Nuclear Structure with a Finite-Range Three-Body Interaction

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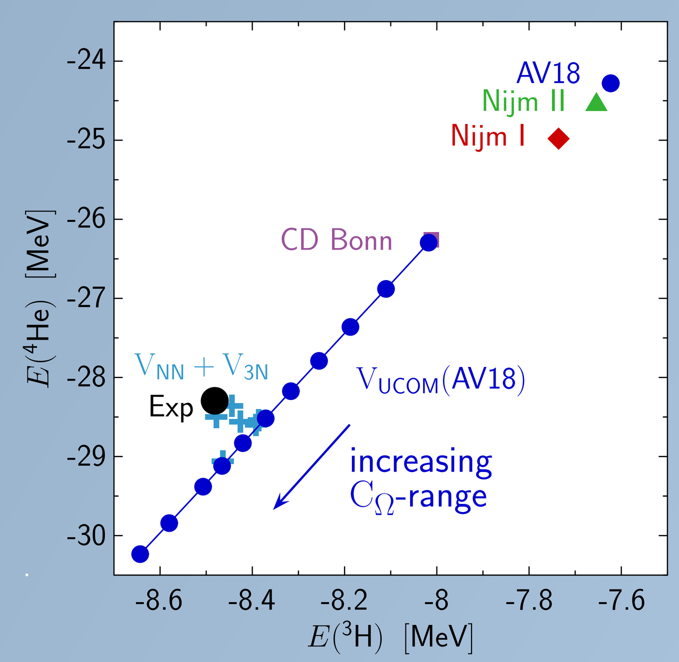
Summary & Motivation

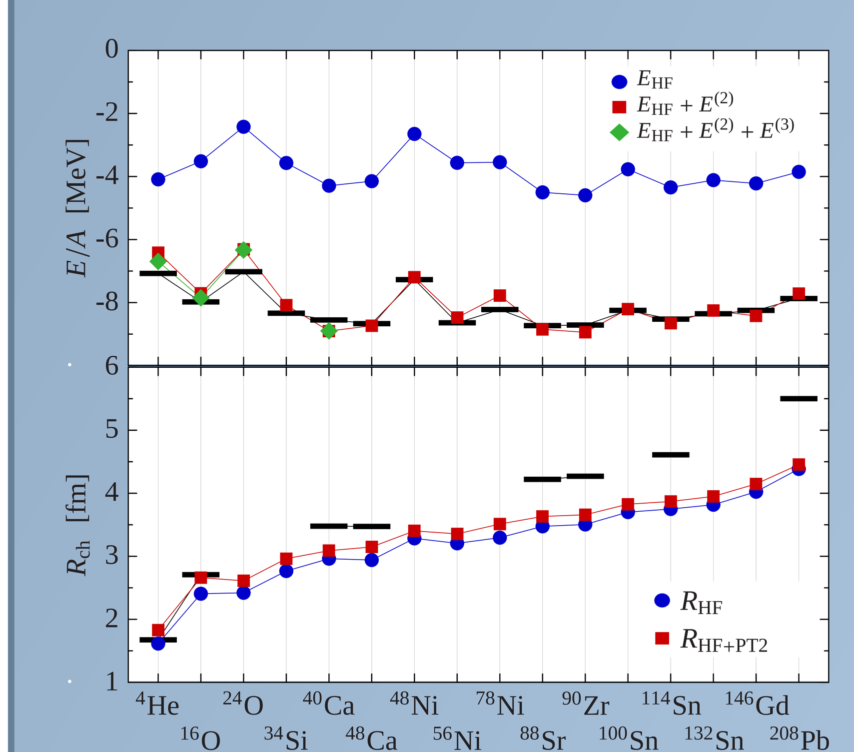
- interest in predictive nuclear structure calculations for stable and exotic nuclei
- solve the nuclear many-body problem, starting from a realistic nuclear interaction
- two problems: construction of a realistic nuclear interaction and solution of quantum many-body problem
- we want to perform nuclear structure calculations across the whole nuclear chart, therefore we incorporate the dominant short-range correlations of the nuclear interaction explicitly by a unitary transformation [1]
- investigations with pure two-body interaction are generally in agreement with experiment [2]
- for further improvement we study the impact of a repulsive three-body interaction
- first investigations with a contact-interaction [3]
- replace contact-interaction by a finite-range three-body interaction which can also be used beyond the mean-field level

