RPA Studies of the Dynamic Response of Ultracold Atoms in 1D Optical Lattices

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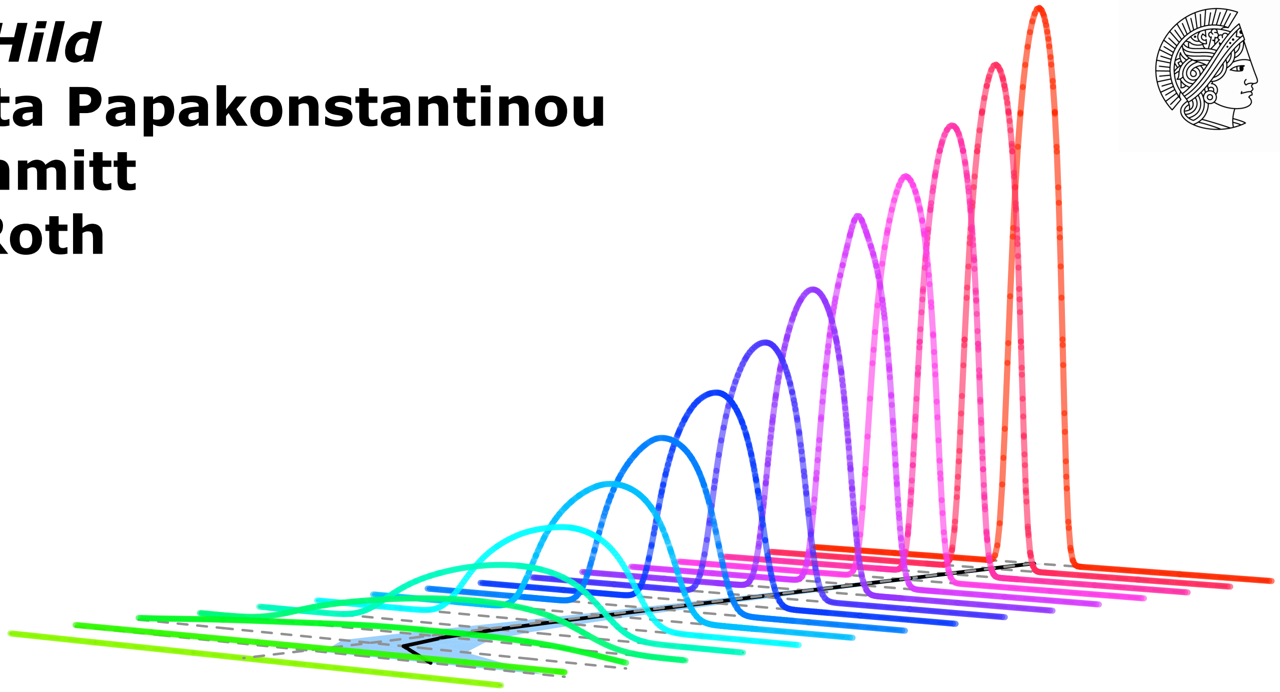

Outline

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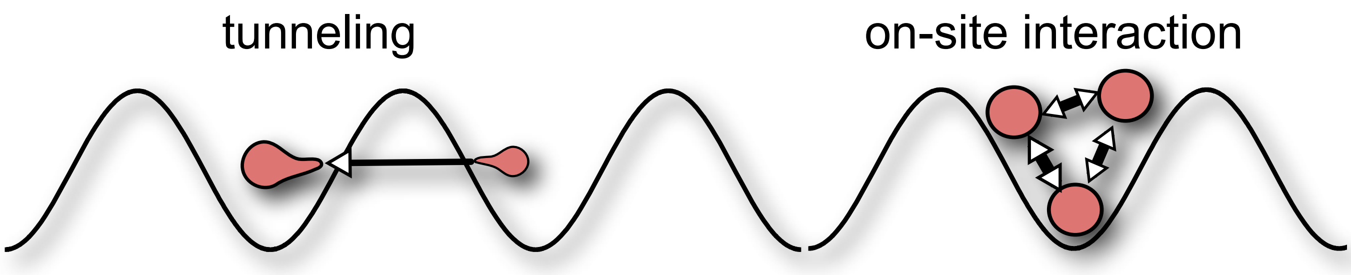
- Bose-Hubbard Model & Spectroscopy
- Equations of Motion (EOM)
- Random Phase Approximation (RPA)...
- ... and the Bose-Hubbard Model
- Results

