Nuclear Structure with a Three-Body Interaction

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Overview

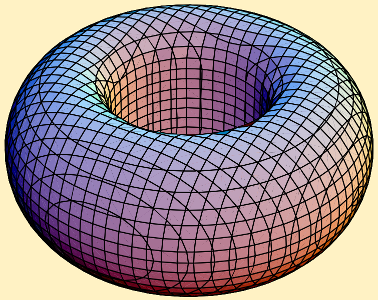
- Introduction
- Three-Body Interaction
- Hartree-Fock Results
- Collective Excitations
- Many-Body Perturbation Theory
- Summary & Outlook

Introduction

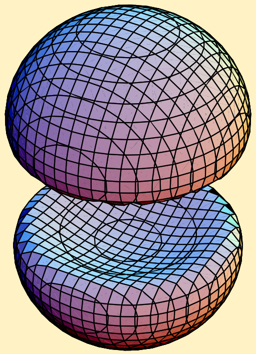
Unitary Correlation Operator Method

- realistic interaction induces correlations:
  - **central correlations:** two-body density is suppressed at low distances
  - **tensor correlations:** angular distribution depends on the relative spin alignments
- treat short-range correlations explicitly by unitary transformation
- correlated interaction $\mathrm{V}_{\mathrm{UCOM}}$ is phase-shift equivalent to the underlying bare nucleon-nucleon interaction

Deuteron:
two-body density



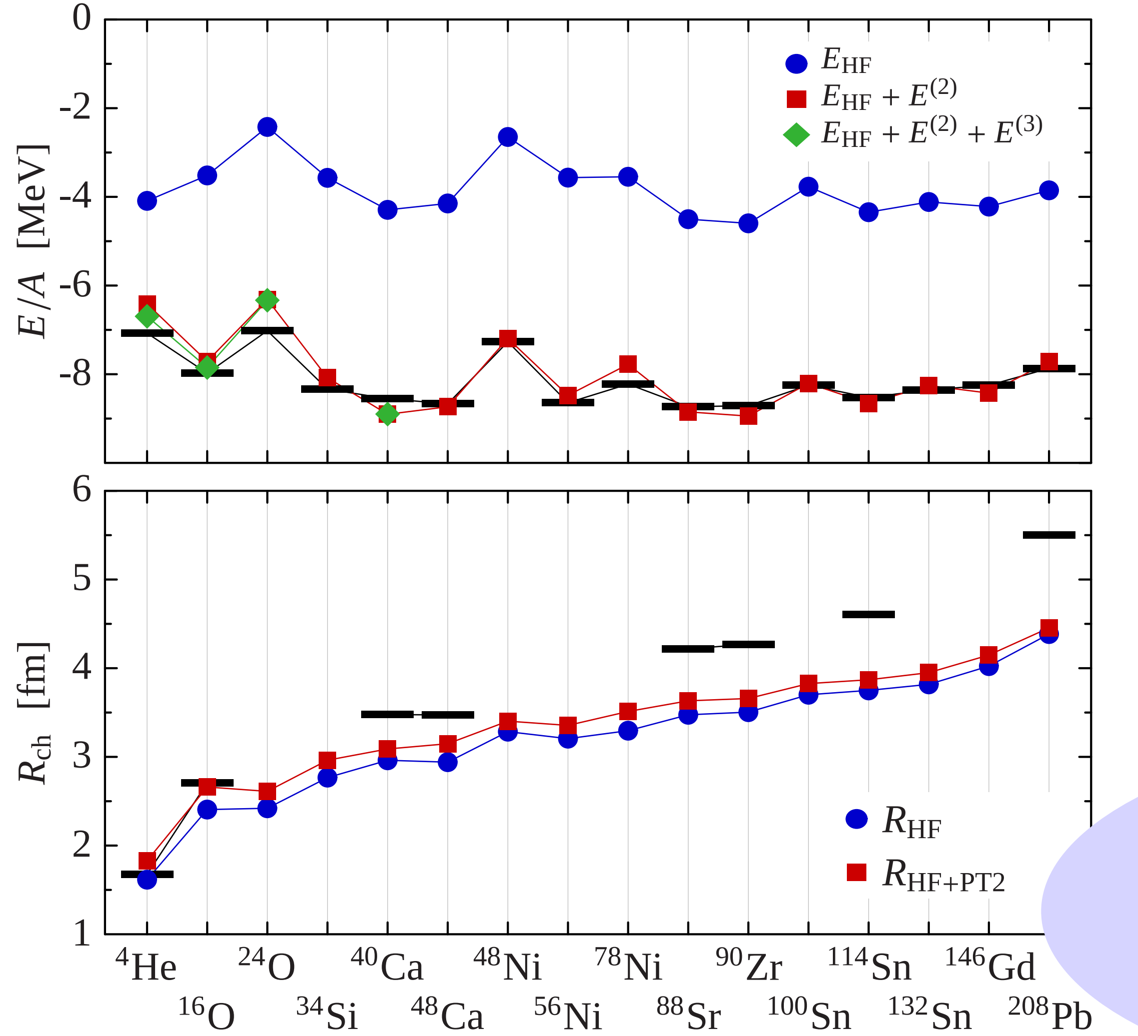
$M_S = 0$
$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$



$M_S = \pm 1$
$|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle$

Motivation

Results with V_{UCOM}



- **binding energies:** good agreement

→ HK 27.4, HK 27.5

- **charge radii:** systematically too small

⇒ repulsive three-body interaction

Three-Body Interaction

Contact Interaction

- repulsive three-body interaction
  $\longrightarrow$ increased charge radii
  $\longrightarrow$ decreased binding energies
  $\longrightarrow$ increased tensor correlation volume
- simplest ansatz: contact interaction

$$\mathbf{V_3} = \boldsymbol{C_3}\, \boldsymbol{\delta}^{(3)}(\vec{\mathbf{x}}_1 - \vec{\mathbf{x}}_2)\, \boldsymbol{\delta}^{(3)}(\vec{\mathbf{x}}_1 - \vec{\mathbf{x}}_3)$$

