Hartree-Fock and Hartree-Fock-Bogoliubov with Modern Effective Interactions

Heiko Hergert
Institut für Kernphysik, TU Darmstadt
Overview

- Motivation

- Modern Effective Interactions
  - Unitary Correlation Operator Method
  - Similarity Renormalization Group

- Few-Body Systems

- Many-Body Systems
  - Hartree-Fock, Many-Body Perturbation Theory & Beyond
  - Hartree-Fock-Bogoliubov

- Conclusions
Nuclear Structure

- Chiral interactions: consistent NN & 3N interaction derived within $\chi$EFT
- Traditional NN-interactions: Argonne V18, CD Bonn,...
- Reproduce experimental NN phase-shifts with high precision
- Induce strong short-range central & tensor correlations

From QCD to Nuclear Structure
From QCD to Nuclear Structure

Nuclear Structure

Exact / Approx. Many-Body Methods

- ‘exact’ solution of the many-body problem for light and intermediate masses (GFMC, NCSM, CC,...)
- controlled approximations for heavier nuclei (HF & MBPT,...)
- rely on restricted model spaces of tractable size
- not suitable for the description of short-range correlations

Realistic Nuclear Interactions

Low-Energy QCD
From QCD to Nuclear Structure

Nuclear Structure

Exact / Approx. Many-Body Methods

Modern Effective Interactions

Realistic Nuclear Interactions

Low-Energy QCD

- adapt realistic potential to the available model space
  - tame short-range correlations
  - improve convergence behavior

- conserve experimentally constrained properties (phase shifts)
  - generate new realistic interaction

- provide consistent effective interaction & effective operators

- unitary transformations most convenient
Deuteron: Manifestation of Correlations

Argonne V18 Deuteron Solution

\[ M_S = 0 \]
\[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \]

\[ M_S = \pm 1 \]
\[ |\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle \]

\[ \rho^{(2)}_{1, M_S}(\vec{r}) \]

short-range repulsion suppresses wavefunction at small distances \( r \)

central correlations

tensor interaction generates D-wave admixture in the ground state

tensor correlations

Modern Effective Interactions I

Unitary Correlation Operator Method (UCOM)
Correlation Operator
define an unitary operator $C$ to describe the effect of short-range correlations

$$C = \exp[-i G] = \exp[-i \sum_{i<j} g_{ij}]$$

Correlated States
imprint short-range correlations onto uncorrelated many-body states

$$|\tilde{\psi}\rangle = C |\psi\rangle$$

Correlated Operators
adapt Hamiltonian and all other observables to uncorrelated many-body space

$$\tilde{O} = C^\dagger O C$$

$$\langle \tilde{\psi} | O | \tilde{\psi}' \rangle = \langle \psi | C^\dagger O C | \psi' \rangle = \langle \psi | \tilde{O} | \psi' \rangle$$
explicit ansatz for the correlation operator motivated by the **physics of short-range central and tensor correlations**

**Central Correlator** \( C_r \)
- radial distance-dependent shift in the relative coordinate of a nucleon pair

\[
g_r = \frac{1}{2} [s(r) \ q_r + q_r \ s(r)]
\]

\[
q_r = \frac{1}{2} [\frac{\mathbf{r}}{r} \cdot \mathbf{q} + \mathbf{q} \cdot \frac{\mathbf{r}}{r}]
\]

**Tensor Correlator** \( C_\Omega \)
- angular shift depending on the orientation of spin and relative coordinate of a nucleon pair

\[
g_\Omega = \frac{3}{2} \vartheta(r) \left( (\mathbf{\sigma}_1 \cdot \mathbf{q}_\Omega)(\mathbf{\sigma}_2 \cdot \mathbf{r}) + (\mathbf{r} \leftrightarrow \mathbf{q}_\Omega) \right)
\]

\[
\mathbf{q}_\Omega = \mathbf{q} - \frac{\mathbf{r}}{r} q_r
\]

