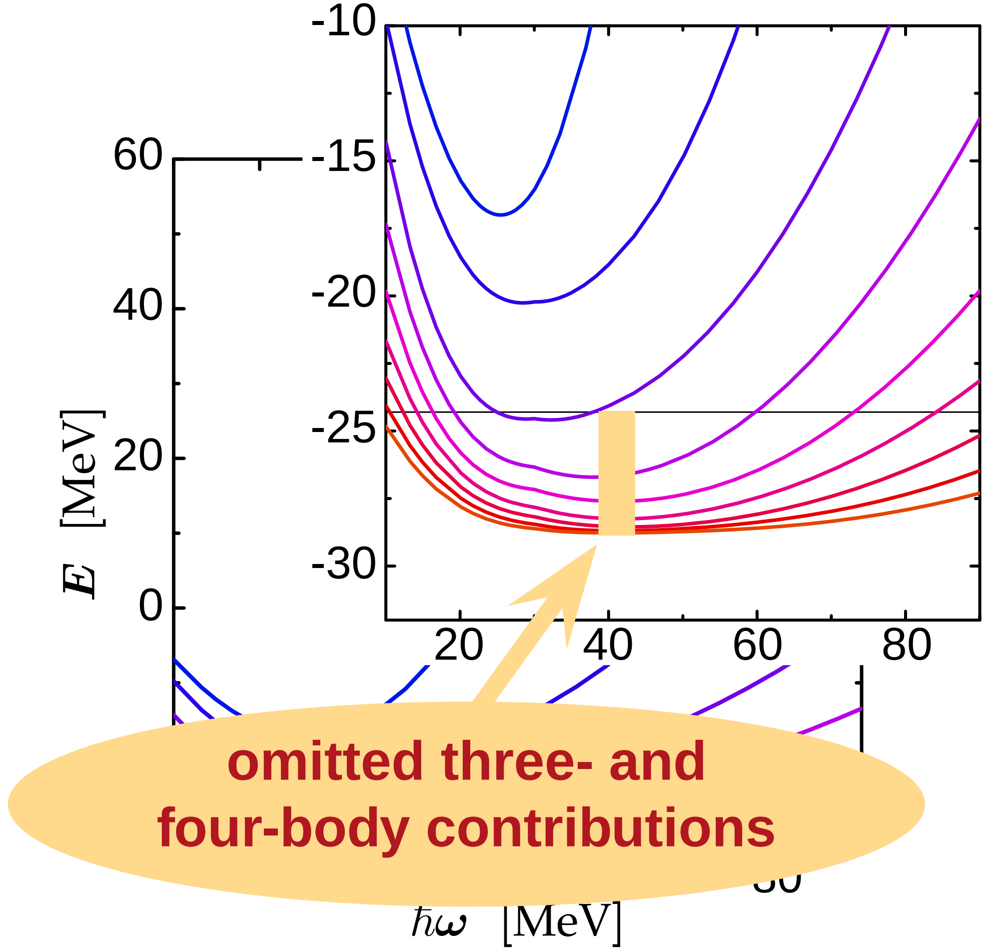
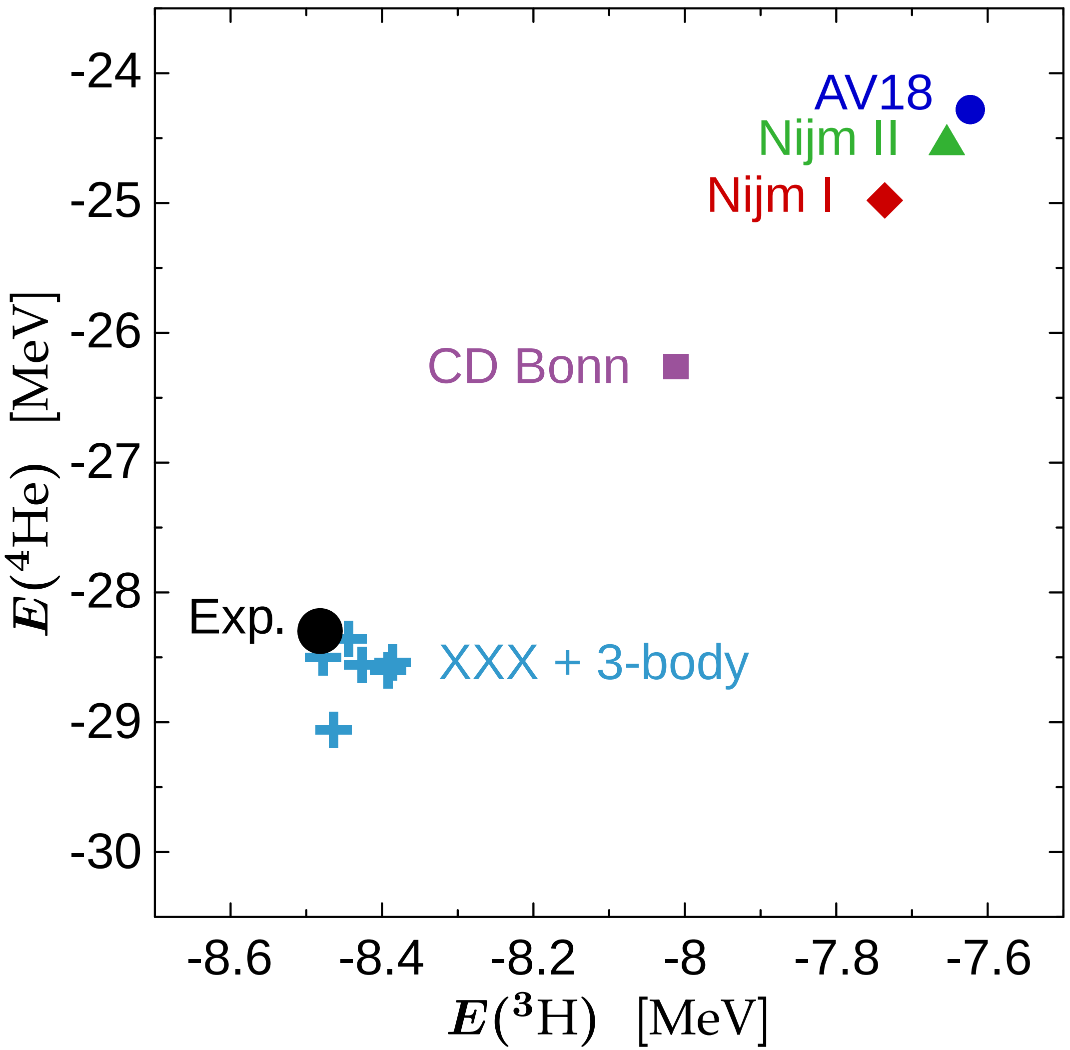