- calculation of matrix elements in harmonic-oscillator basis

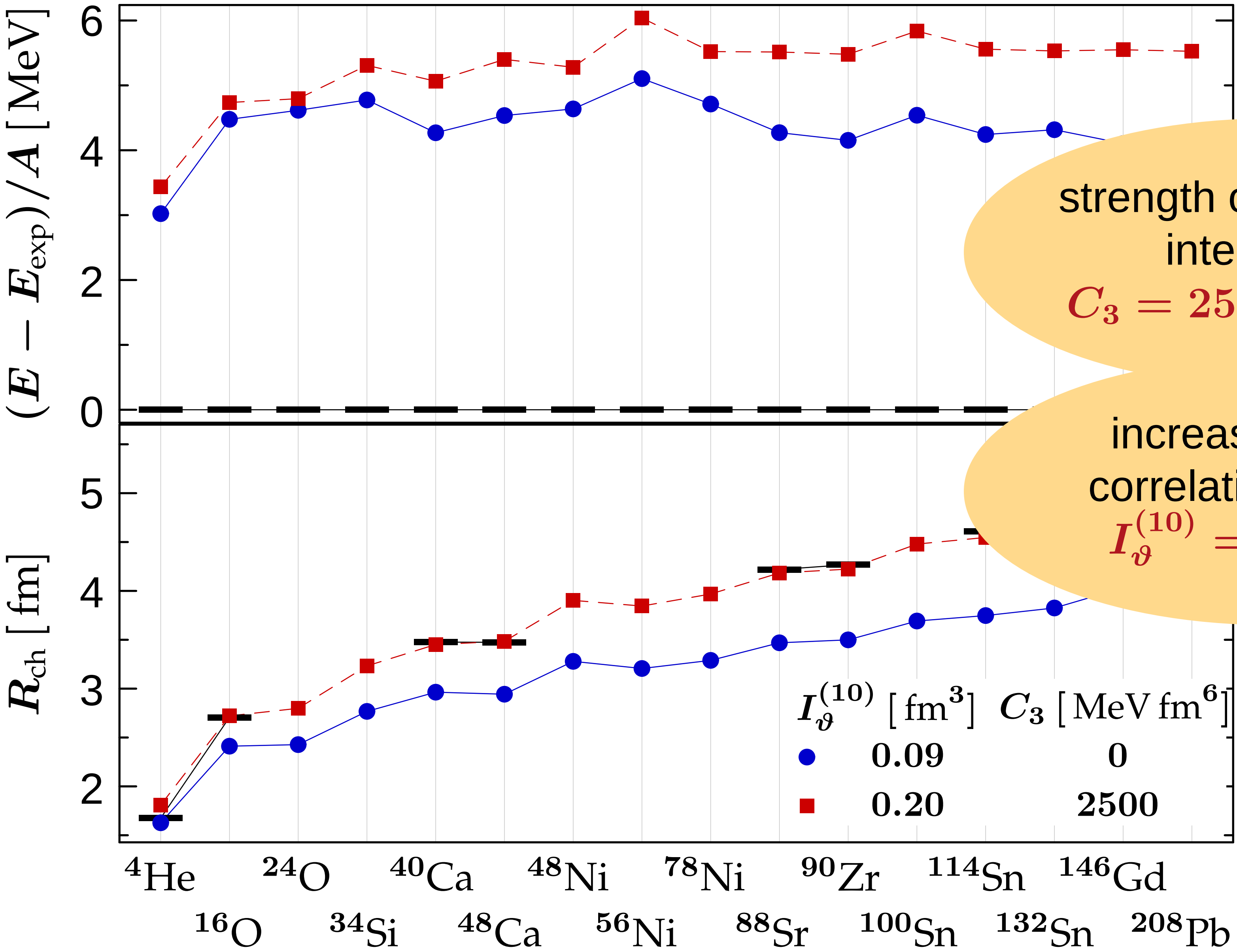
Three-Body Interaction

Matrix Elements in Harmonic-Oscillator Basis

$$
\begin{aligned}
&\langle n_1 l_1 j_1 m_1 m_{t_1}, n_2 l_2 j_2 m_2 m_{t_2}, n_3 l_3 j_3 m_3 m_{t_3} | \mathrm{V}_3 | n_4 l_4 j_4 m_4 m_{t_4}, n_5 l_5 j_5 m_5 m_{t_5}, n_6 l_6 j_6 m_6 m_{t_6} \rangle \\
&\quad = C_3\, \delta_{m_{t_1} m_{t_4}} \delta_{m_{t_2} m_{t_5}} \delta_{m_{t_3} m_{t_6}} \\
&\qquad \times \frac{1}{16\pi^2} \sqrt{(2l_1+1)(2l_2+1)(2l_3+1)(2l_4+1)(2l_5+1)(2l_6+1)} \\
&\qquad \times \int dx x^2 R_{n_1 l_1}(x) R_{n_2 l_2}(x) R_{n_3 l_3}(x) R_{n_4 l_4}(x) R_{n_5 l_5}(x) R_{n_6 l_6}(x) \\
&\qquad \times \sum_{m_{s_1} m_{s_2} m_{s_3}} \mathbf{c}\left(\begin{array}{cc|c} l_1 & \frac{1}{2} & j_1 \\ m_1 - m_{s_1} & m_{s_1} & m_1 \end{array} \right) \mathbf{c}\left(\begin{array}{cc|c} l_2 & \frac{1}{2} & j_2 \\ m_2 - m_{s_2} & m_{s_2} & m_2 \end{array} \right) \mathbf{c}\left(\begin{array}{cc|c} l_3 & \frac{1}{2} & j_3 \\ m_3 - m_{s_3} & m_{s_3} & m_3 \end{array} \right) \\
&\qquad \times \mathbf{c}\left(\begin{array}{cc|c} l_4 & \frac{1}{2} & j_4 \\ m_4 - m_{s_1} & m_{s_1} & m_4 \end{array} \right) \mathbf{c}\left(\begin{array}{cc|c} l_5 & \frac{1}{2} & j_5 \\ m_5 - m_{s_2} & m_{s_2} & m_5 \end{array} \right) \mathbf{c}\left(\begin{array}{cc|c} l_6 & \frac{1}{2} & j_6 \\ m_6 - m_{s_3} & m_{s_3} & m_6 \end{array} \right) \\
&\qquad \times \sum_{L_1 L_2 L_3} \frac{1}{(2L_2+1)} \mathbf{c}\left(\begin{array}{cc|c} l_1 & l_2 & L_1 \\ 0 & 0 & 0 \end{array} \right) \mathbf{c}\left(\begin{array}{cc|c} L_1 & l_3 & L_2 \\ 0 & 0 & 0 \end{array} \right) \mathbf{c}\left(\begin{array}{cc|c} l_4 & l_5 & L_3 \\ 0 & 0 & 0 \end{array} \right) \mathbf{c}\left(\begin{array}{cc|c} L_3 & l_6 & L_2 \\ 0 & 0 & 0 \end{array} \right) \\
&\qquad \times \mathbf{c}\left(\begin{array}{cc|c} l_1 & l_2 & L_1 \\ m_1 - m_{s_1} & m_2 - m_{s_2} & M_{L_1} \end{array} \right) \mathbf{c}\left(\begin{array}{cc|c} L_1 & l_3 & L_2 \\ M_{L_1} & m_3 - m_{s_3} & M_{L_2} \end{array} \right) \\
&\qquad \times \mathbf{c}\left(\begin{array}{cc|c} l_4 & l_5 & L_3 \\ m_4 - m_{s_1} & m_5 - m_{s_2} & M_{L_3} \end{array} \right) \mathbf{c}\left(\begin{array}{cc|c} L_3 & l_6 & L_2 \\ M_{L_3} & m_6 - m_{s_3} & M_{L_2} \end{array} \right)
\end{aligned}
$$

