# The Unitary Correlation Operator Method from a Similarity Renormalization Group Perspective

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## Summary


- construct phase-shift equivalent effective nucleon-nucleon interactions with the SRG [1,2]
- SRG evolution leads to generators with the same structure as used in the UCOM
- momentum-space matrix elements confirm the similarities of both approaches
- construct UCOM correlation functions by using SRG evolved interactions [3]
- no-core shell model calculations show good convergence for light nuclei
- realistic systematics of binding energies of heavier nuclei on the Hartree-Fock level


## Unitary Correlation Operator Method (UCOM)

- define unitary operators to describe the effect of short-range correlations [4-6]

$$C_\Omega C_r = \exp\{-\mathrm{i}\Sigma_{i<j} g_{\Omega,ij}\}\exp\{-\mathrm{i}\Sigma_{i<j} g_{r,ij}\}$$

- central correlator $C_r$: radial distance-dependent shift of two nucleons

$$g_r = \frac{1}{2}\left[s(r)\,q_r + q_r\,s(r)\right], \qquad q_r = \frac{1}{2}\left[\frac{\boldsymbol{r}}{r}\boldsymbol{q} + \boldsymbol{q}\frac{\boldsymbol{r}}{r}\right]$$

- tensor correlator $C_\Omega$: angular shift depending on spin-orientation of two nucleons

$$g_\Omega = \vartheta(r)\frac{3}{2}\left[(\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_\Omega)(\boldsymbol{\sigma}_2\cdot\boldsymbol{r}) + (\boldsymbol{\sigma}_1\cdot\boldsymbol{r})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_\Omega)\right], \qquad \boldsymbol{q}_\Omega = \boldsymbol{q} - \frac{\boldsymbol{r}}{r}q_r$$

- parameters to determine: $R_+(r) \approx r + s(r)$ and $\vartheta(r)$ describe strenght and distance dependence of the transformations

## Similarity Renormalization Group (SRG)

- unitary transformation of the Hamiltonian towards a band-diagonal structure through RG flow equation

$$H_\alpha = U_\alpha^\dagger\, H\, U_\alpha \quad \Rightarrow \quad \frac{\mathrm{d}}{\mathrm{d}\alpha}H_\alpha = [\eta_\alpha,\, H_\alpha]$$

- dynamical generator defined as commutator with relative kinetic energy

$$\eta_\alpha = [T_{\mathrm{rel}},\, H_\alpha] = \frac{1}{2\mu}\left[\boldsymbol{q}^2,\, H_\alpha\right]$$