- \( s(r) \) and \( \vartheta(r) \) for given potential determined by energy minimization in the two-body system (for each \( S, T \))
Correlated States: The Deuteron

\[ L = 0 \]

\[ L = 0 \]

\[ L = 2 \]

\[ s(r) \]

\[ \vartheta(r) \]

central correlations

tensor correlations

only short-range tensor correlations treated by \( C_\Omega \)
Correlated Interaction: $V_{UCOM}$

$^3S_1$

$^3S_1 - ^3D_1$

$V_{AV18}$

pre-diagonalization of Hamiltonian

$V_{UCOM}$

Modern Effective Interactions II

Similarity Renormalization Group (SRG)
unitary transformation of the Hamiltonian to a band-diagonal form with respect to a given uncorrelated many-body basis

Flow Equation for Hamiltonian

- Evolution equation for Hamiltonian

\[ \tilde{H}(\alpha) = C^\dagger(\alpha) H C(\alpha) \quad \rightarrow \quad \frac{d}{d\alpha} \tilde{H}(\alpha) = [\eta(\alpha), \tilde{H}(\alpha)] \]

- Dynamical generator defined as commutator with the operator in whose eigenbasis \(H\) shall be diagonalized

\[ \eta(\alpha) = [T_{\text{int}}, \tilde{H}(\alpha)] \equiv \frac{2B}{2\mu} [\vec{q}^2, \tilde{H}(\alpha)] \]

[Bogner et al., PRC75 061001(R) (2007); Hergert & Roth, PRC75 051001(R) (2007)]
The SRG Generator: A Closer Look

- typical $NN$ interaction operators:
  \[ O_p \in \{1, \vec{\sigma}_1 \cdot \vec{\sigma}_2, \vec{l}^2, \vec{l} \cdot \vec{s}, s_{12}(\vec{r}, \vec{r}'), \ldots \} \otimes \{1, \vec{r}_1 \cdot \vec{r}_2, \ldots \} \]

Radial Kinetic Energy

\[ \eta_r(0) \sim [q_r^2, V] = \sum_p [q_r^2, \nu_p(r)O_p] = \sum_p \left( q_r \nu'_p(r)O_p + O_p \nu'_p(r)q_r \right) \]

Angular Kinetic Energy

\[ \eta_\Omega(0) \sim [\vec{l}^2, V] = [\vec{l}^2, \nu_t(r)s_{12}(\vec{r}, \vec{r}')] = -4i\nu_t(r)s_{12}(\vec{r}, \vec{q}_\Omega) \]

☞ $\eta(0)$ has the same structure as the UCOM generators $g_r$ and $g_\Omega$
SRG Evolution: The Deuteron

Argonne V18

strong off-diagonal contributions

short-range central & tensor correlations

\[ \langle r | \phi_{SRG}^{L=0} \rangle \]

\[ \langle r | \phi_{SRG}^{L=2} \rangle \]
SRG Evolution: The Deuteron

\[ V_{\text{SRG}}(q, q') \]

\[ \langle r | \phi_{\text{SRG}}^{L=0} \rangle \]

\[ \langle r | \phi_{\text{SRG}}^{L=2} \rangle \]

\[ \alpha = 0.0004 \text{ fm}^4 \]

SRG Evolution: The Deuteron

\[ \alpha = 0.0010 \text{ fm}^4 \]
SRG Evolution: The Deuteron

\[ \alpha = 0.0020 \text{ fm}^4 \]

\[ \langle r \mid \phi_{SRG}^{L=0} \rangle \]
\[ \langle r \mid \phi_{SRG}^{L=2} \rangle \]
SRG Evolution: The Deuteron

\[ V_{\text{SRG}}(q, q') \]

\[ 3S_1 - 3D_1 \]

\[ \langle r | \phi_{\text{SRG}}^L=0 \rangle \]

\[ \langle r | \phi_{\text{SRG}}^L=2 \rangle \]