V_{UCOM}

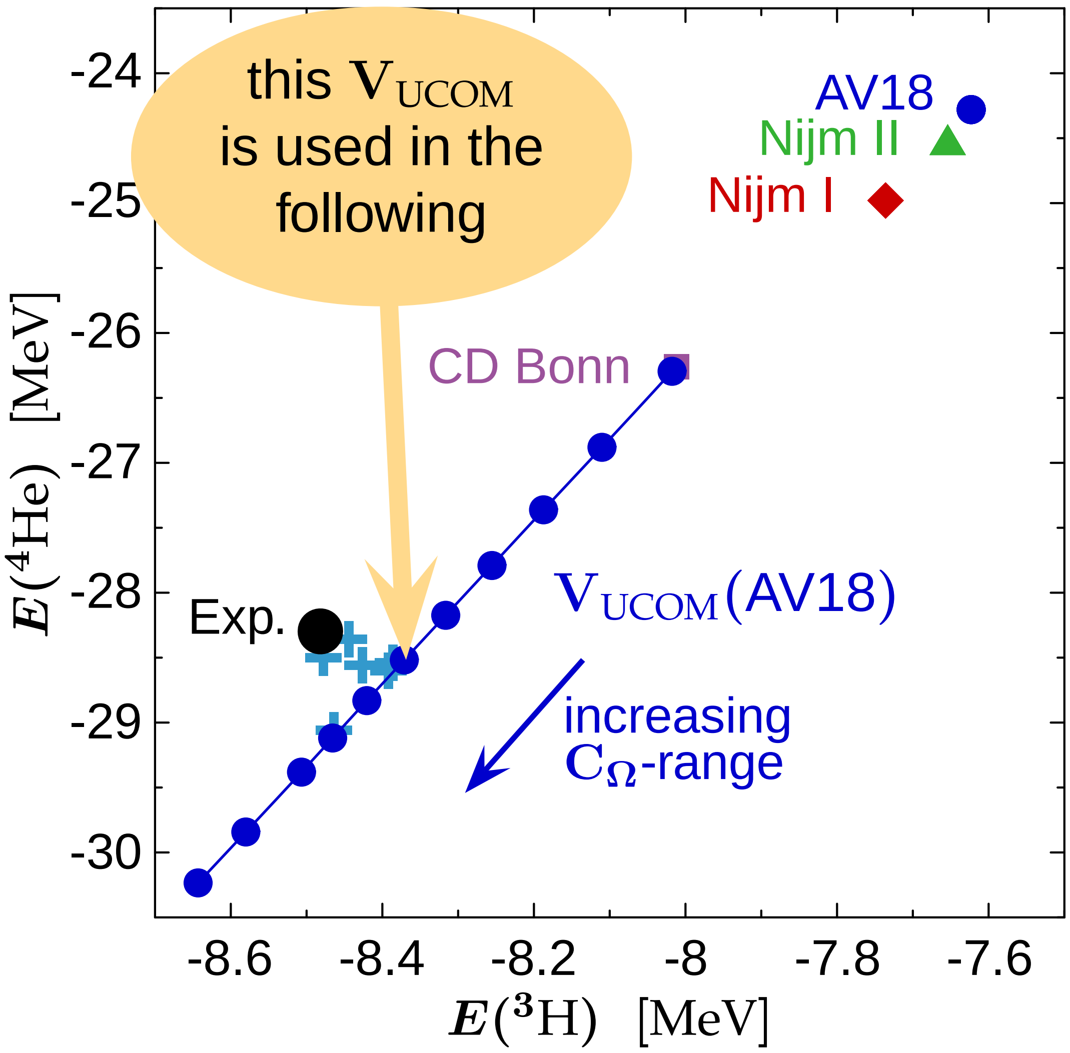


Tjon-Line and Correlator Range



- **Tjon-line**: $E(^4\mathrm{He})$ vs. $E(^3\mathrm{H})$ for phase-shift equivalent NN-interactions