Results

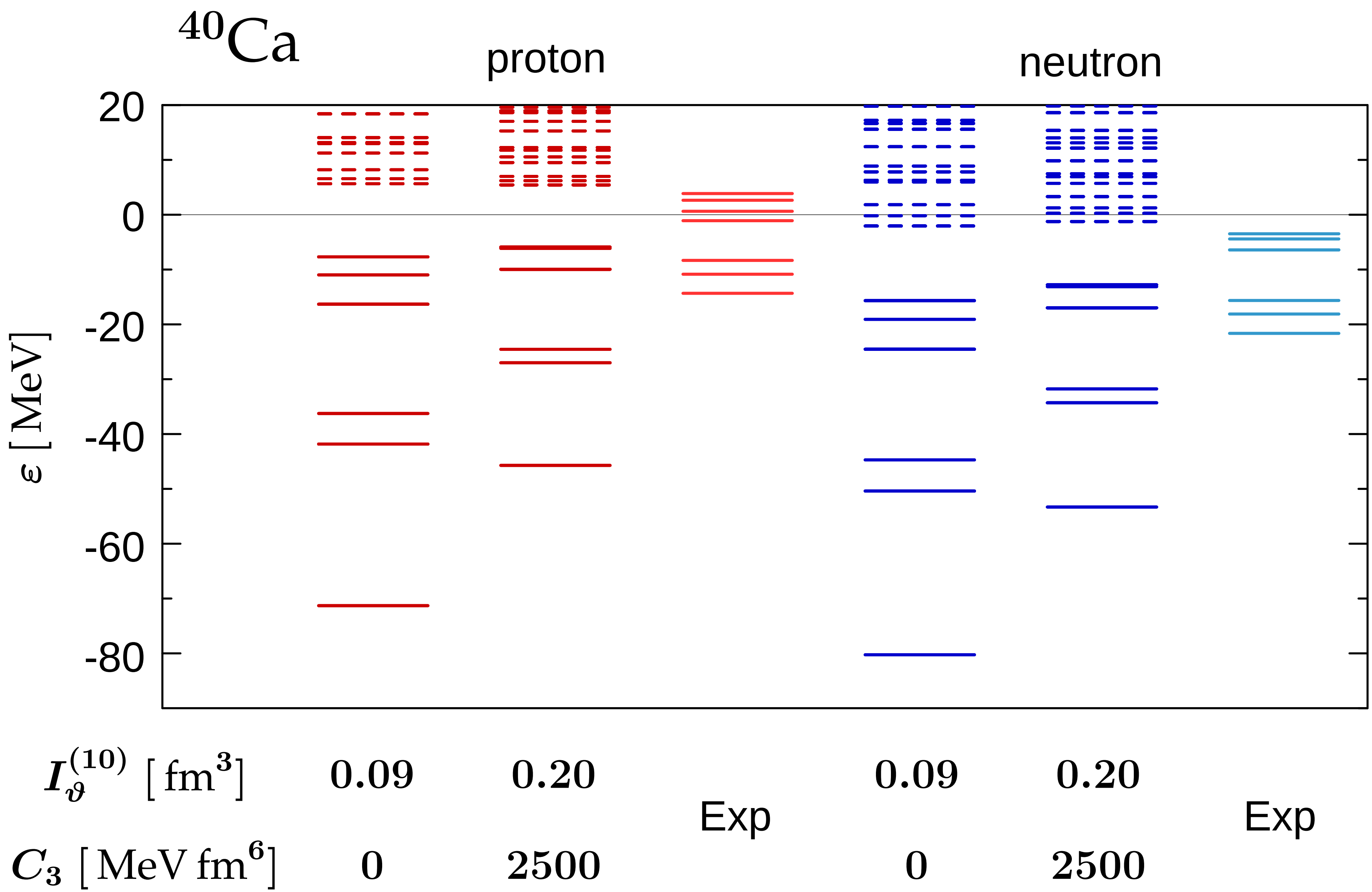
Hartree-Fock Results