Unitary Correlation Operator Method (UCOM)

- the nuclear interaction induces strong central and tensor correlations, the short-range part of these correlations is treated explicitly by a unitary transformation
- transform the Argonne v_{18} potential into a phase-shift equivalent correlated interaction $\mathrm{V}_{\mathrm{UCOM}}$ [1]
- variation of the range of the tensor correlator leads to a shift along the Tjon line
- range parameter fixed to $I_\vartheta^{(10)} = 0.09\,\mathrm{fm}^3$ for calculations with pure two-body interaction



- binding energies are underestimated by Hartree-Fock calculations, since long-range correlations cannot be described [2]



- long-range correlations are included within many-body perturbation theory

⟶ good agreement with experiment

- charge radii are systematically too small, especially for heavier nuclei
- indications for missing three-body forces
- Similarity Renormalization Group (SRG) can be applied to derive correlation functions which improve the Hartree-Fock results [5]

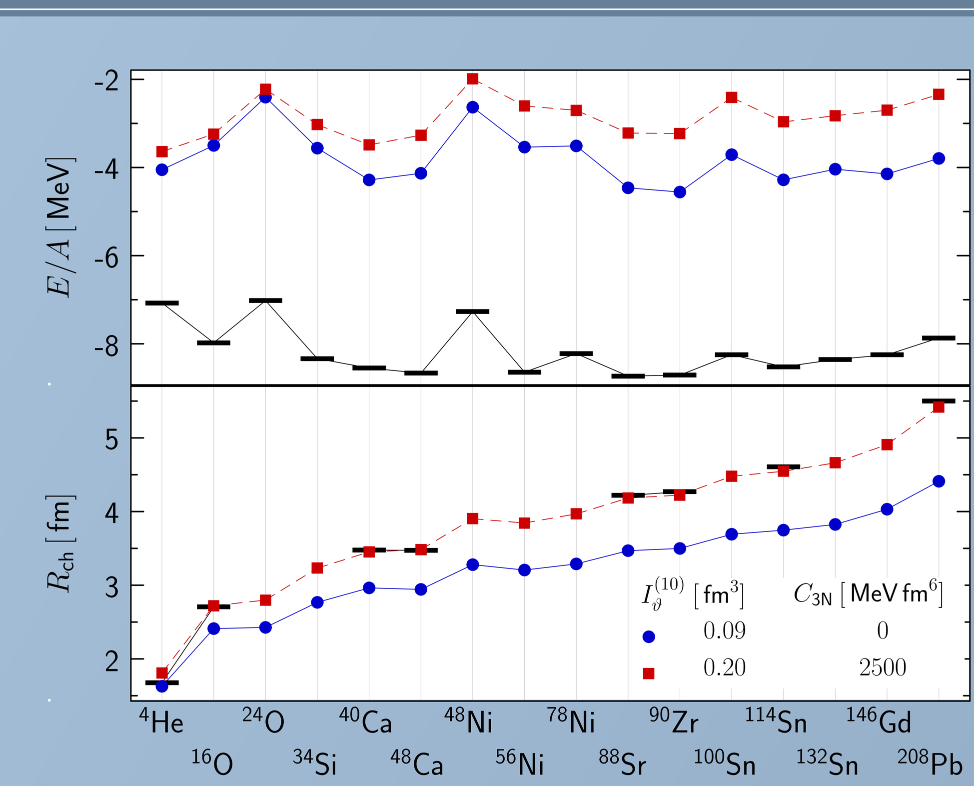
⟶ S. Reinhardt, HK 34.71

Three-Body Contact-Interaction

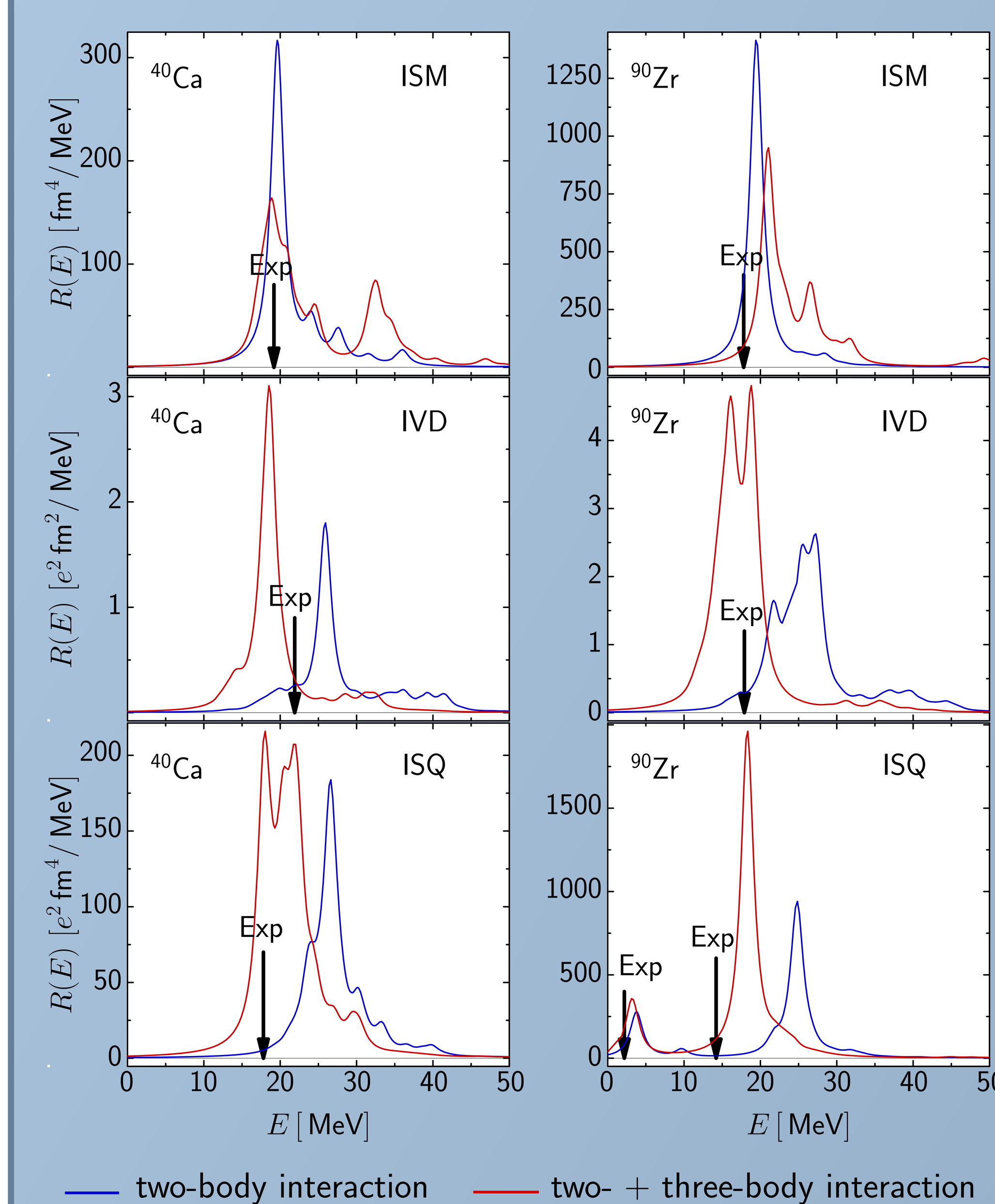
- repulsive three-body interaction
- simplest ansatz: contact-interaction with variable strength $C_{3\mathrm{N}}$