Bose-Hubbard Model

I sites, N particles, interaction strength U, tunneling strength J

$T = 0$

optical lattice



$$\mathbf{H} = -J \sum_{i=1}^{I} (\mathbf{a}_i^\dagger \mathbf{a}_{i+1} + \mathbf{a}_{i+1}^\dagger \mathbf{a}_i) + \frac{U}{2} \sum_{i=1}^{I} \mathbf{n}_i (\mathbf{n}_i - 1)$$

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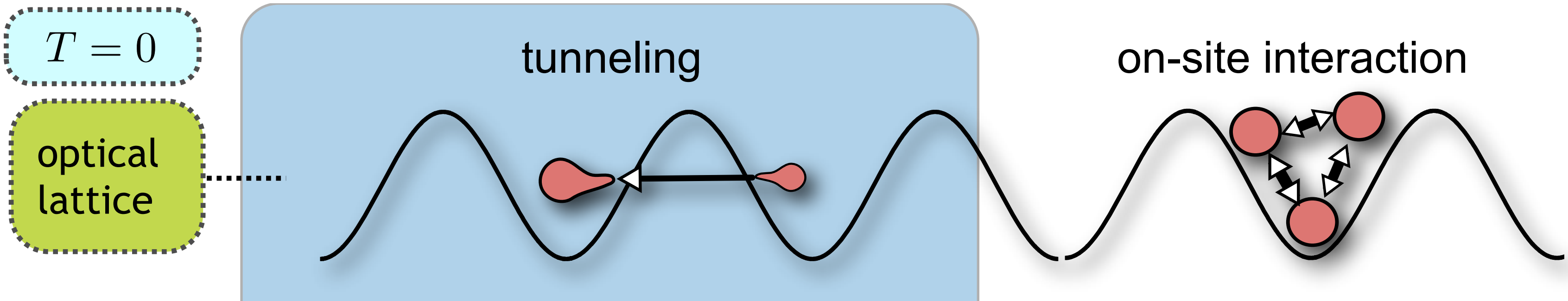

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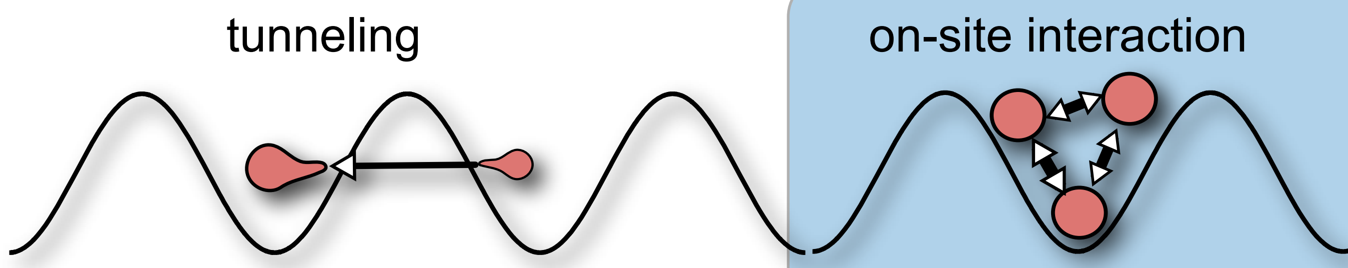