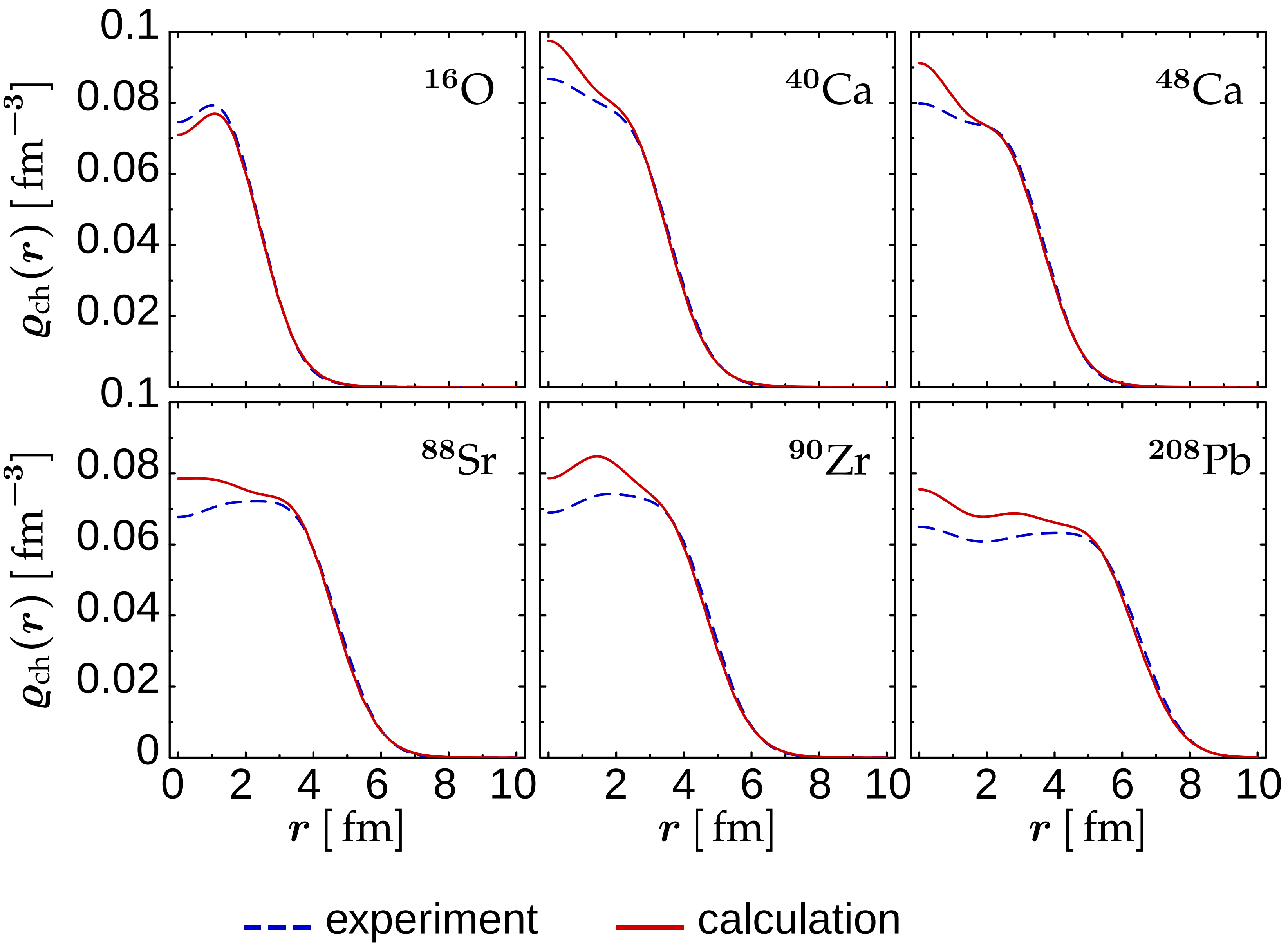
strength of three-body interaction:
$C_3 = 2500\,\text{MeV fm}^6$