$$\mathrm{V}_{3\mathrm{N}} = C_{3\mathrm{N}}\ \delta^{(3)}(\vec{\mathrm{r}}_1 - \vec{\mathrm{r}}_2)\ \delta^{(3)}(\vec{\mathrm{r}}_1 - \vec{\mathrm{r}}_3)$$

- calculation of matrix-elements in harmonic oscillator basis
- optimal strength of the three-body interaction is determined on the basis of Hartree-Fock calculations: $C_{3\mathrm{N}} = 2500\,\mathrm{MeV\,fm}^6$ [3]



- increase range of the tensor correlator ($I_\vartheta^{(10)} = 0.20\,\mathrm{fm}^3$) to compensate the additional repulsion
- binding energies are again underestimated, similar to the results obtained with the pure two-body interaction while charge radii are well reproduced across the nuclear chart



- study of collective excitations within RPA framework [4]
- isoscalar monopole giant resonance: fragmentation of response function
- isovector dipole and isoscalar quadrupole giant resonances: shift to lower excitation energies
- further improvement can be achieved by increasing the single-particle space
- inclusion of a simple three-body interaction cures some of the discrepancies observed with the pure two-body interaction
- problem of contact-interaction: not suitable for calculations beyond mean-field, need for regulators which complicate the calculations

