Bose Gases in Oscillating 1D Superlattices : Response & Quasi-Momentum Structure

Q24.6 (Bosonische Gitter II)

M. Hild F. Schmitt I. Türschmann R. Roth

Institut für Kernphysik • Technische Universität Darmstadt

20.03.2007 / Frühjahrstagung 2007

Overview

- Bose-Hubbard Model & Truncation Scheme
- Time Evolution & Oscillating Lattice Potential
- Linear Response Analysis
- Interference Pattern

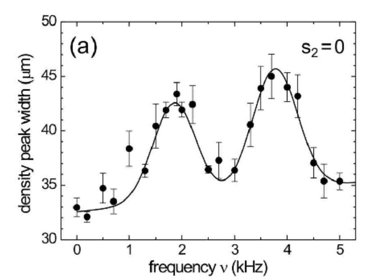
- Results

- Summary

Motivation

- quasi-momentum distribution directly connected to interference pattern
- most observables are measured indirectly via the interference pattern
- modulation experiments: broadening of central interference peak indicates energy transfer

Lattice Oscillation



L. Fallani *et al.*, cond-mat/0603655 v2 (2006)

Framework

Bose-Hubbard Hamiltonian

$$\mathbf{H} = \underbrace{-J \sum_{i=1}^{I} \left(\mathbf{a}_i^\dagger \mathbf{a}_{i+1} + \mathbf{a}_{i+1}^\dagger \mathbf{a}_i \right)}_{\text{hopping } \mathbf{H}_J} + \underbrace{\frac{U}{2} \sum_{i=1}^{I} \mathbf{n}_i \left(\mathbf{n}_i - 1 \right)}_{\text{interaction } \mathbf{H}_U}$$

tunneling strength J interaction strength U sites I particles N

Basis Representation

$$|\Psi\rangle = \sum_{\alpha}^{D} c_\alpha |\{n_1 n_2 \ldots n_I\}_\alpha\rangle$$

- states are defined by coefficients c_α
- coefficients $c_\alpha^{(\nu)}$ of eigenstates $|\nu\rangle$ are obtained by diagonalisation of Hamilton matrix

Adaptive Basis Truncation

Problem

- basis dimension increases rapidly with number of atoms & lattice-sites

Answer: Basis Truncation

- few number states contribute to low-lying eigenstates
- diagonal elements of Hamiltonian provide estimate for importance of basis states
- relevant number states $|\{n_1\ n_2\ \cdots\ n_I\}_\alpha\rangle$ satisfy the inequality

$$E_{trunc} \geq \langle\{n_1\ n_2\ \cdots\ n_I\}_\alpha|\mathbf{H}|\{n_1\ n_2\ \cdots\ n_I\}_\alpha\rangle$$

  with the truncation energy E_{trunc}

➜ precise description in the vicinity of the Mott insulating phase

Time Evolution & Oscillating Lattice Potential

Probing the Excitation Spectrum by lattice oscillation

Optical Lattice

$$V(x,t) = V_0(x)\left(1+\mathcal{F}\sin(\omega t)\right)$$

amplitude $\mathcal{F}$, frequency ω

$\Rightarrow$

Hubbard Parameters

$$J(t) \approx J_0 \exp\left(-\mathcal{F}\sin(\omega t)\right)$$

$$U(t) \approx U_0 \left(1+\mathcal{F}\sin(\omega t)\right)^{1/4}$$

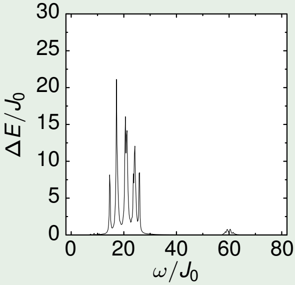
Energy Transfer

Setup

- 10 sites / 10 bosons
- interaction strength $U_0/J_0 = 20$

Time Evolution

1. choose frequency ω/J_0
2. evolve ground state in time
3. evaluate $\Delta E/J_0$ each timestep



Linear Response Analysis

Linearisation of the Hamiltonian

lowest-order terms of a Taylor expansion in the oscillation amplitude $\mathcal{F}$

$$\mathbf{H}_{\text{lin}}(t) = \mathbf{H}_0 + FV_0 \sin(\omega t) \left[\left.\frac{d \ln U}{dV}\right|_{F=0} \mathbf{H}_0 - J \left(\left.\frac{d \ln J}{dV}\right|_{F=0} - \left.\frac{d \ln U}{dV}\right|_{F=0} \right) \mathbf{H}_J \right]$$