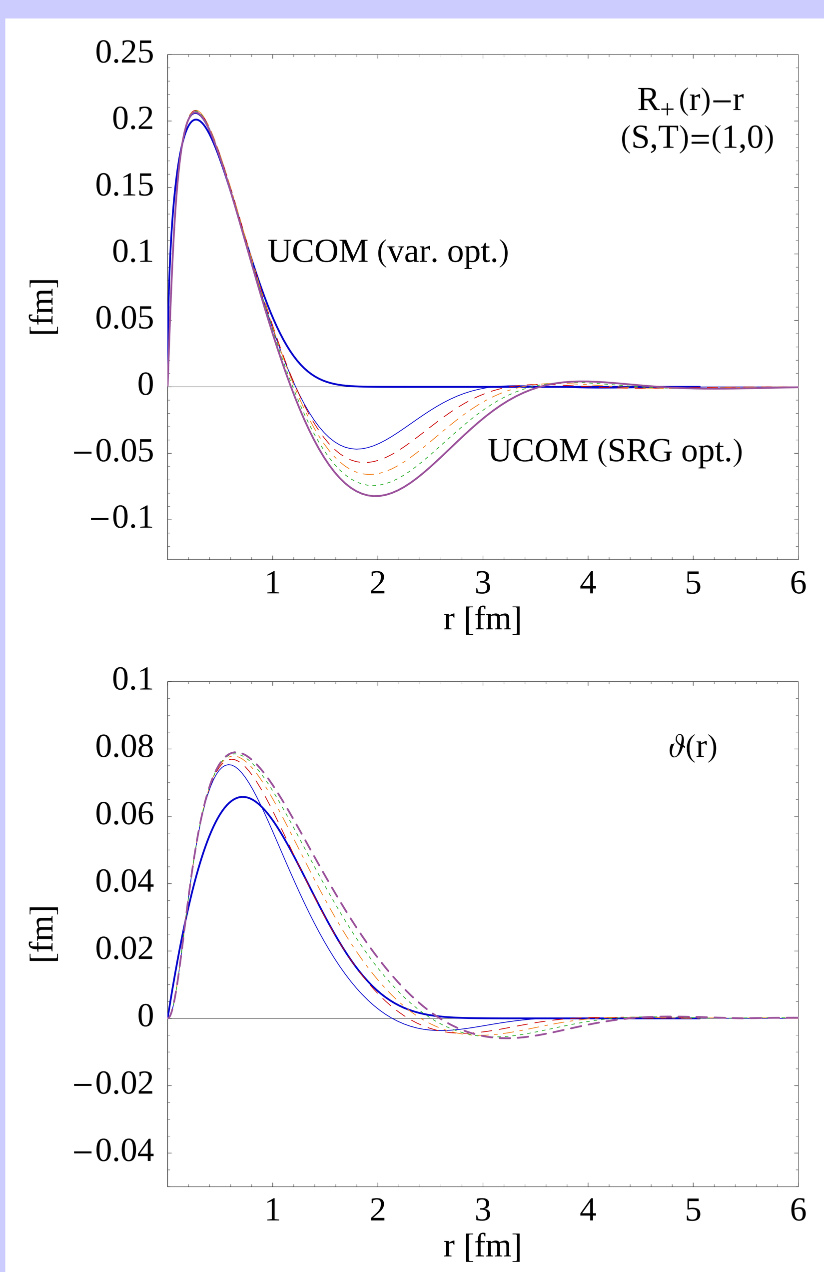
- initial generator with typical $NN$-interaction operators with similar structure as the UCOM generators $g_r$ and $g_\Omega$

$$\eta(0) = \frac{\mathrm{i}}{2}(q_r\,S(r) + S(r)\,q_r) + \mathrm{i}\Theta(r)\,S_{12}(\boldsymbol{r},\,\boldsymbol{q}_\Omega)$$

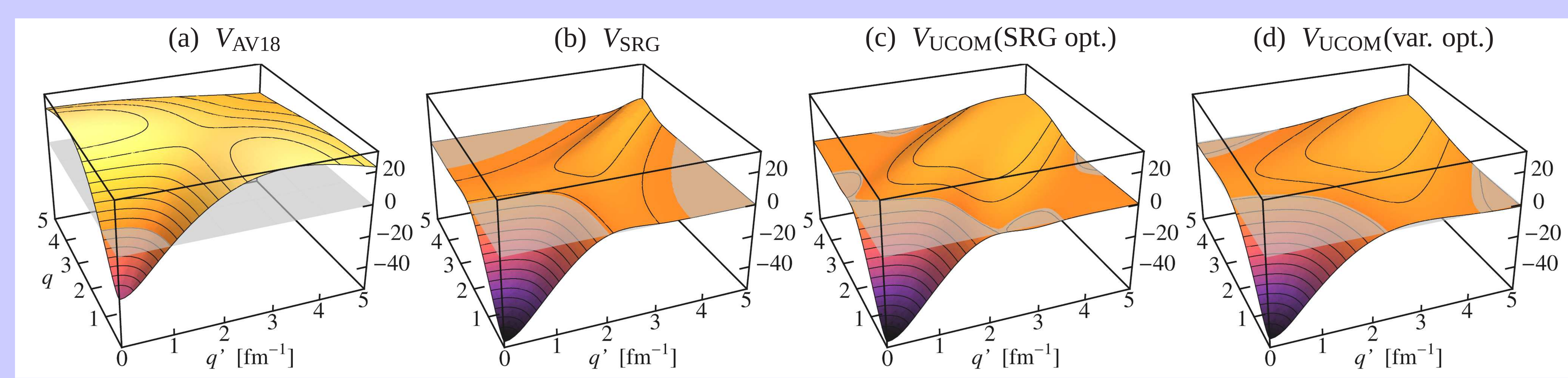
$S(r)$ and $\Theta(r)$: operator valued functions containing radial dependencies