\[ \alpha = 0.0040 \text{ fm}^4 \]
SRG Evolution: The Deuteron

\[ V_{\text{SRG}}(q, q') \]

\[ \langle r | \phi_{\text{SRG}}^{L=0} \rangle \]

\[ \langle r | \phi_{\text{SRG}}^{L=2} \rangle \]

\[ \alpha = 0.0100 \text{ fm}^4 \]
SRG Evolution: The Deuteron

\[ V_{\text{SRG}}(q, q') \]

\[ \langle r \mid \phi_{L=0}^{ \text{SRG} } \rangle \]

\[ \langle r \mid \phi_{L=2}^{ \text{SRG} } \rangle \]

\[ \alpha = 0.0200 \text{ fm}^4 \]
SRG Evolution: The Deuteron

\[ \langle r | \phi_{SRG} \rangle \]

\[ \langle r | \phi_{SRG}^{L=2} \rangle \]

\[ \alpha = 0.0400 \text{ fm}^4 \]
SRG Evolution: The Deuteron

\[ V_{\text{SRG}}(q, q') \]

\[ \alpha = 0.1000 \text{ fm}^4 \]

\[ \langle r \mid \phi_{\text{SRG}}^L=0 \rangle \]

\[ \langle r \mid \phi_{\text{SRG}}^L=2 \rangle \]
SRG Evolution: The Deuteron

\[ \langle r | \phi_{SRG}^{L=0} \rangle \]
\[ \langle r | \phi_{SRG}^{L=2} \rangle \]

\[ \alpha = 0.1000 \text{ fm}^4 \]

suppression of off-diagonal contributions

elimination of short-range correlations
SRG Evolution: The Deuteron

\[ \langle r | \phi_{SRG}^{L=0} \rangle \]
\[ \langle r | \phi_{SRG}^{L=2} \rangle \]

\[ \alpha = 0.1000 \text{ fm}^4 \]

extract UCOM correlation functions \( s(r) \) and \( \vartheta(r) \)
Few-Body Systems
\( ^4\text{He}: \text{Convergence} \)

**AV18**

\[ N_{\text{max}} \]

\[ 0 \quad 2 \quad 4 \]

\[ E_{\text{AV18}} \]

\[ E \quad [\text{MeV}] \]

\[ 20 \quad 40 \quad 60 \quad 80 \]

\[ \hbar \omega \quad [\text{MeV}] \]

**\(^4\text{He}\)**

**\( V_{\text{UCOM}} \)**

\[ \text{residual state-dependent long-range correlations} \]

NCSM code by P. Navrátil [PRC 61, 044001 (2000)]
$^4\text{He}$: Convergence

**AV18**

<table>
<thead>
<tr>
<th>$N_{\text{max}}$</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
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<tr>
<td>$E_{\text{AV18}}$</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>$\omega$ [MeV]</th>
</tr>
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<tbody>
<tr>
<td>20</td>
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</table>

$E$ [MeV]

**$^4\text{He}$**

$V_{\text{UCOM}}$

$E$ [MeV]

NCSM code by P. Navrátil [PRC 61, 044001 (2000)]

omitted higher-order cluster contributions
Three-Body Interactions — Strategies

Correlated Hamiltonian in Many-Body Space

\[ \tilde{H} = C^\dagger (T + V_{NN} + V_{3N}) C \]
\[ = \tilde{T}^{[1]} + (\tilde{T}^{[2]} + \tilde{V}_{NN}^{[2]}) + (\tilde{T}^{[3]} + \tilde{V}_{NN}^{[3]} + \tilde{V}_{3N}^{[3]}) + \cdots \]
\[ = T + V_{UCOM} + V_{UCOM}^{[3]} + \cdots \]