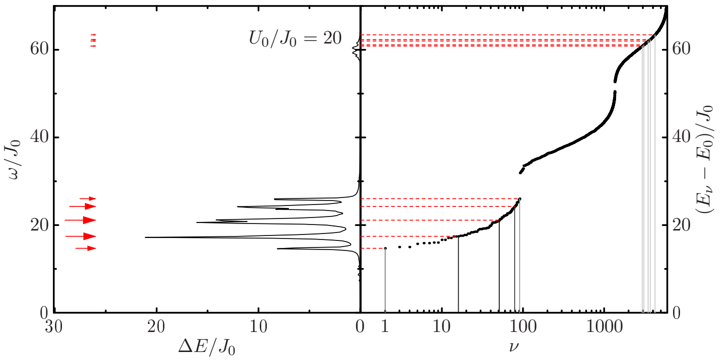
Starting at the Ground State of $\mathbf{H}_0$

☞ look for strong matrixelements $\langle 0|\mathbf{H}_J|\nu\rangle$

K. Braun-Munzinger, PhD thesis, Oxford (2004)
Clark et al., New J. Phys. **8** 160 (2006)
M. Hild et al., J. Phys. B **39** 4547 (2006)

Linear Response Analysis and Time Evolution

10 Bosons / 10 Sites, Interaction Strength $U/J = 20$



- strong matrix elements $\langle 0|\mathbf{H}_J|\nu\rangle$ connect to higher eigenstates
- ☞ prediction of the resonance spectrum and fine-structure

Clark et al., New J. Phys. **8** 160 (2006), M. Hild et al., J. Phys. B **39** 4547 (2006)

Matter Wave Interference Pattern

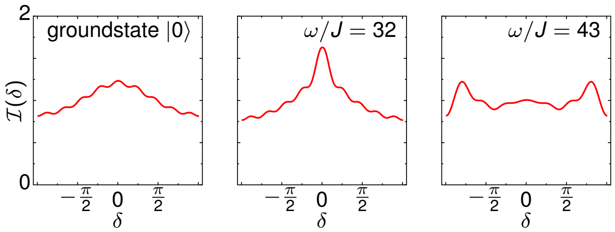
Example: 10 Bosons / 10 Sites, Interaction Strength $U/J = 40$

Intensity as Function of the Relative Phase δ

$$\mathcal{I}(\delta) = \frac{1}{I} \sum_{k,k'}^{I} e^{i(k-k')\delta} \langle\psi|\mathbf{a}_k^\dagger \mathbf{a}_{k'}|\psi\rangle$$

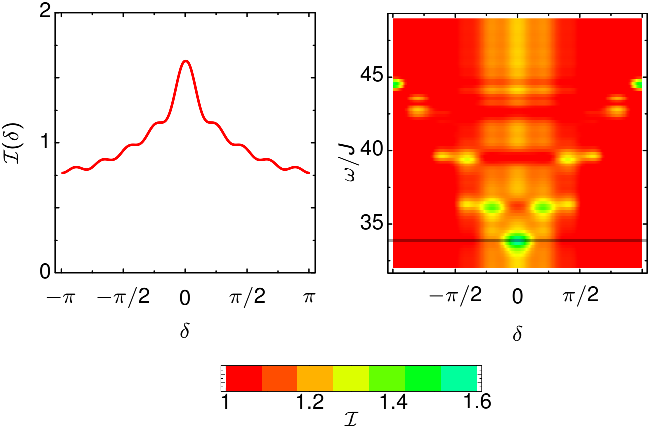
corresponds to occupation numbers n_q of quasi-momenta $q = \frac{\delta I}{2\pi}$

interference pattern is extracted instanteneously (without re-thermalisation)