Tjon-Line and Correlator Range



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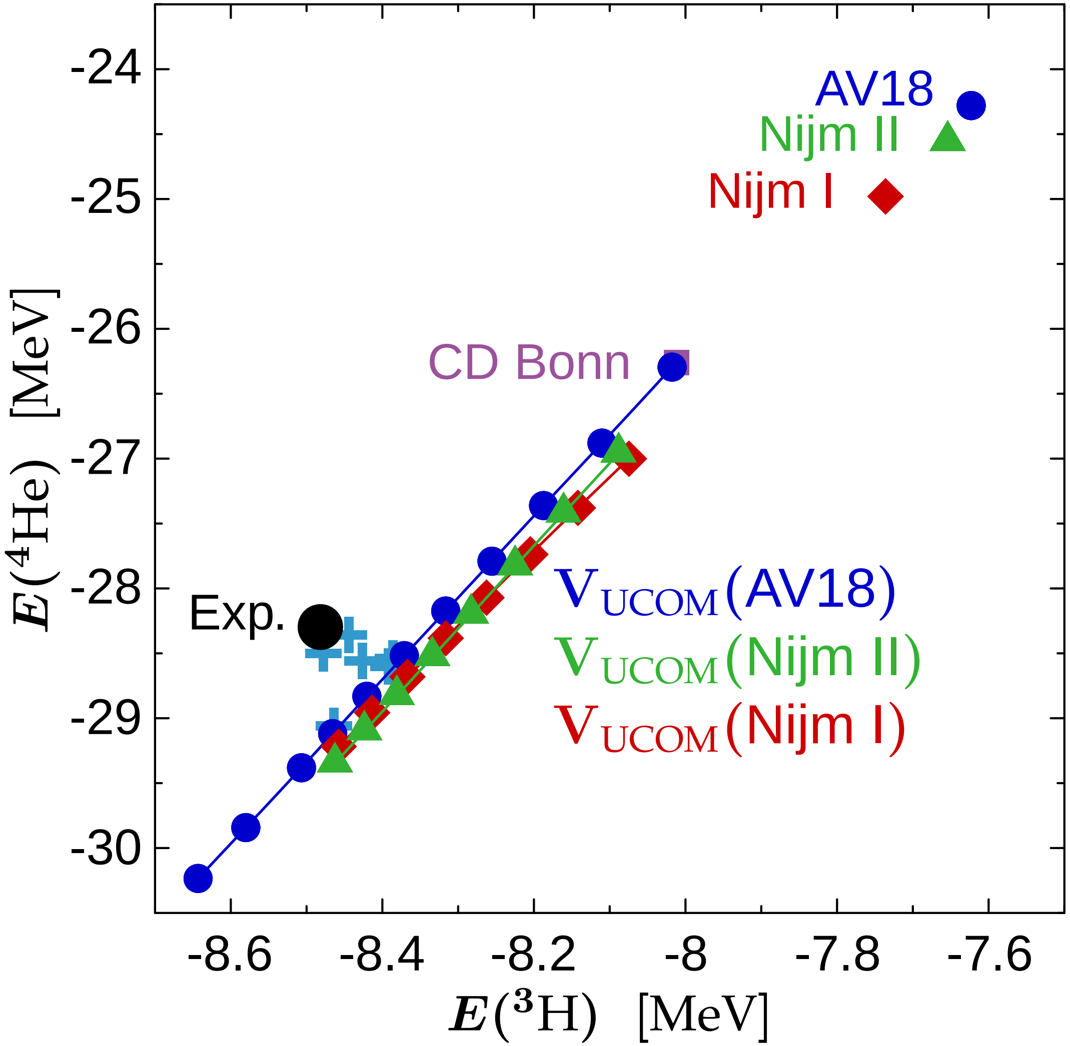
- change of C_{Ω}-correlator range results in shift along Tjon-line

minimise net three-body force by choosing correlator with energies close to experimental value