- strategies for treating the three-body contributions:
  1. include full \( V_{UCOM}^{[3]} \) consisting of genuine and induced 3N terms
  2. replace \( V_{UCOM}^{[3]} \) by “phenomenological” three-body force
  3. minimize \( V_{UCOM}^{[3]} \) by proper choice of unitary transformation
Three-Body Interactions — Tjon Line

\[
E(4\text{He}) \text{ vs. } E(3\text{H})
\]

for phase-shift equivalent NN-interactions

\[
E(4\text{He}) \quad [\text{MeV}]
\]

\[
E(3\text{H}) \quad [\text{MeV}]
\]

- Tjon-line: $E(4\text{He})$ vs. $E(3\text{H})$

AV18
Nijm II
Nijm I
CD Bonn

Exp.

$V_{NN} + V_{3N}$

Three-Body Interactions — Tjon Line

-8.6 -8.4 -8.2 -8 -7.8 -7.6

\[ E(3\text{H}) \text{[MeV]} \]

-30 -29 -28 -27 -26 -25 -24

\[ E(4\text{He}) \text{[MeV]} \]

Exp.

CD Bonn

AV18

Nijm II

Nijm I

\textbf{Tjon-line:} \( E(4\text{He}) \) vs. \( E(3\text{H}) \)

for phase-shift equivalent NN-interactions

increasing \( C_\Omega \)-range
Three-Body Interactions — Tjon Line

- **Tjon-line**: $E(^4\text{He})$ vs. $E(^3\text{H})$ for phase-shift equivalent NN-interactions

- Use $\alpha$ / range of $C_\Omega$ to
  - test dependence of $V_\alpha$ or $V_{UCOM}$
  - tune contributions of net 3N force
Three-Body Interactions — Tjon Line

- Tjon-line: $E(^4\text{He})$ vs. $E(^3\text{H})$
  for phase-shift equivalent NN-interactions
  - use $\alpha$ / range of $C_\Omega$ to
    - test dependence of $V_\alpha$ or $V_{UCOM}$
    - tune contributions of net 3N force

$V_{UCOM}$

- min. net 3N force:
  $I_\vartheta = 0.09 \text{ fm}^3$
- with phen. 3N force:
  $I_\vartheta = 0.20 \text{ fm}^3$
Many-Body Systems
Hartree-Fock with $V_{UCOM}$

\[ E = \langle \Psi | H_{int} | \Psi \rangle \]

- $E/A$ [MeV]
- $R_{ch}$ [fm]

Elements:
- $^4\text{He}$
- $^{16}\text{O}$
- $^{34}\text{Si}$
- $^{40}\text{Ca}$
- $^{48}\text{Ni}$
- $^{56}\text{Ni}$
- $^{68}\text{Ni}$
- $^{78}\text{Ni}$
- $^{88}\text{Sr}$
- $^{90}\text{Zr}$
- $^{100}\text{Sn}$
- $^{114}\text{Sn}$
- $^{132}\text{Sn}$
- $^{146}\text{Gd}$
- $^{208}\text{Pb}$

$I_9 [\text{fm}^3]$: 0.09

Experiment
Hartree-Fock with $V_{UCOM}$
Hartree-Fock with SRG Potentials

need $3N$ force for saturation

$E/A$ [MeV]

$R_{ch}$ [fm]

experiment

$\alpha$ [fm$^4$]:  
- Blue circle: 0.025
- Red square: 0.030
- Green diamond: 0.035

$^4$He, $^{16}$O, $^{24}$O, $^{34}$Si, $^{40}$Ca, $^{48}$Ca, $^{48}$Ni, $^{56}$Ni, $^{68}$Ni, $^{78}$Ni, $^{88}$Sr, $^{90}$Zr, $^{100}$Sn, $^{114}$Sn, $^{132}$Sn, $^{146}$Gd, $^{208}$Pb
Hartree-Fock with $V_{UCOM}$