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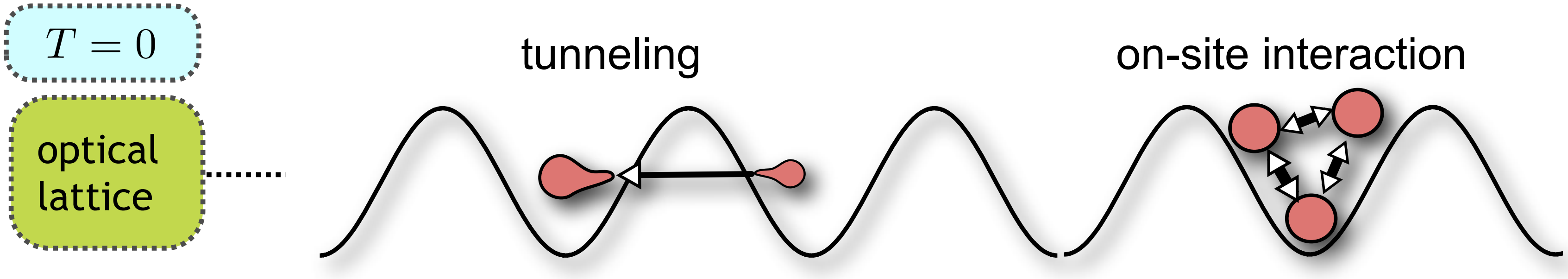

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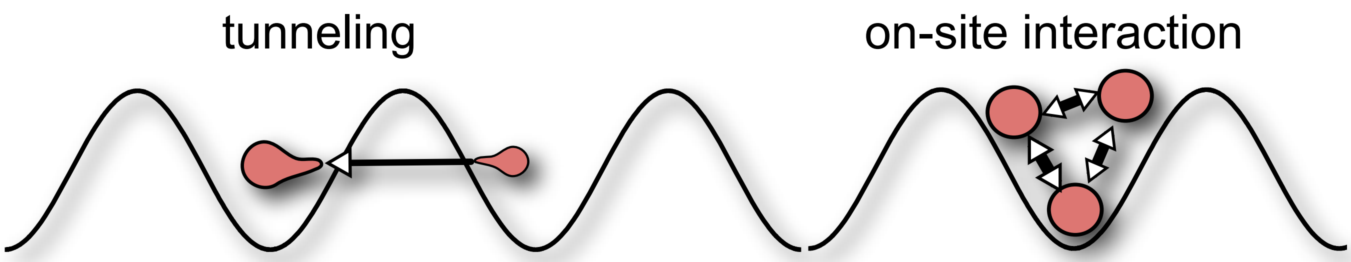

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number-state representation

$$|\phi\rangle = \sum_l c_l^\phi |n_1, ..., n_I\rangle_l, \quad \sum_i n_i = N$$

Spectroscopy via Lattice Modulation

static lattice potential

$$V(x) = V_0 \sin^2(kx)$$

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$$V(x,t) = V_0[1 + F\sin(\omega t)]\sin^2(kx)$$

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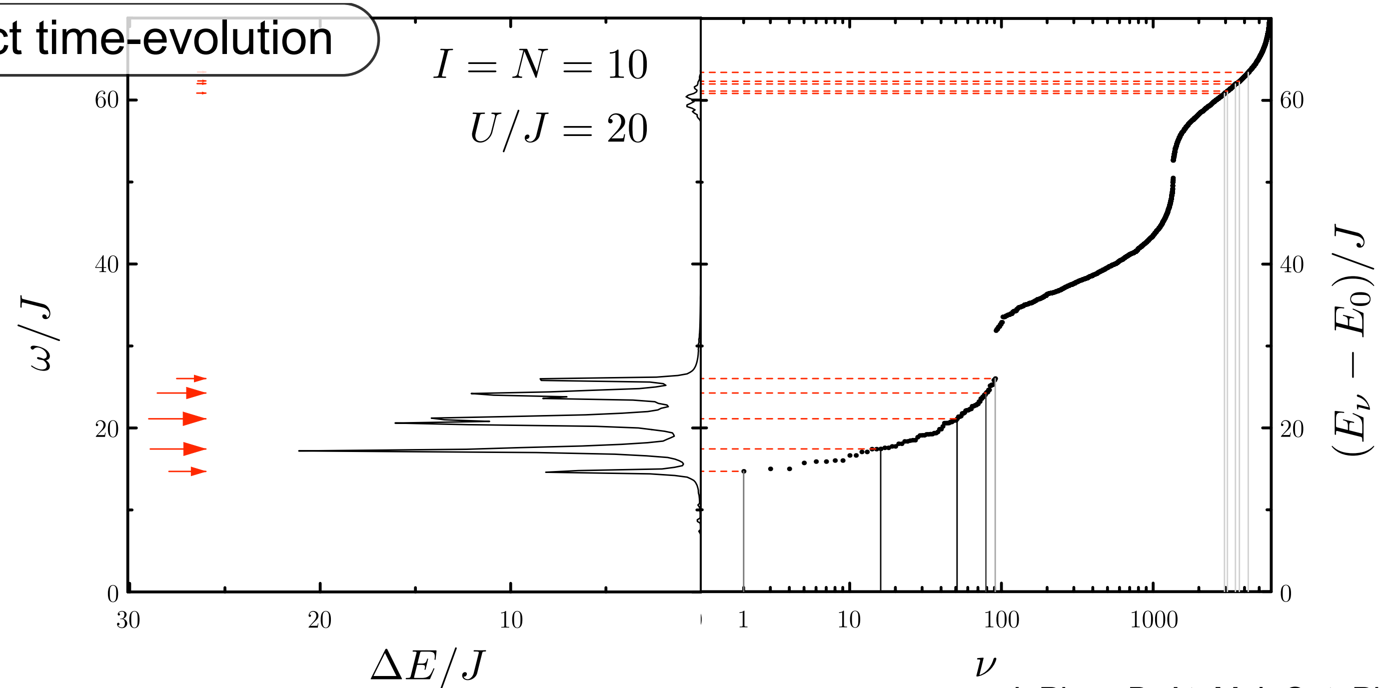
exact time-evolution