increased tensor correlation volume:
$I_{\vartheta}^{(10)} = 0.20\,\text{fm}^3$

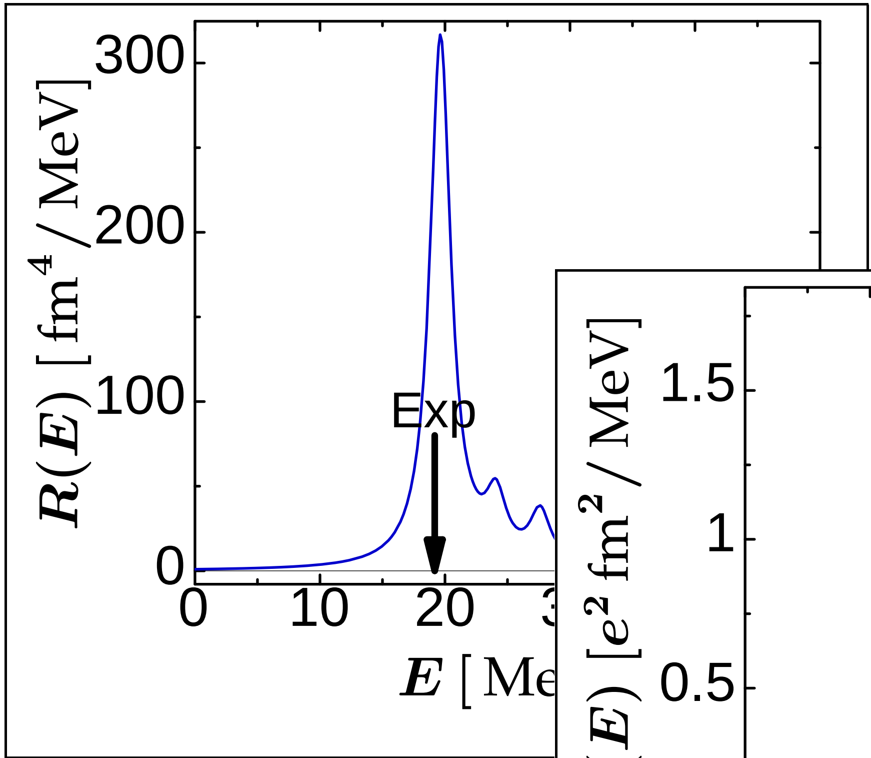
Hartree-Fock Results



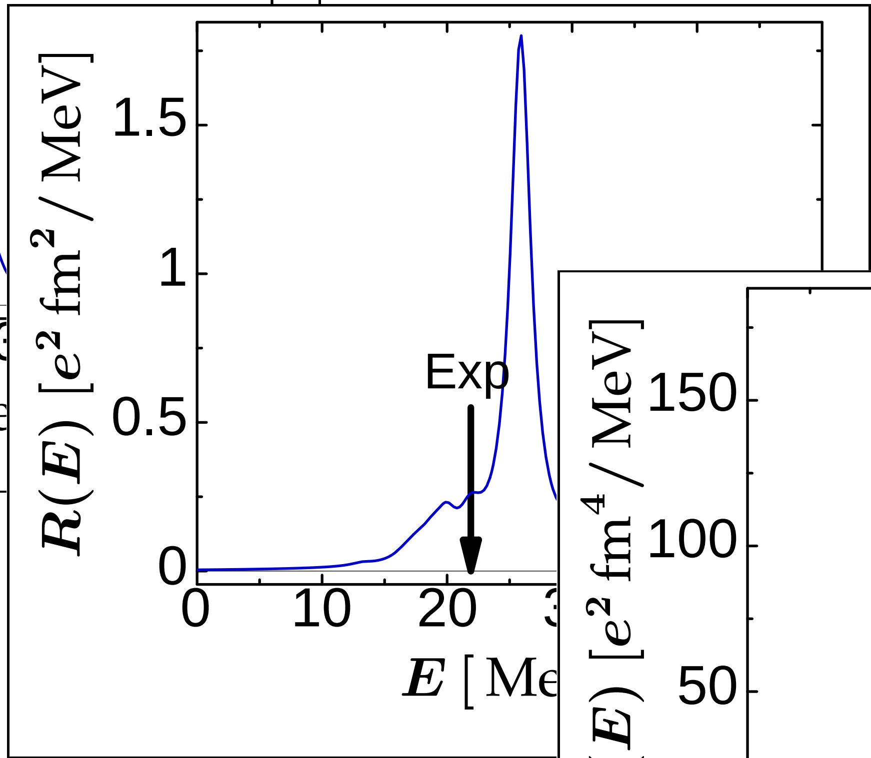
Hartree-Fock Results



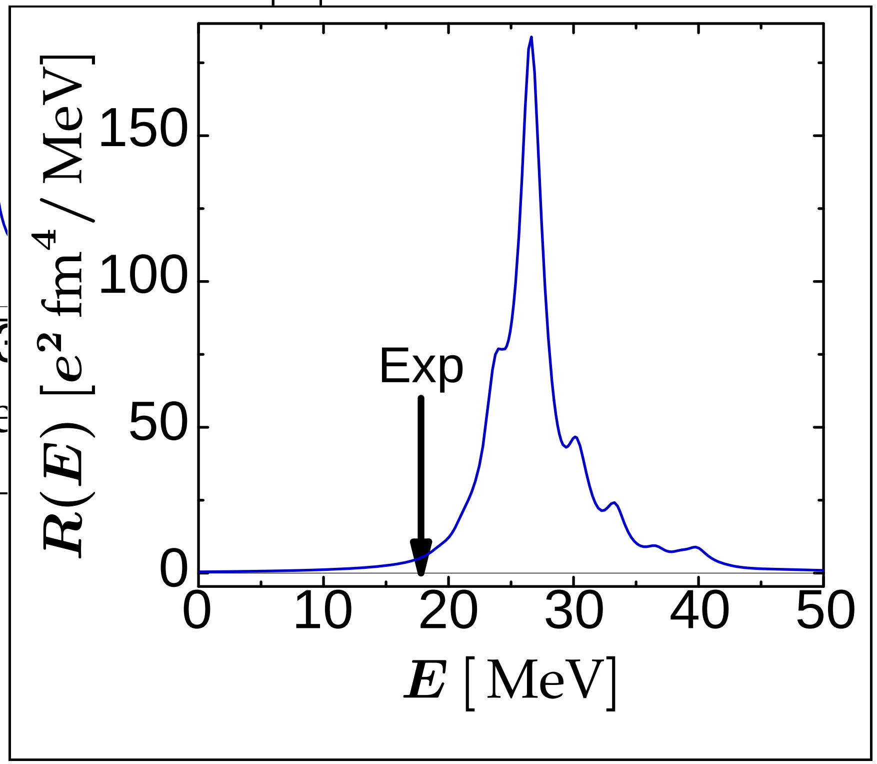
Collective Excitations: ^{40}Ca



Isoscalar giant monopole resonance



Isovector giant dipole resonance



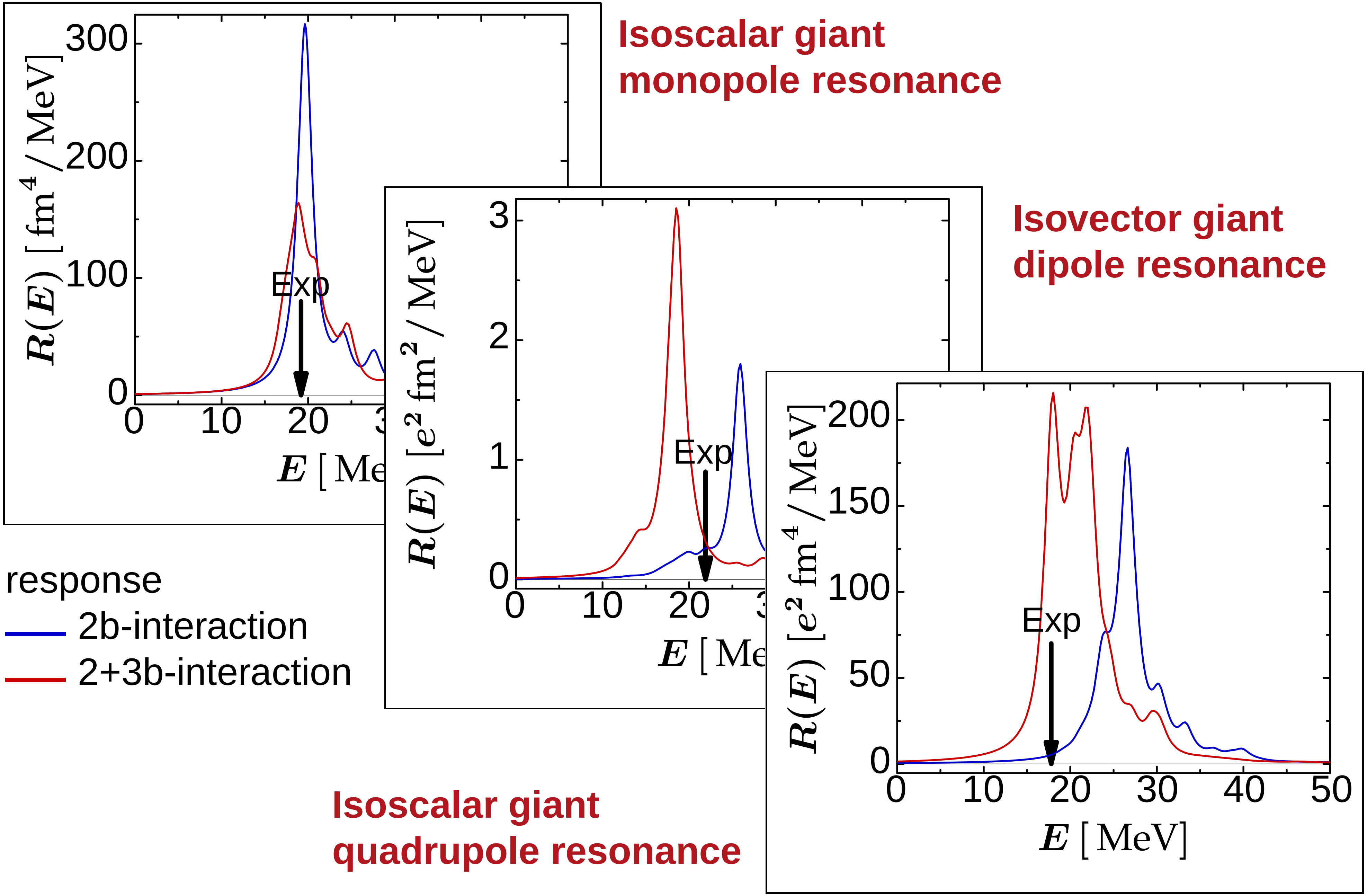
Isoscalar giant quadrupole resonance

response

—— 2b-interaction

→ HK 32.9

Collective Excitations: ^{40}Ca