- long-range correlations are missing
Perturbation Theory with $V_{UCOM}$

The diagram shows the energy per nucleon ($E/A$) and the charge radius ($R_{ch}$) for various isotopes as a function of atomic mass number. The isotopes include $^4\text{He}$, $^{16}\text{O}$, $^{24}\text{O}$, $^{34}\text{Si}$, $^{40}\text{Ca}$, $^{48}\text{Ca}$, $^{48}\text{Ni}$, $^{56}\text{Ni}$, $^{68}\text{Ni}$, $^{78}\text{Ni}$, $^{88}\text{Sr}$, $^{90}\text{Zr}$, $^{100}\text{Sn}$, $^{114}\text{Sn}$, $^{132}\text{Sn}$, $^{146}\text{Gd}$, and $^{208}\text{Pb}$.

The graph compares experimental data (black line) with calculations using Hartree-Fock (HF) and Hartree-Fock plus Perturbation Theory (HF + PT2) models. The energy per nucleon is given in MeV, and the charge radius is given in fm.
RPA Ring Summation with $V_{UCOM}$

- Indication for $3N$ force
- Long-range correlations are perturbative
- Tractable within PT, BHF, CC, RPA, ...

The diagram shows the energy per nucleon ($E/A$) and charge radius ($R_{ch}$) for various nuclei, comparing experimental data with different theoretical models (HF, HF + PT2, HF + ring sum). The nuclei considered range from $^4$He to $^{208}$Pb.
3N Forces: Energies & Radii

\[ \frac{E}{A} \text{ [MeV]} \]

\[ R_{ch} \text{ [fm]} \]

\[ (I_\theta \text{ [fm}^3], C_{3N} \text{ [GeV fm}^6]) : \quad (0.09, -) \]
3N Forces: Energies & Radii

![Graph showing the relationship between energy per particle (E/A) and charge radius (R_{ch}) for various isotopes.](image)

- **E/A [MeV]**
- **R_{ch} [fm]**

Isotopes: ²⁴⁰Ca, ³⁴⁰Si, ⁴⁰⁰Ca, ⁴⁸⁰Ni, ⁵⁶⁰Ni, ⁶⁸⁰Ni, ⁷⁸⁰Sr, ⁹⁰⁰Zr, ¹⁰⁰⁰Sn, ¹¹⁴⁰Sn, ¹³²⁰Sn, ¹⁴⁶⁰Gd, ²⁰⁸⁰Pb

- **Experimental data** (I₉ [fm³], C₃N [GeV fm⁶]):
  - Blue circle: (0.09, -)
  - Red square: (0.20, 1.5)
  - Green diamond: (0.20, 2.5)
# 3N Forces: HF Single-Particle Energies

![Graph showing single-particle energies for protons and neutrons](image)

- **$\epsilon [\text{MeV}]$:**
  - Protons: $-40, -30, -20, -10, 0$
  - Neutrons: $-40, -30, -20, -10, 0$

- **$I_\theta [\text{fm}^3]$:**
  - Protons: $0.09, 0.20, 0.20, \text{Exp.}$
  - Neutrons: $0.09, 0.20, 0.20, \text{Exp.}$

- **$C_{3N} [\text{GeV fm}^6]$:**
  - Protons: $-1.5, 2.5$
  - Neutrons: $1.5, 2.5$

---

**Notes:**

- Occupied states are represented by solid lines.
- Unoccupied states are represented by dashed lines.

---

Forces: HF Single-Particle Energies

\( \varepsilon \) [MeV]

\( I_\theta \) [fm\(^3\)]

\( C_{3N} \) [GeV fm\(^6\)]

0

0.09

0.20

0.20

Exp.