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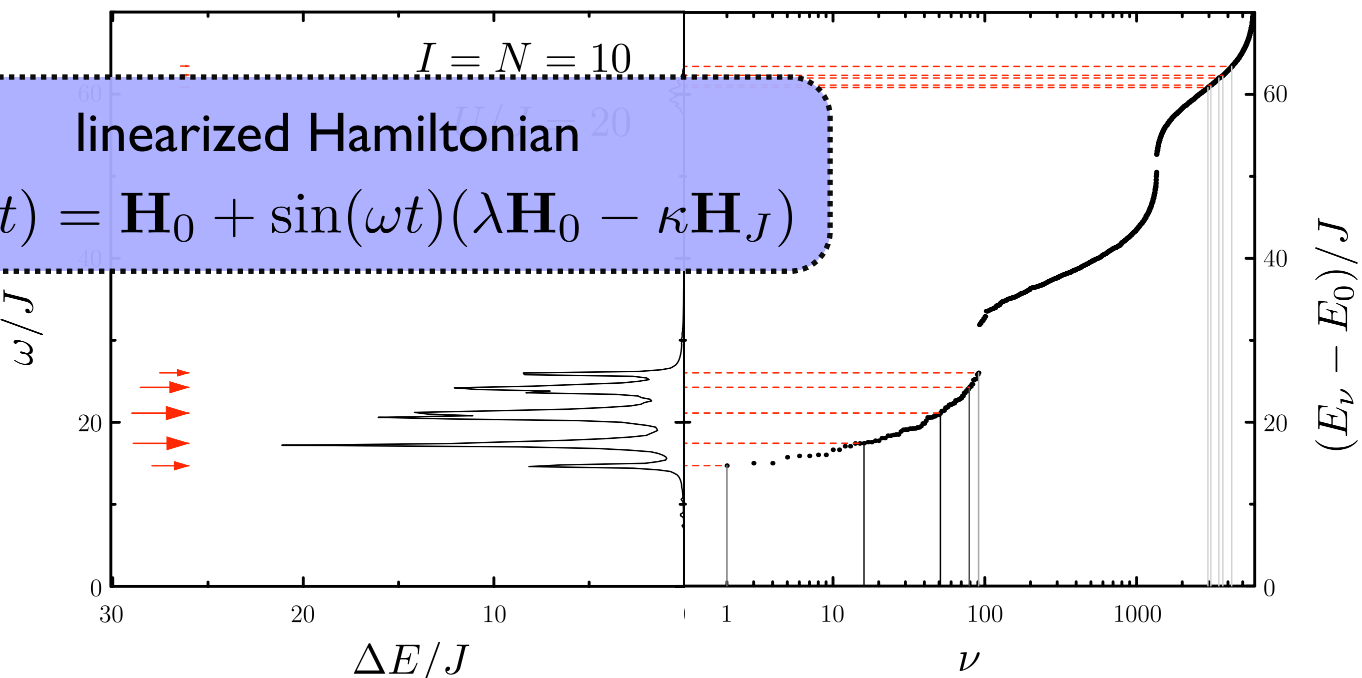
$$\mathbf{H}_{\text{lin}}(t) = \mathbf{H}_0 + \sin(\omega t)(\lambda \mathbf{H}_0 - \kappa \mathbf{H}_J)$$