Isoscalar giant monopole resonance

Isovector giant dipole resonance

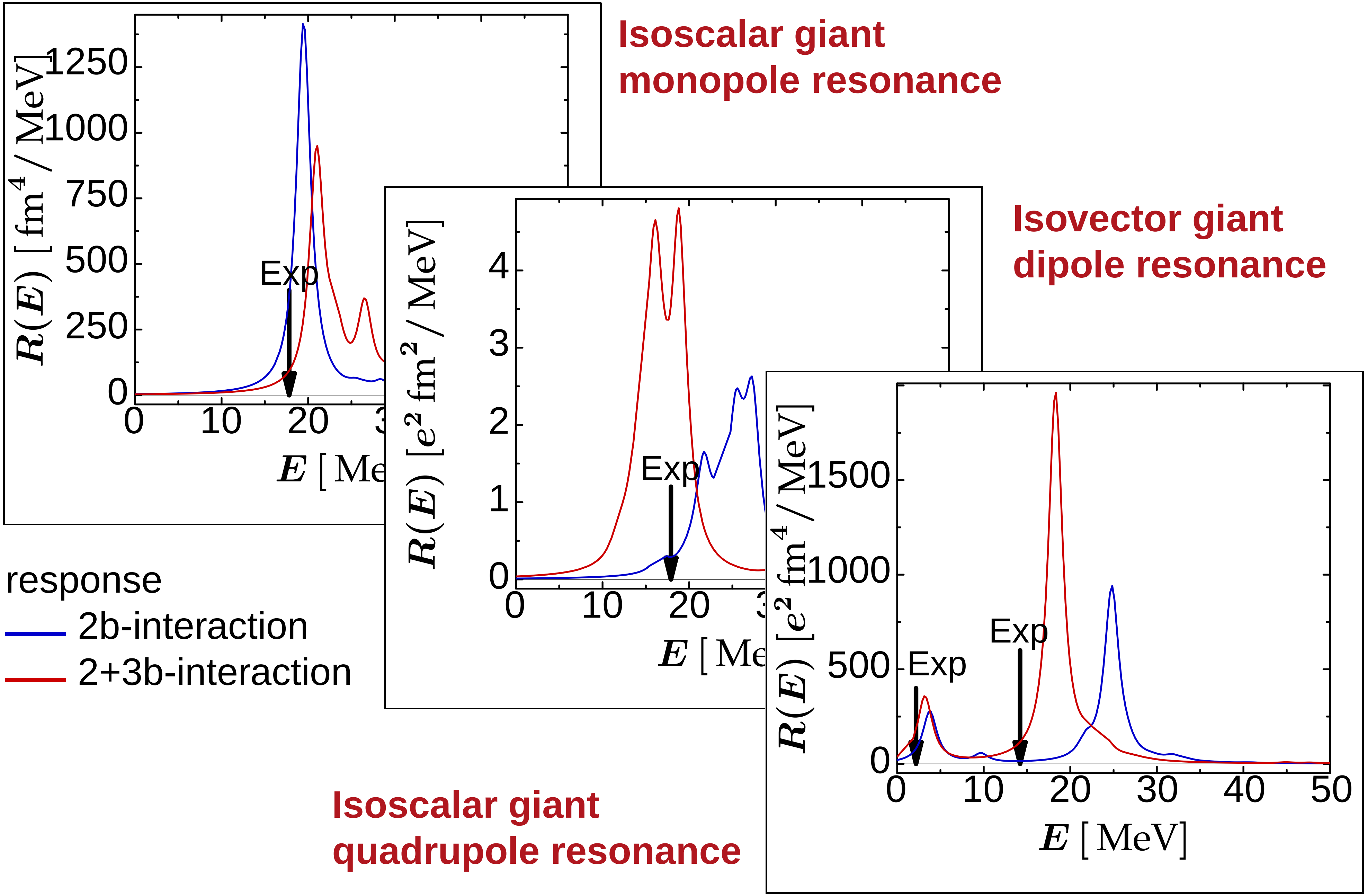
Isoscalar giant quadrupole resonance

response
- 2b-interaction
- 2+3b-interaction

Collective Excitations: ^{90}Zr

Isoscalar giant monopole resonance

Isovector giant dipole resonance

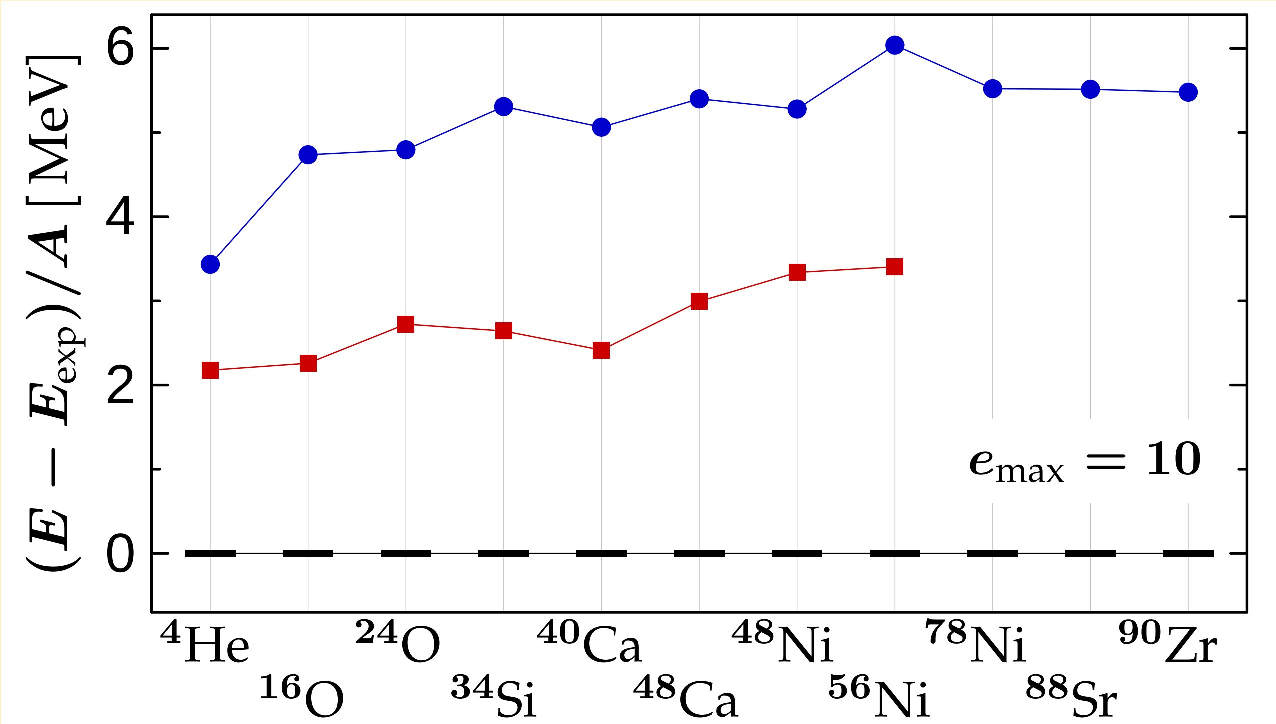


response
- 2b-interaction
- 2+3b-interaction

Isoscalar giant quadrupole resonance

Many-Body Perturbation Theory

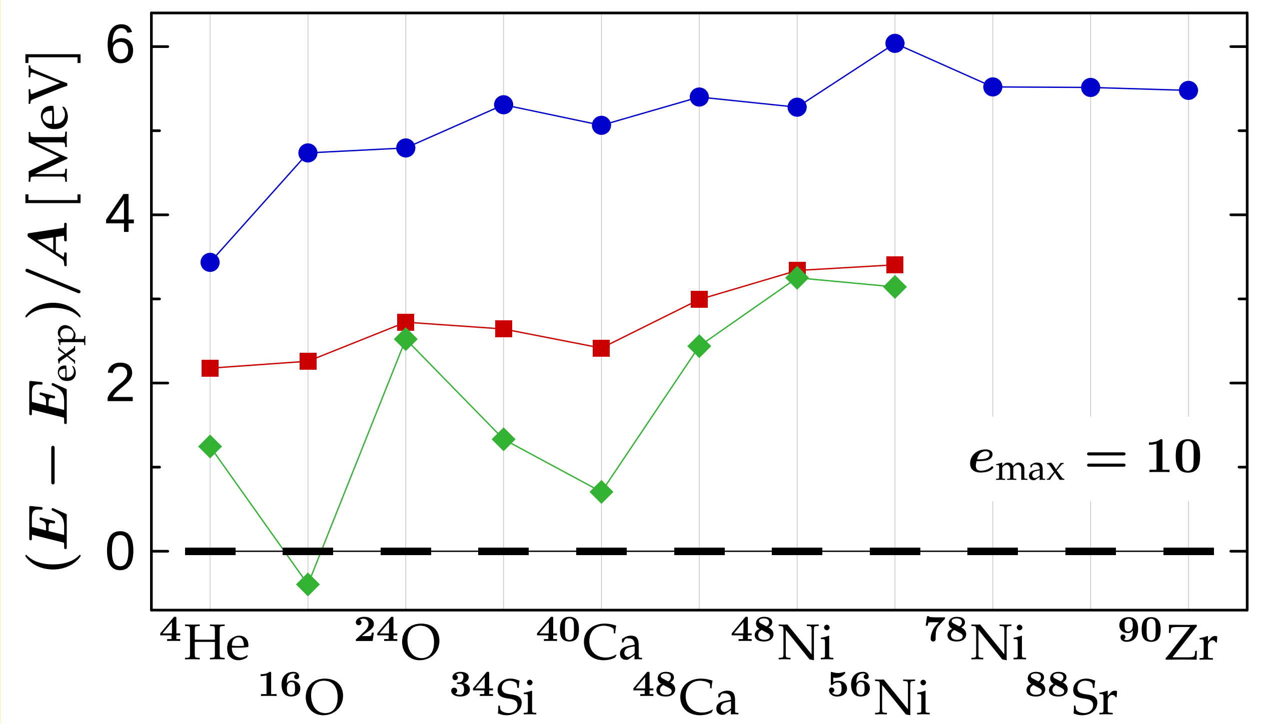