0.09

0.20

1.5

2.5

Occupied

Unoccupied

\( 3N \) force improves level density

208\(^{\text{Pb}}\)

Beyond Hartree-Fock

Hartree-Fock (Mean-Field)

- Perturbation theory, Brueckner-HF, RPA...
- Long-Range Correlations
- Pairing Correlations
- Hartree-Fock-Bogoliubov
Bogoliubov Transformation

\[ \beta_k^\dagger = \sum_q U_{qk} c_q^\dagger + V_{qk} c_q \]
\[ \beta_k = \sum_q U_{qk}^* c_q + V_{qk}^* c_q^\dagger \]

where

\[ \{ \beta_k, \beta_{k'} \} = \{ \beta_k^\dagger, \beta_{k'}^\dagger \} = 0 \]
\[ \{ \beta_k, \beta_{k'}^\dagger \} = \delta_{kk'} \]

HFB Densities & Fields

\[ \rho_{kk'} \equiv \langle \Psi | c_{k'}^\dagger c_k | \Psi \rangle = (V^* V^T)_{kk'} \]
\[ \kappa_{kk'} \equiv \langle \Psi | c_k c_{k'} | \Psi \rangle = (V^* U^T)_{kk'} \]
\[ \Gamma_{kk'} = \sum_{qq'} \left( \frac{2}{A} \bar{t}_{rel} + \bar{v} \right)_{kq',k'q} \rho_{qq'} \]
\[ \Delta_{kk'} = \sum_{qq'} \left( \frac{2}{A} \bar{t}_{rel} + \bar{v} \right)_{kk',qq'} \kappa_{qq'} \]

Energy

\[ E[\rho, \kappa, \kappa^*] = \frac{\langle \Psi | \mathcal{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle} \equiv \frac{1}{2} (\text{tr} \Gamma \rho - \text{tr} \Delta \kappa^*) \]

HFB Equations

\[ (\mathcal{H} - \lambda \mathcal{N}) \begin{pmatrix} U \\ V \end{pmatrix} \equiv \begin{pmatrix} \Gamma - \lambda & \Delta \\ -\Delta^* & -\Gamma^* + \lambda \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix} = E \begin{pmatrix} U \\ V \end{pmatrix} \]
Variation of Projected Energy

\[ \delta E(N_0) = \frac{1}{2\pi\langle P_{N_0} \rangle} \int_0^{2\pi} d\phi \left\{ \delta \langle H \rangle_{\phi} - \left( E(N_0) - \langle H \rangle_{\phi} \right) \delta \log \langle e^{i\phi} \rangle \right\} \]

\[ \langle H \rangle_{\phi} \equiv \frac{\langle He^{i\phi} N \rangle}{\langle e^{i\phi} N \rangle} \]

Lipkin-Nogami + PAV

- power series expansion
- expansion coefficients not varied
- indeterminate / numerically unstable at shell closures
- exact PNP after variation

VAP

- higher (but managable) computational effort
- implement with care: subtle cancellations between divergences of direct, exchange, and pairing terms

Structure of HFB equations is preserved by both methods!

Sn Isotopes: Binding & Pairing Energies

\[ E/A \quad [\text{MeV}] \]

\[ E_{\text{pair}} \quad [\text{MeV}] \]

\[ ^{A}\text{Sn} \]

Graph showing the binding and pairing energies for Sn isotopes, with experimental (exp.) and HFB results.
Sn Isotopes: Binding & Pairing Energies

$E/A [\text{MeV}]$

$E_{\text{pair}} [\text{MeV}]$

100 104 108 112 116 120 124 128 132

exp. HFB PAV LN+PAV VAP
Density-Dependent Force: Pairing

- **Linear density dependence**, \( t_0 = \frac{1}{6} C_{3N} \):

  \[
  V_\rho = t_0 \left( 1 + P_\sigma \right) \rho \left( \frac{1}{2} (\vec{r}_1 + \vec{r}_2) \right) \delta^3 (\vec{r}_1 - \vec{r}_2)
  \]

- **Mixed density** for PAV/VAP: \( \rho(\vec{R}) \longrightarrow \rho_{\phi_p,\phi_n}(\vec{R}) \)

- Phenomenological VAP calculations: \( E_{\text{pair}} \sim 10 - 20 \text{ MeV} \)