Tjon-Line and Correlator Range



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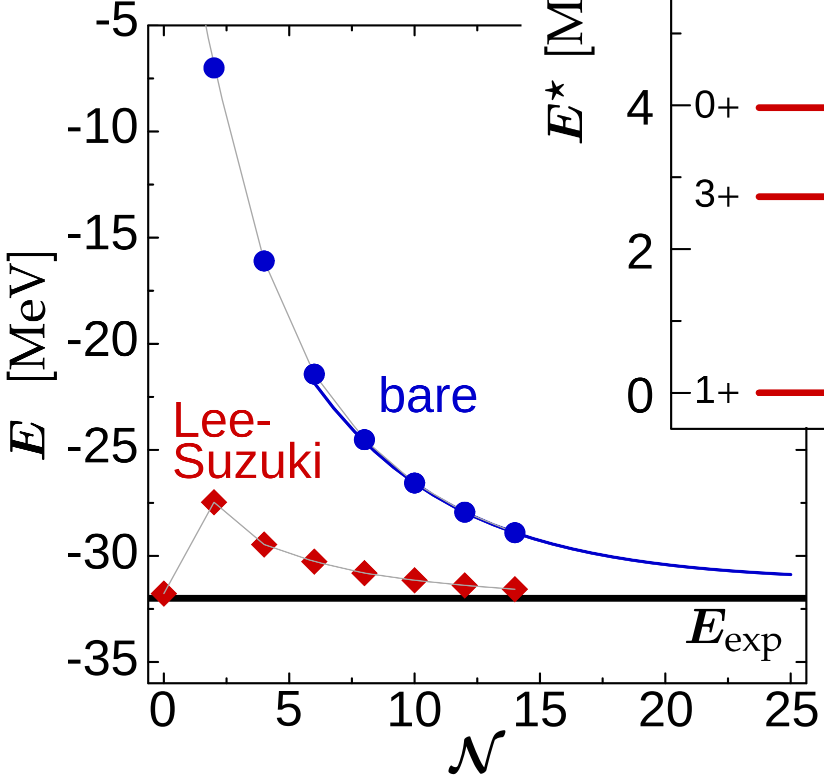
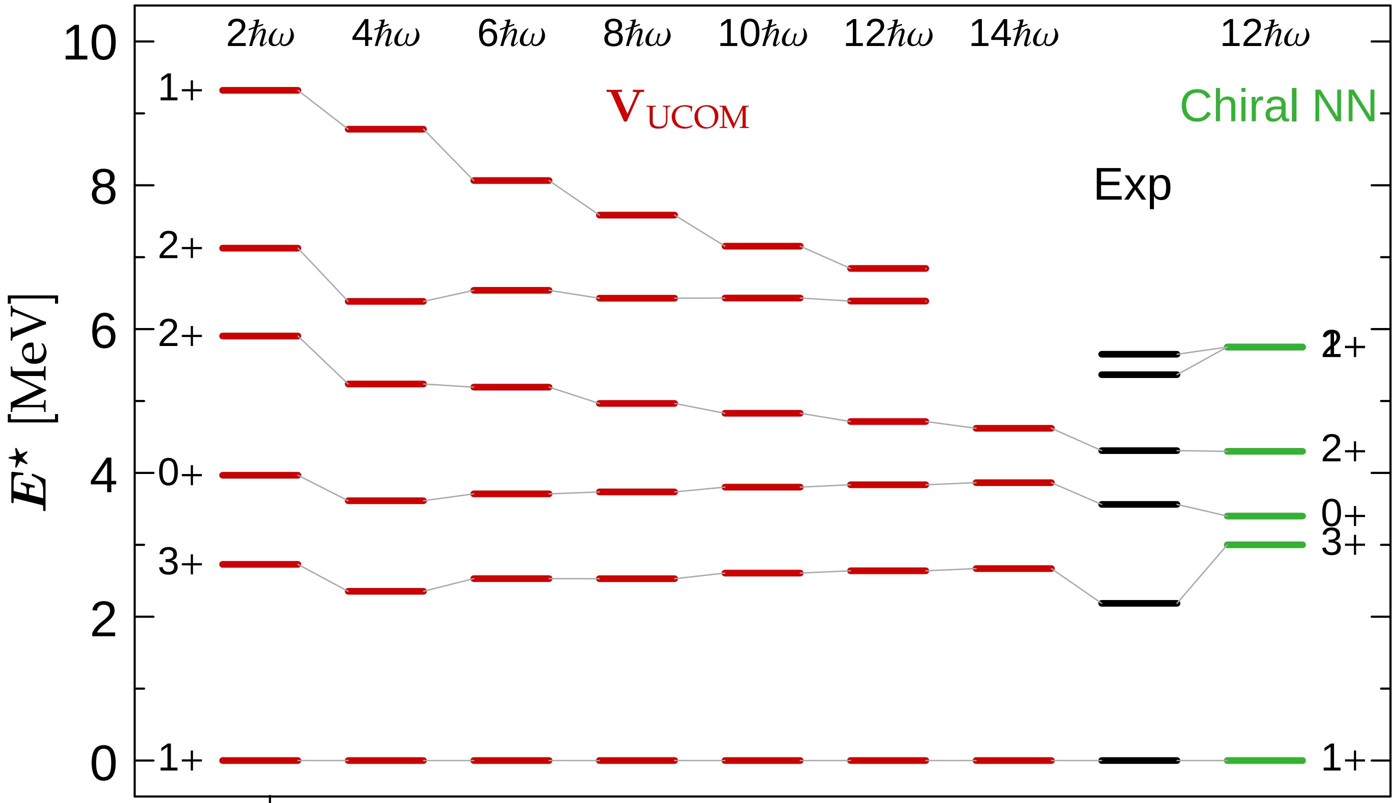
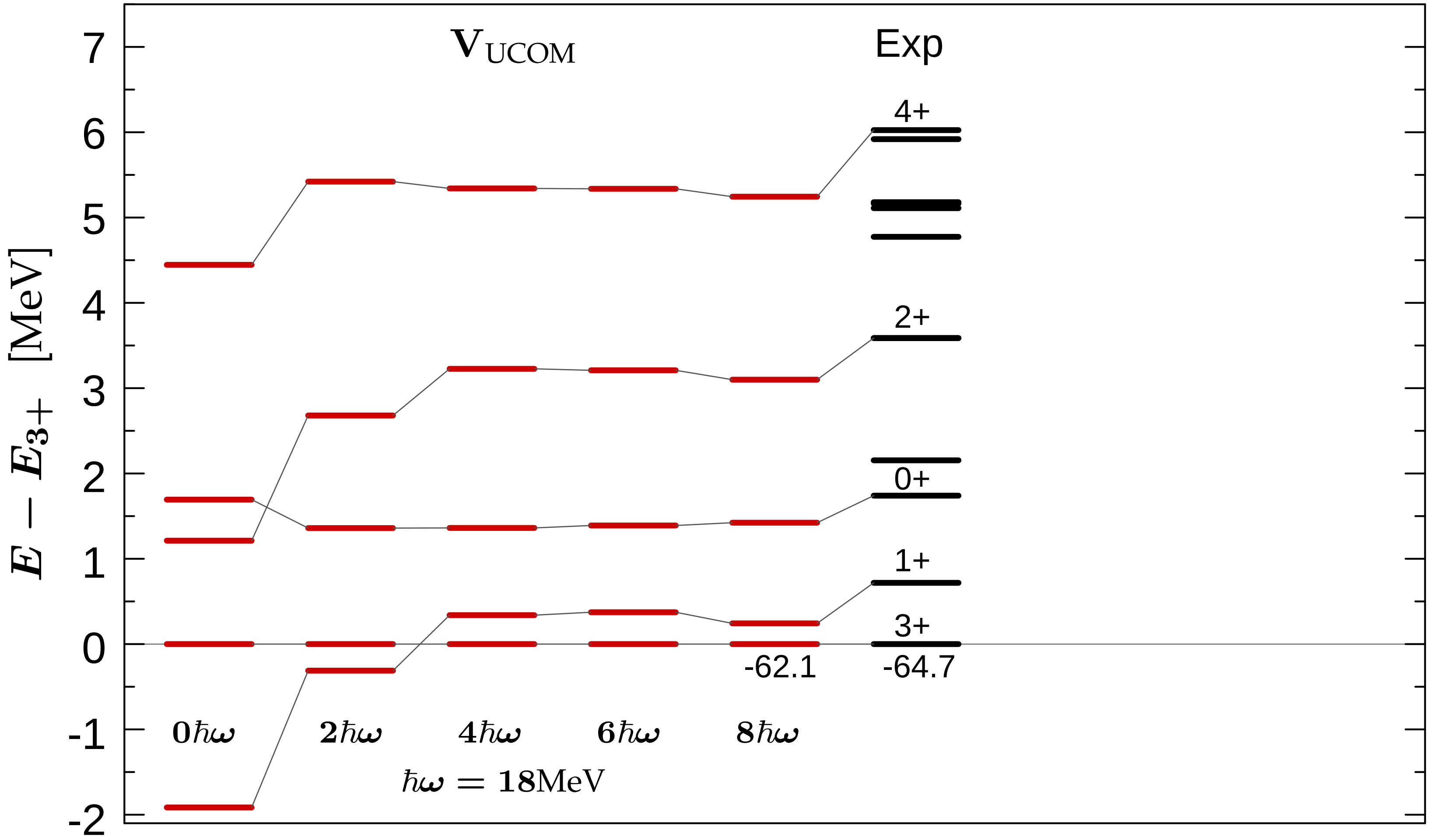
^{6}Li: Survey of the p-Shell