Energy Corrections for the Two-Body Interaction



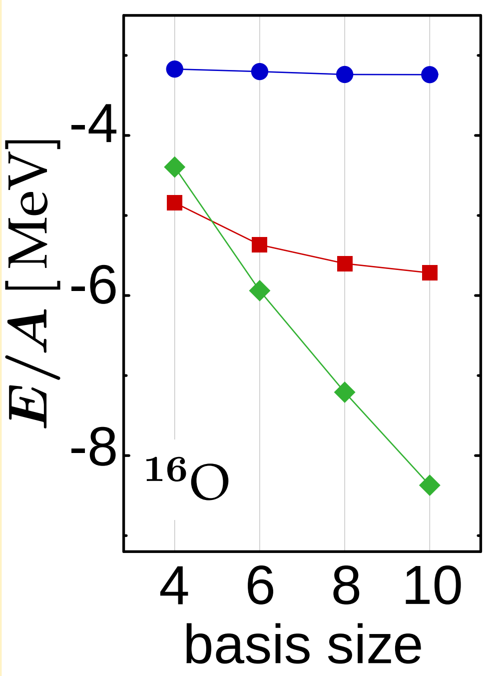
- Hartree-Fock with 3b-interaction
- perturbation theory for 2b-interaction

Many-Body Perturbation Theory

Energy Corrections for the Two- plus Three-Body Interaction



- Hartree-Fock with 3b-interaction
- perturbation theory for 2b-interaction
- perturbation theory for 2+3b-interaction



- varying energy corrections
- no convergence

$\Rightarrow$ problems of contact interaction

Summary & Outlook

Summary

- three-body contact interaction

$\Rightarrow$ improved results:

- charge radii & charge density distributions
- single-particle spectra
- collective excitations

- problem: contact interaction

Outlook

- renormalize the contact interaction
- finite-range three-body interaction

Epilogue...

My Collaborators

- R. Roth, P. Papakonstantinou, H. Hergert, P. Hedfeld

  Institut für Kernphysik, TU Darmstadt

- T. Neff, H. Feldmeier

  Gesellschaft für Schwerionenforschung (GSI)

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