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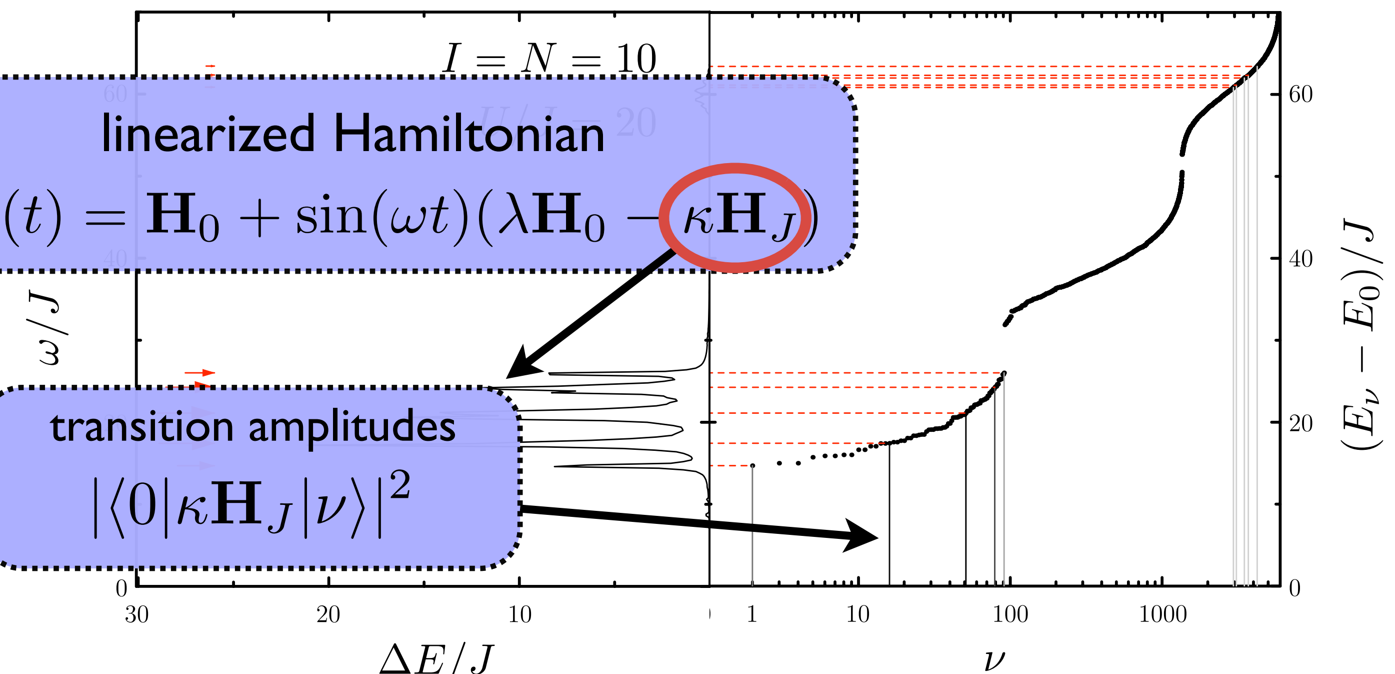
transition amplitudes

$$|\langle 0|\kappa \mathbf{H}_J|\nu\rangle|^2$$



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From the Schrödinger Equation to the Equations of Motion (EOM)