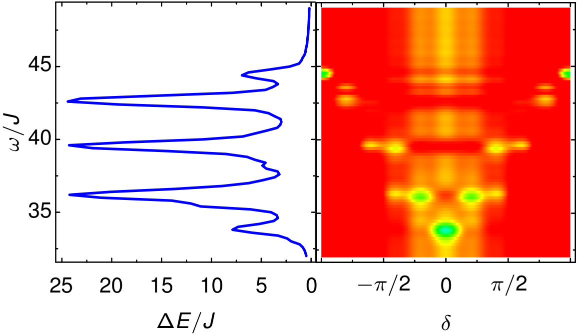
Energy Transfer & Interference Pattern

10 Bosons / 10 Sites, Interaction Strength $U/J = 40$



Energy Transfer & Interference Pattern

10 Bosons / 10 Sites, Interaction Strength $U/J = 40$



Focusing on the 1U-Resonance

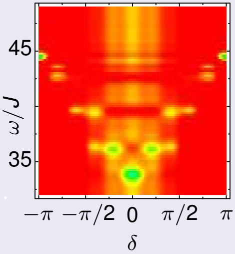
higher oscillation frequencies cause occupation of higher quasi-momenta

Benchmarking Different Basis Truncations

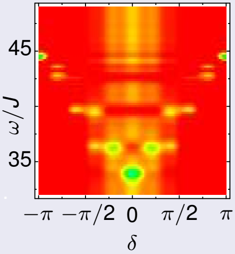
Interference Pattern in the Vicinity of the 1U-Resonance

$N = 10$ bosons, $I = 10$ sites, interaction strength $U/J = 40$

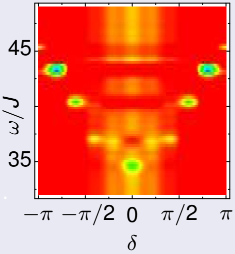
up to 3p3h



up to 2p2h

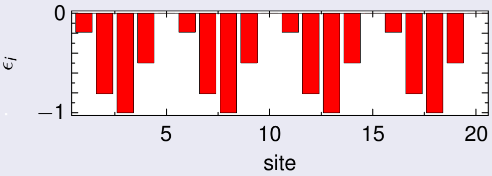


up to 1p1h



- 1p1h-basis shows all features in the deep Mott regime
- ☞ this allows evolution of 20 bosons on 20 sites

Two-Colour Superlattices



- superposition of two optical standing waves
- commensurate wavelengths

Modification of the Hamiltonian

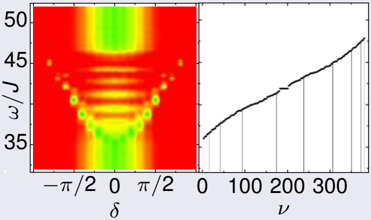
$$\mathbf{H} = \mathbf{H}_{\text{hopping}} + \mathbf{H}_{\text{interaction}} + \Delta \sum_{i=1}^{I} \epsilon_i \, \mathbf{n}_i$$

superlattice modulation amplitude Δ as additional parameter

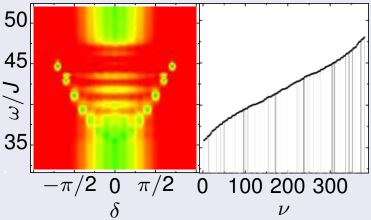
Results I

20 Bosons / 20 Sites, $U/J = 40$, Different Superlattice Amplitudes Δ/J

$\Delta/J = 0$



$\Delta/J = 1$

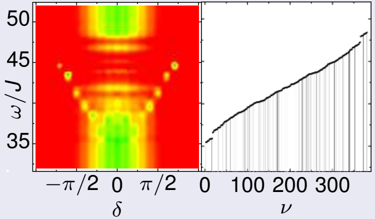


- interference pattern determined by strong matrix elements $\langle 0|\mathbf{H}_J|\nu\rangle$
- emerging superlattice provides more possibilities to connect to higher eigenstates

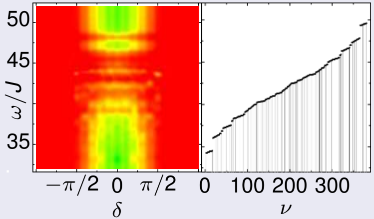
Results II

20 Bosons / 20 Sites, $U/J = 40$, Different Superlattice Amplitudes Δ/J

$\Delta/J = 2$



$\Delta/J = 4 \ll U/J$



- interference structure gets *blurred* but is still visible

- interference structure disappears far below transition to the Bose-Glass phase ($\Delta \approx U$)

Summary

- linear response predicts excitation energies / fine-structure
- resonances: higher modulation frequencies cause occupation of higher quasi-momenta
- stronger superlattice amplitudes cause blurring of the interference pattern
- interference structure disappears far below transition to the Bose-Glass phase