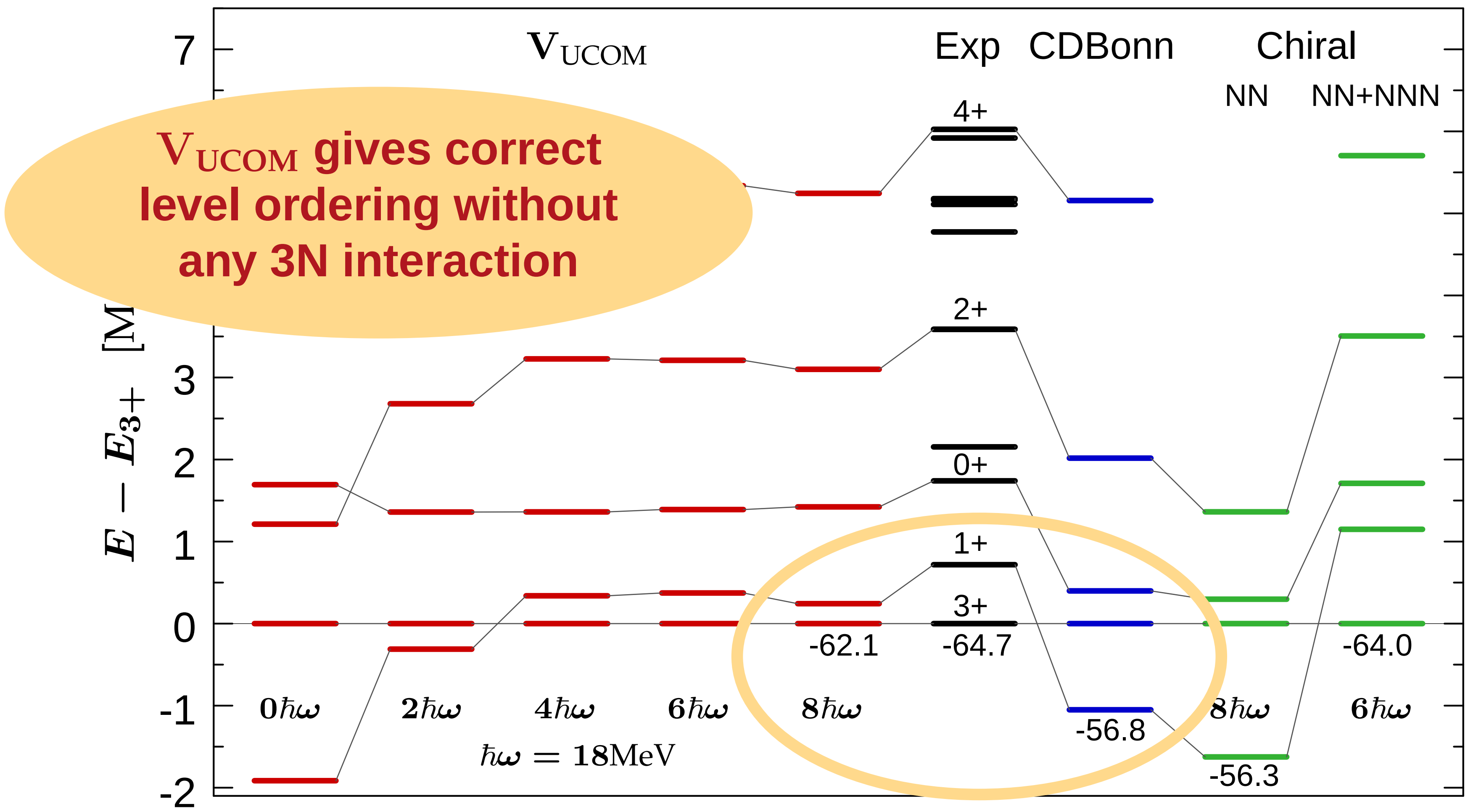


convergent g.s. energies and spectra in good agreement with experiment

^{10}B: Hallmark of the 3N-Interaction?



^{10}B: Hallmark of the 3N Interaction?