## Extracting UCOM Correlation Functions from SRG Calculations

- SRG confirms that all relevant generators are included in UCOM scheme
- derive UCOM correlators $\vartheta(r)$ and $R_+(r)$ from SRG calculations
- mapping of the SRG-evolved states onto the initial states
  1. Solve SRG flow equation for an initial interaction with a certain value of the flow parameter → matrix elements for each partial wave
  2. Solve two-body problem with these
  3. Map two-body eigenstates of SRG-evolved interaction onto corresponding states of initial interaction → UCOM correlation functions
- UCOM (SRG opt.): $R_+(r) - r$ changes sign, correlators have negative part



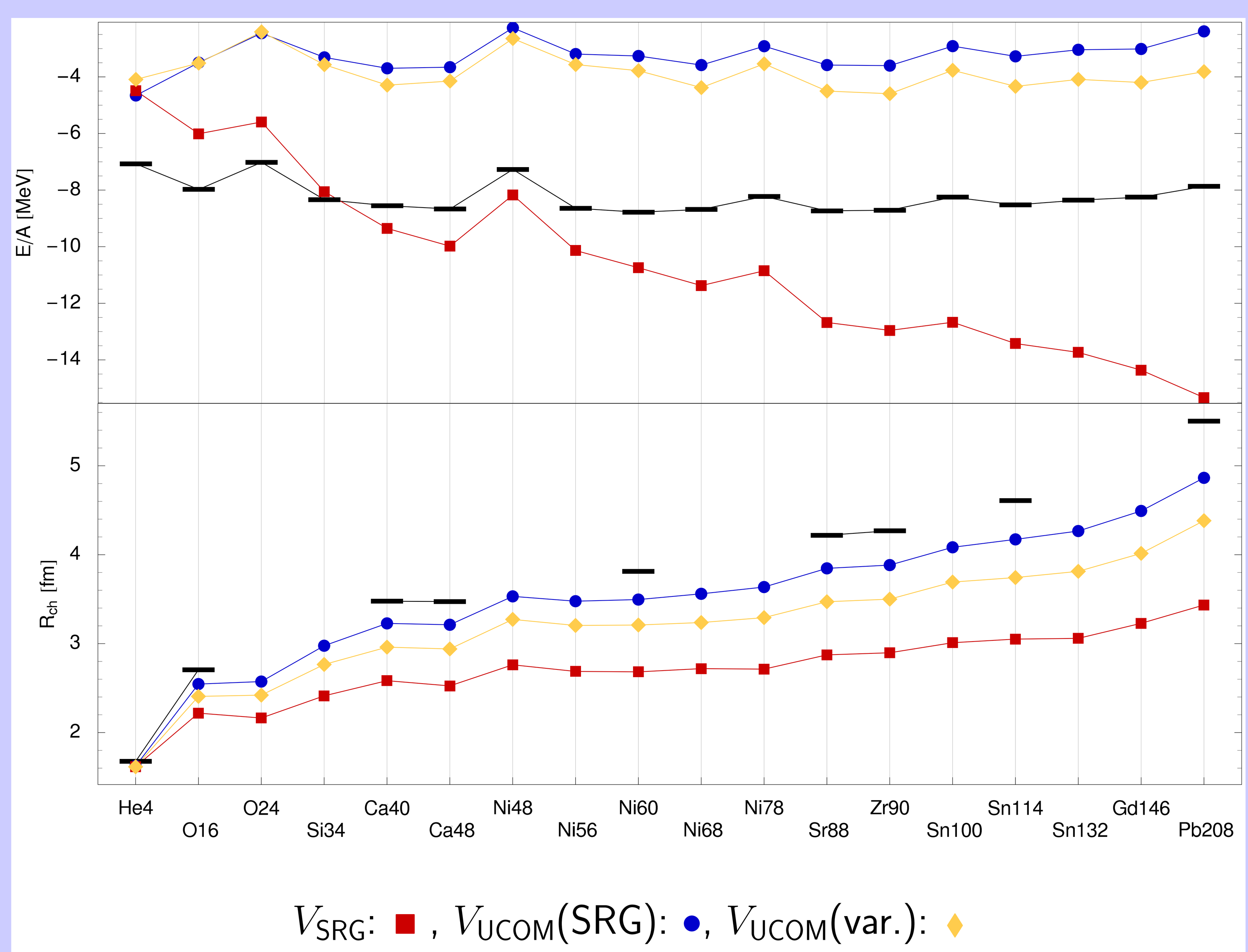

## Momentum-Space Matrix Elements for $^1S_0$ Partial Wave




- strong reduction of off-diagonal matrix elements
- narrow band-diagonal structure for SRG