Finite-Range Three-Body Interaction

- introduce a phenomenological finite-range three-body interaction which is also suitable for calculations beyond the mean-field level
- finite-range three-body interaction with gaussian shape with variable strength $C_{3\mathrm{N}}$ and variable width $a_{3\mathrm{N}}$

$$\mathrm{V}_{3\mathrm{N}} = C_{3\mathrm{N}}\ \exp\left\{ -\frac{1}{a_{3\mathrm{N}}^2}\{(\vec{\mathrm{r}}_1-\vec{\mathrm{r}}_2)^2 + (\vec{\mathrm{r}}_2-\vec{\mathrm{r}}_3)^2 + (\vec{\mathrm{r}}_3-\vec{\mathrm{r}}_1)^2\}\right\}$$

- matrix-elements are calculated in the basis of the cartesian harmonic oscillator
- the full 3-dimensional three-body matrix-element can be separated into a product of three one-dimensional three-body matrix-elements, since the interaction as well as the states can be separated into the three cartesian coordinates
- separation of the full three-body matrix-element:

$$\begin{aligned}&\langle n_{x_1}\, n_{y_1}\, n_{z_1}, n_{x_2}\, n_{y_2}\, n_{z_2}, n_{x_3}\, n_{y_3}\, n_{z_3} | \mathrm{V}_{3\mathrm{N}} | \bar{n}_{x_1}\, \bar{n}_{y_1}\, \bar{n}_{z_1}, \bar{n}_{x_2}\, \bar{n}_{y_2}\, \bar{n}_{z_2}, \bar{n}_{x_3}\, \bar{n}_{y_3}\, \bar{n}_{z_3}\rangle \\ &= \langle n_{x_1}, n_{x_2}, n_{x_3} | \mathrm{V}_{3\mathrm{N}}^{(\mathrm{x})} | \bar{n}_{x_1}, \bar{n}_{x_2}, \bar{n}_{x_3}\rangle \langle n_{y_1}, n_{y_2}, n_{y_3} | \mathrm{V}_{3\mathrm{N}}^{(\mathrm{y})} | \bar{n}_{y_1}, \bar{n}_{y_2}, \bar{n}_{y_3}\rangle \langle n_{z_1}, n_{z_2}, n_{z_3} | \mathrm{V}_{3\mathrm{N}}^{(\mathrm{z})} | \bar{n}_{z_1}, \bar{n}_{z_2}, \bar{n}_{z_3}\rangle\end{aligned}$$

- transform the states of the 3-dimensional cartesian into the spherical harmonic oscillator:

$$\begin{aligned}&\langle n_r, l, m | n_x, n_y, n_z\rangle \\ &= 2\pi^{3/2}(-1)^{n_r} \frac{n_x!\,n_y!\,n_z!\,N_{n_x n_y n_z}}{\Gamma(n_r+l+3/2)\,N_{n_r,l}} \sqrt{\frac{2l+1}{4\pi}(l+m)!(l-m)!} \\ &\times \sum_{a=0}^{n_r} \sum_{p=0}^{\frac{1}{2}(l-m)} \sum_{b=0}^{p} \frac{(-1)^{n_r-n_z-a-b}\, i^{m-n_x}}{p!(p+m)!(l-m-2p)!\,2^{2p+m}} \binom{n_r}{a} \binom{a}{(n_z+2p-l+m)/2} \binom{p}{b} \binom{p+m}{2n_r+2p-2a+m-n_x-b}\end{aligned}$$

- transformation brackets were derived by means of generating functions [6]
- calculation of matrix-elements is time-consuming due to the transformation
- currently development of a suitable storage scheme in order to handle the large number of non-vanishing matrix-elements efficiently

[1] R. Roth et al., *Phys. Rev.* **C 72**, 034002 (2005)
[2] R. Roth et al., *Phys. Rev.* **C 73**, 044312 (2006)
[3] A. Zapp, diploma thesis, TU Darmstadt, 2006
[4] N. Paar et al., *Phys. Rev.* **C 74**, 014318 (2006)
[5] R. Roth et al., nucl-th/0802.4239
[6] L. Chaos-Cador et al., *Int. J. Quantum Chem.* **97**, 844 (2004)