Application II:

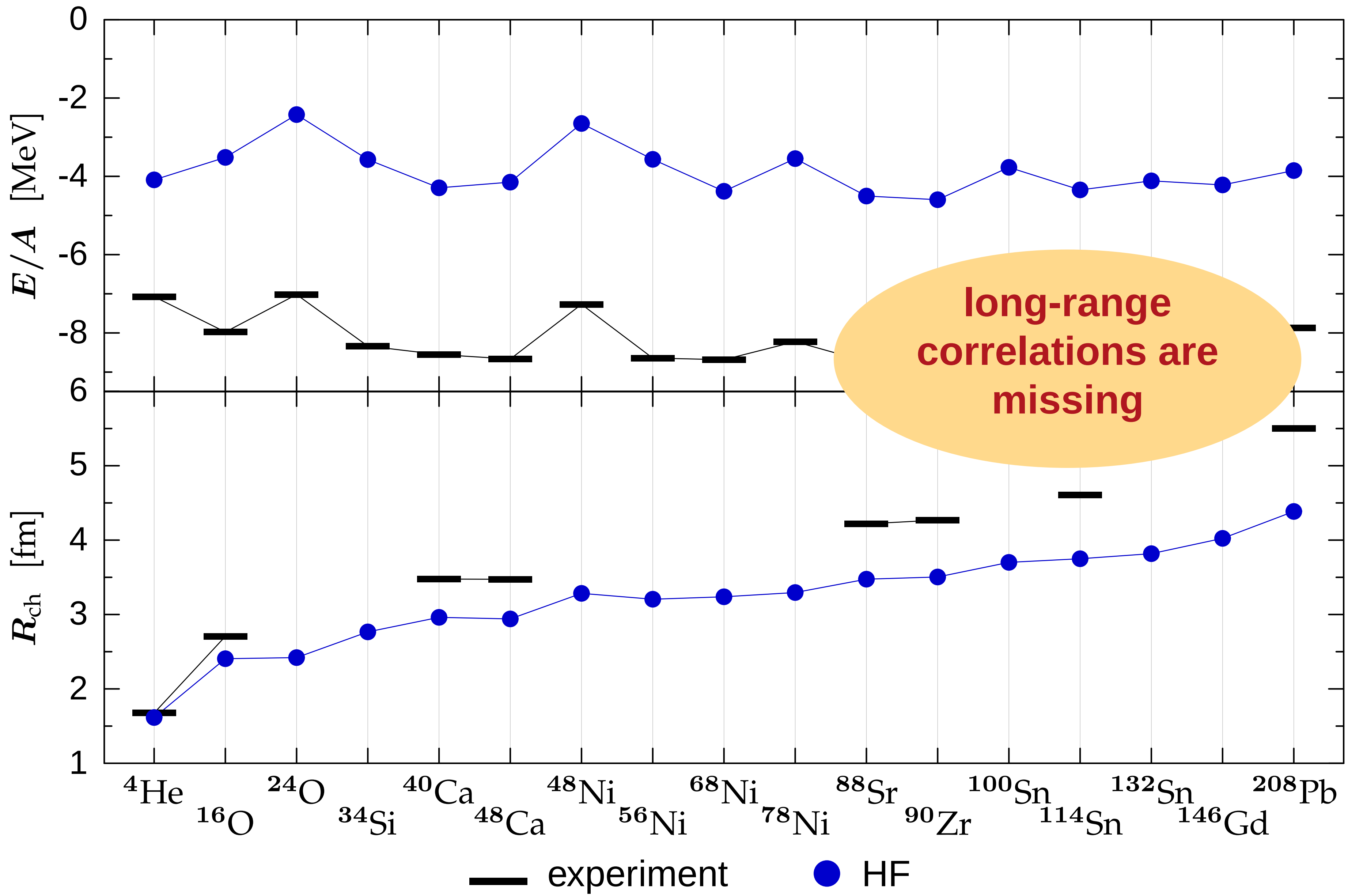
Hartree-Fock & Beyond

UCOM + HF

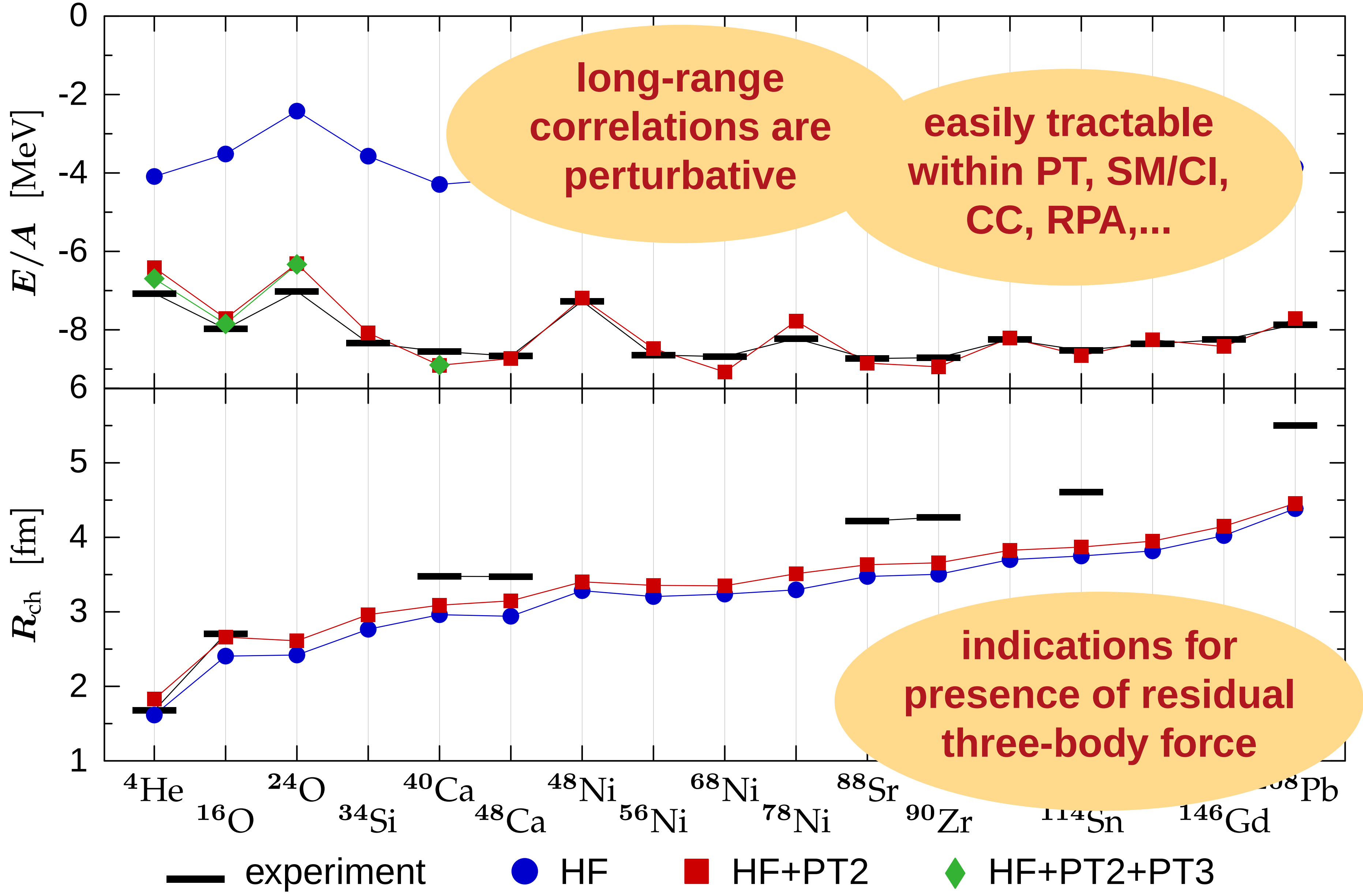
Standard Hartree-Fock
+
Matrix Elements of Correlated Realistic NN-Interaction V_{UCOM}

- many-body state is a **Slater determinant** of single-particle states expanded in oscillator basis (typically 13 major shells)
- **correlations cannot be described** by Hartree-Fock states