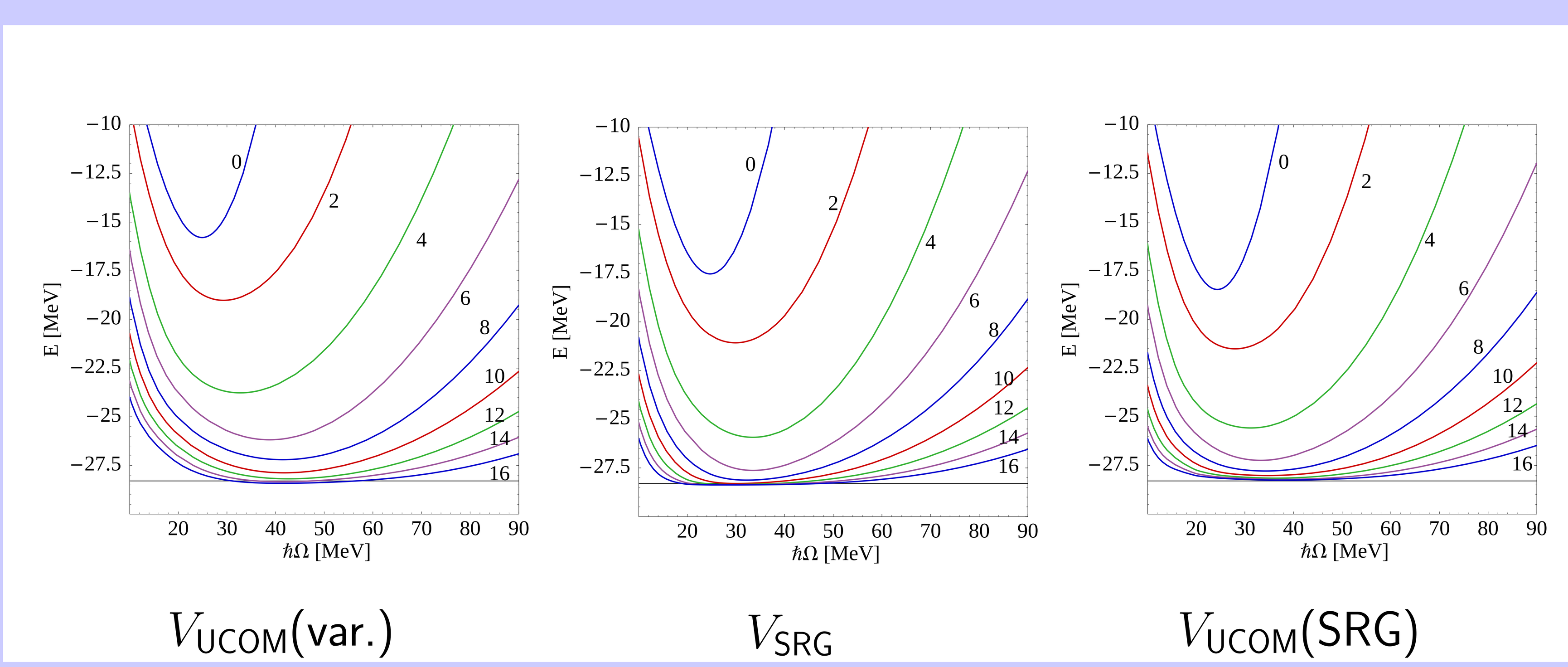
## Hartree-Fock Calculations

- $V_{\mathrm{UCOM}}$(SRG) and $V_{\mathrm{UCOM}}$(var.): realistic trend of binding energies
- $V_{\mathrm{SRG}}$: strong overbinding for heavier nuclei → needs three-body interaction




## No-Core Shell Model Calculations for $^4$He

- $V_{\mathrm{UCOM}}$(SRG) and $V_{\mathrm{SRG}}$: good convergence behavior, better than $V_{\mathrm{UCOM}}$(var.)





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[3] R. Roth, S. Reinhardt, H. Hergert (2008) arXiv:0802.4239v1 [nucl-th]
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