Density-Dependent Force: Pairing

$E_{\text{pair}}^p$ [MeV] vs. systematics:
- HFB
- PAV
- LN+PAV
- VAP

systematics: more realistic $3N$ force
Conclusions
Modern Effective Interactions

**Status**

- treatment of **short-range central** and **tensor correlations** by unitary transformations:
  - Unitary Correlation Operator Method
  - Similarity Renormalization Group

- **universal phase-shift equivalent** correlated interaction $V_{UCOM}$

**Outlook**

- **connections** between UCOM and SRG
- inclusion & treatment of **3N Forces**, in particular...
- **chiral interactions**
Mean-Field (Hartree-Fock)

- Perturbation theory, Brueckner-HF, RPA, ...
- Long-Range Correlations
- Pairing Correlations
- Hartree-Fock-Bogoliubov

QRPA
Status

- **fully consistent** HFB calculations with particle number projection
- inclusion of $3N$-forces: contact & finite range matrix elements for HF, density-dependent force for HFB, RPA, ...
- Like-particle- & $pn$-QRPA (benchmarked)

Outlook

- $pn$ pairing, Isobaric Analog & Gamow-Teller Resonances
- deformation and symmetry restoration by projection (isospin, parity, angular momentum)
- **caveat**: analytic structure of density-dependent forces is very important in symmetry-projected HFB (J. Dobaczewski, arXiv: 0708.0441)
Modern Effective Interactions

- treatment of short-range central and tensor correlations by unitary transformations: UCOM, SRG, Lee-Suzuki,...

- phase-shift equivalent correlated interaction $V_{UCOM}$

- universal input for...

Innovative Many-Body Methods

- No-Core Shell Model,...

- Importance Truncated NCSM, Coupled Cluster Method,...

- Hartree-Fock plus MBPT, Padé Resummed MBPT, BHF, HFB, RPA,...

- Fermionic Molecular Dynamics,...
My Collaborators

- R. Roth, P. Papakonstantinou, A. Zapp, P. Hedfeld, S. Reinhardt
  Institut für Kernphysik, TU Darmstadt

- T. Neff, H. Feldmeier
  Gesellschaft für Schwerionenforschung (GSI)

- N. Paar
  Department of Physics — Faculty of Science, University of Zagreb, Croatia

References


- http://crunch.ikp.physik.tu-darmstadt.de/tnp/
Appendix
$^{10}$B: Hallmark of a 3$N$ Interaction?
$^{10}$B: Hallmark of a $3N$ Interaction?

$V_{UCOM}$ gives correct level ordering without any $3N$ interaction.
RPA, ERPA & SRPA
+
Matrix Elements of Correlated Realistic Interaction $V_{UCOM}$

- **fully self-consistent RPA** based on the Hartree-Fock orbits using the same $V_{UCOM}$
- recovering sum rules with high precision, spurious center-of-mass mode fully decoupled at $\sim 10$ keV
- **Extended-RPA and Second-RPA** to include effects of ground state correlations and complex configurations
RPA with $V_{\text{UCOM}}$

ISGMR in good agreement with experiment

IVGDR & ISGQR centroid energies too high

need for additional correlations & three-nucleon force

SRPA: Complex Configurations

Complex configurations have significant impact on response.

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**40Ca**

**IVD**

- RPA (full 1p1h space)
- SRPA (full 1p1h+2p2h space)

(11 major shells, $\Gamma = 2$ MeV)
complex configurations have significant impact on response

case study: 

(11 major shells, $\Gamma = 50$ keV)

possibility to investigate fine structure

Outlook: RPA with Three-Body Forces

- long-range tensor correlator & repulsive three-body contact interaction

- systematic improvement of
  - rms-radii
  - single-particle spectra
  - strength distributions

\[ \text{standard } V_{\text{UCOM}} \]

\[ V_{\text{UCOM}} \text{ with long-range tensor & three-body contact interaction} \]