- starting point for **improved many-body calculations**: MBPT, RPA, SM/CI, CC,...

Hartree-Fock with V_{UCOM}

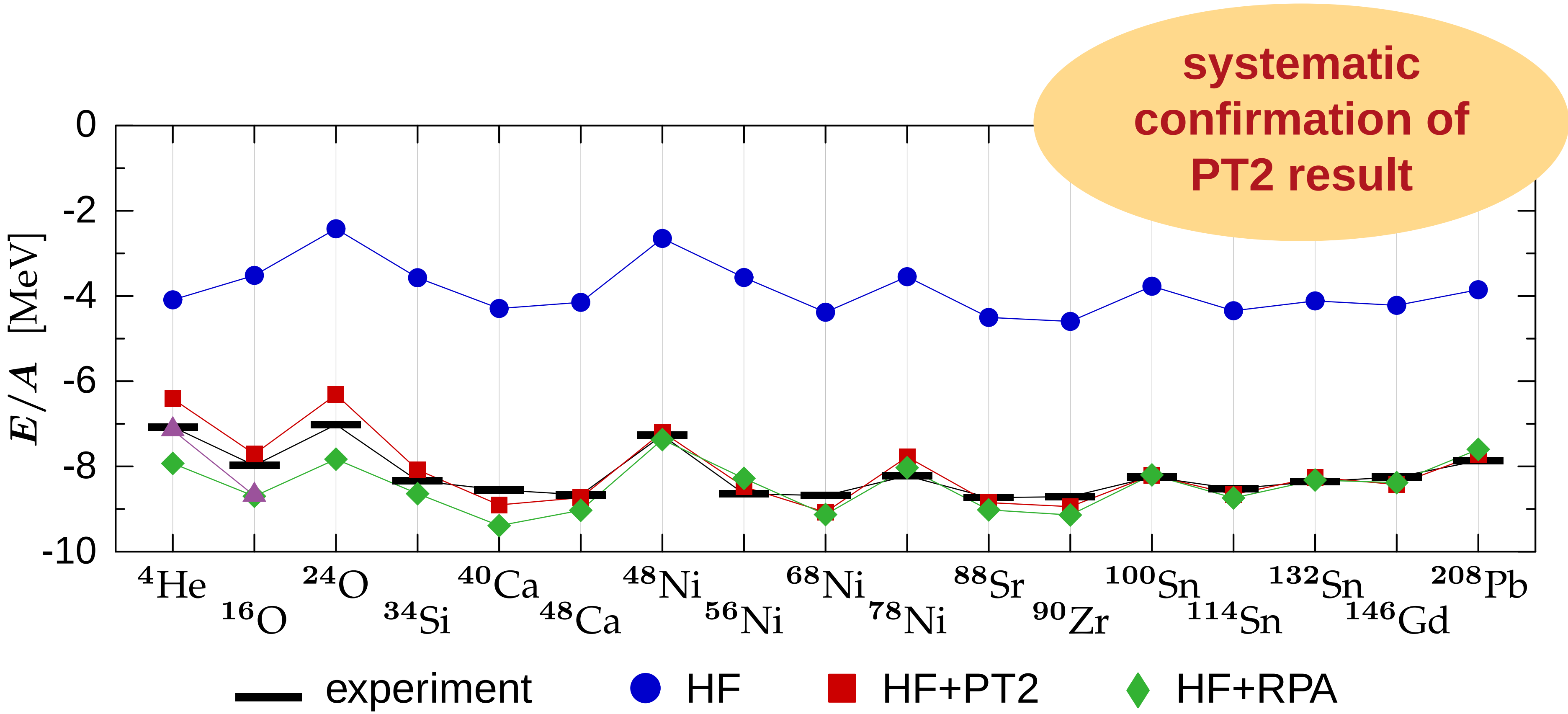


Perturbation Theory with V_{UCOM}



RPA Ground State Correlations

- evaluate correlation energy beyond Hartree-Fock via **ring summation** using RPA amplitudes
- include all parities and charge exchange and correct for double-counting of 2nd order term



Next Steps...

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Collective Excitations

- RPA description of collective modes based on V_{UCOM}
- impact of correlations on response & s.p. properties

→ **C. Barbieri** & **N. Paar**

Continuum & Reactions

- inclusion of continuum and scattering states
- description structure and reactions on the same footing

→ **S. Bacca** & **H. Feldmeier**

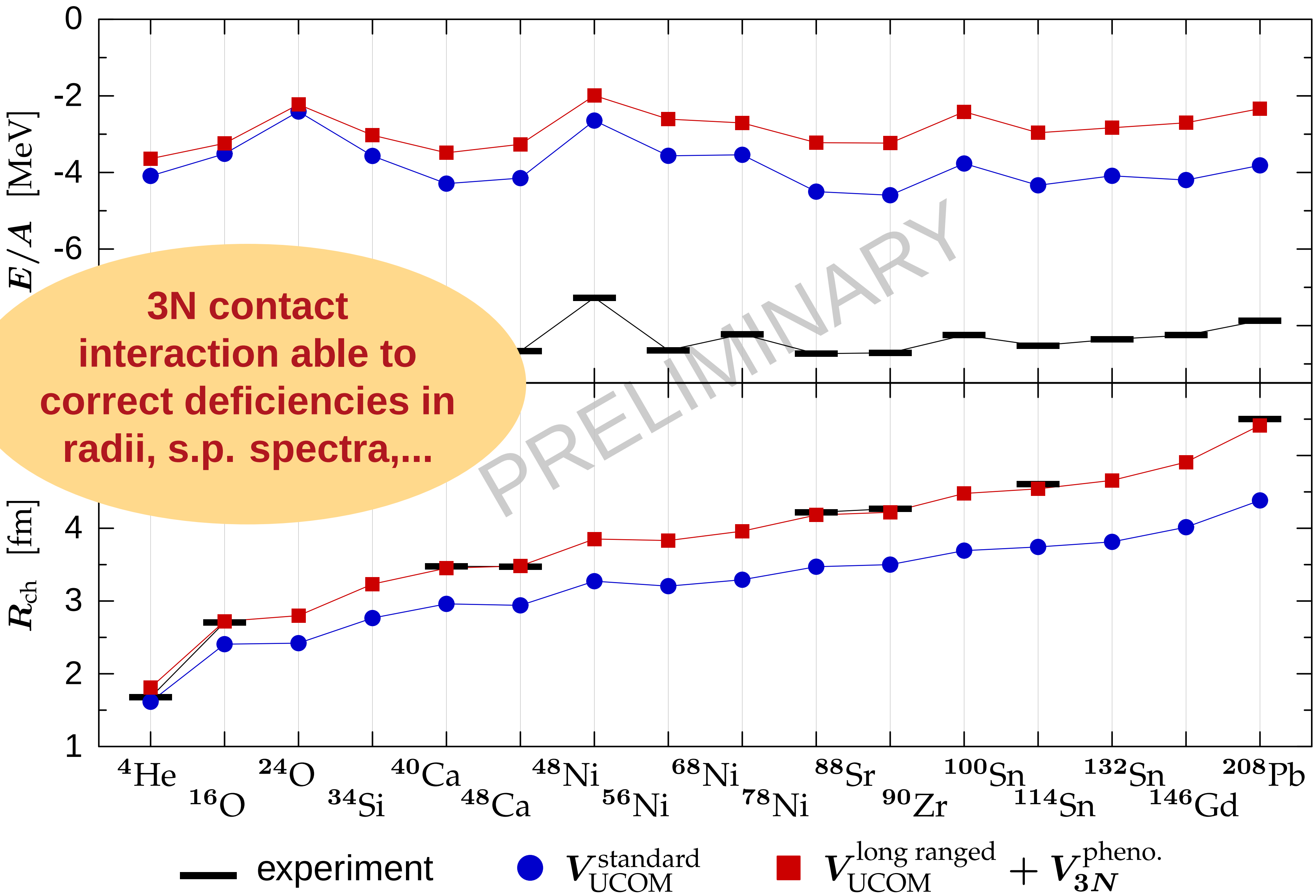
Beyond HF + MBPT

- pairing phenomena with realistic interactions (HFB)
- Padé-MBPT & Adaptive NCSM: innovative many-body methods beyond the p-shell

3N Interaction

- inclusion of phenom. zero- or finite-range 3N interaction to supplement V_{UCOM}
- quantitative nuclear structure studies in NCSM, HF, RPA,...

Phenomenological 3N Interactions



3N contact interaction able to correct deficiencies in radii, s.p. spectra,...

Conclusions

- **Unitary Correlation Operator Method (UCOM)**
  - explicit description of short-range central and tensor correlations
  - universal phase-shift equivalent correlated interaction $\mathbf{V}_{\mathrm{UCOM}}$

- **Innovative Many-Body Methods**
  - No-Core Shell Model,...
  - Hartree-Fock, MBPT, CI, CC,...
  - RPA, ERPA, SRPA, SCGF,...
  - Fermionic Molecular Dynamics

unified description of nuclear structure across the whole nuclear chart is within reach

Epilogue

- **thanks to my group & my collaborators**

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