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Schrödinger equation

$$\mathbf{H}|\nu\rangle = E_\nu|\nu\rangle$$

phonon operators

$$\mathbf{Q}_\nu^\dagger|0\rangle = |\nu\rangle$$
$$\mathbf{Q}_\nu|0\rangle = 0$$

variations $\delta\mathrm{Q}$ exhaust the whole Hilbert space

$$\langle 0|[\delta\mathbf{Q},[\mathbf{H},\mathbf{Q}_\nu^\dagger]]|0\rangle = (E_\nu - E_0)\langle 0|[\delta\mathbf{Q},\mathbf{Q}_\nu^\dagger]|0\rangle$$

- **"complicated" reformulation of the Schrödinger equation**
- **requires the groundstate**

- **offers "handles" for approximations: *phonon operators* and *restriction of the variations***

Approximation of the Phonon Operators

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systematic approach

expand phonon operators in terms of particle-hole (de-) excitations $\mathbf{c}_k^\dagger \; (\mathbf{c}_k)$

simplest expansions of phonon operators

Tamm-Dancoff Approximation (TDA)

$$\mathbf{Q}_\nu^\dagger = \sum_k X_k^{(\nu)} \mathbf{c}_k^\dagger$$

1 particle - 1 hole excitations

Random Phase Approximation (RPA)

$$\mathbf{Q}_\nu^\dagger = \sum_k X_k^{(\nu)} \mathbf{c}_k^\dagger - \sum_k Y_k^{(\nu)} \mathbf{c}_k$$

1 particle - 1 hole excitations + de-excitations

→ higher RPAs

Particle-Hole Operators and the Bose-Hubbard Model

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system properties

- filling factor $N/I = 1$
- strongly interacting $U \gg J$

→

ground-state approximation

$$|\mathrm{RPA}, 0\rangle \approx |1, 1, \cdots, 1\rangle$$

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naive ansatz

$$\mathbf{c}^\dagger_{ij} = \mathbf{a}^\dagger_i \mathbf{a}_j$$

$$\mathbf{c}_{ij} = \mathbf{a}^\dagger_j \mathbf{a}_i$$

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just hopping in opposite directions

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Fail

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$$\mathbf{c}_{ij} = \mathbf{a}_j^\dagger \mathbf{a}_i$$

→ Fail

just hopping in opposite directions

$$\mathbf{c}_{ij}^\dagger = \frac{1}{\sqrt{2}} \mathbf{a}_i^\dagger \mathbf{a}_i^\dagger \mathbf{a}_i \mathbf{a}_j$$

→ creates ph-excitation

$$\mathbf{c}_{ij} = \frac{1}{\sqrt{2}} \mathbf{a}_j^\dagger \mathbf{a}_i^\dagger \mathbf{a}_i \mathbf{a}_i$$

→ destroys ph-excitation

Solving RPA

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EOM formulated as generalized eigenproblem

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X_\nu \\ Y_\nu \end{pmatrix} = E_{\nu 0} \begin{pmatrix} S & -T \\ -T & S \end{pmatrix} \begin{pmatrix} X_\nu \\ Y_\nu \end{pmatrix}$$

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$$\begin{aligned} A_{kl} &= \langle 0 | [\mathbf{c}_k, \mathbf{H}, \mathbf{c}_l^\dagger] | 0 \rangle &= A_{lk}, \\ B_{kl} &= -\langle 0 | [\mathbf{c}_k, \mathbf{H}, \mathbf{c}_l] | 0 \rangle &= B_{lk}, \\ S_{kl} &= \langle 0 | [\mathbf{c}_k, \mathbf{c}_l^\dagger] | 0 \rangle &= S_{lk}, \\ T_{kl} &= \langle 0 | [\mathbf{c}_k, \mathbf{c}_l] | 0 \rangle &= -T_{lk} \end{aligned}$$

$$[\mathbf{A}, \mathbf{H}, \mathbf{B}] := \frac{1}{2} \left([\mathbf{A}, [\mathbf{H}, \mathbf{B}]] + [[\mathbf{A}, \mathbf{H}], \mathbf{B}] \right)$$

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- excitation energies $E_{\nu 0}$
- excited states $|E_{\nu_0}\rangle$ via (X_ν, Y_ν)

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strength function

$$R(\omega) = \sum_{(\nu)} \delta(\omega - E_{\nu 0}) \left|\langle 0|\kappa \mathbf{H}_J|E_{\nu_0}\rangle\right|^2$$

Solving RPA

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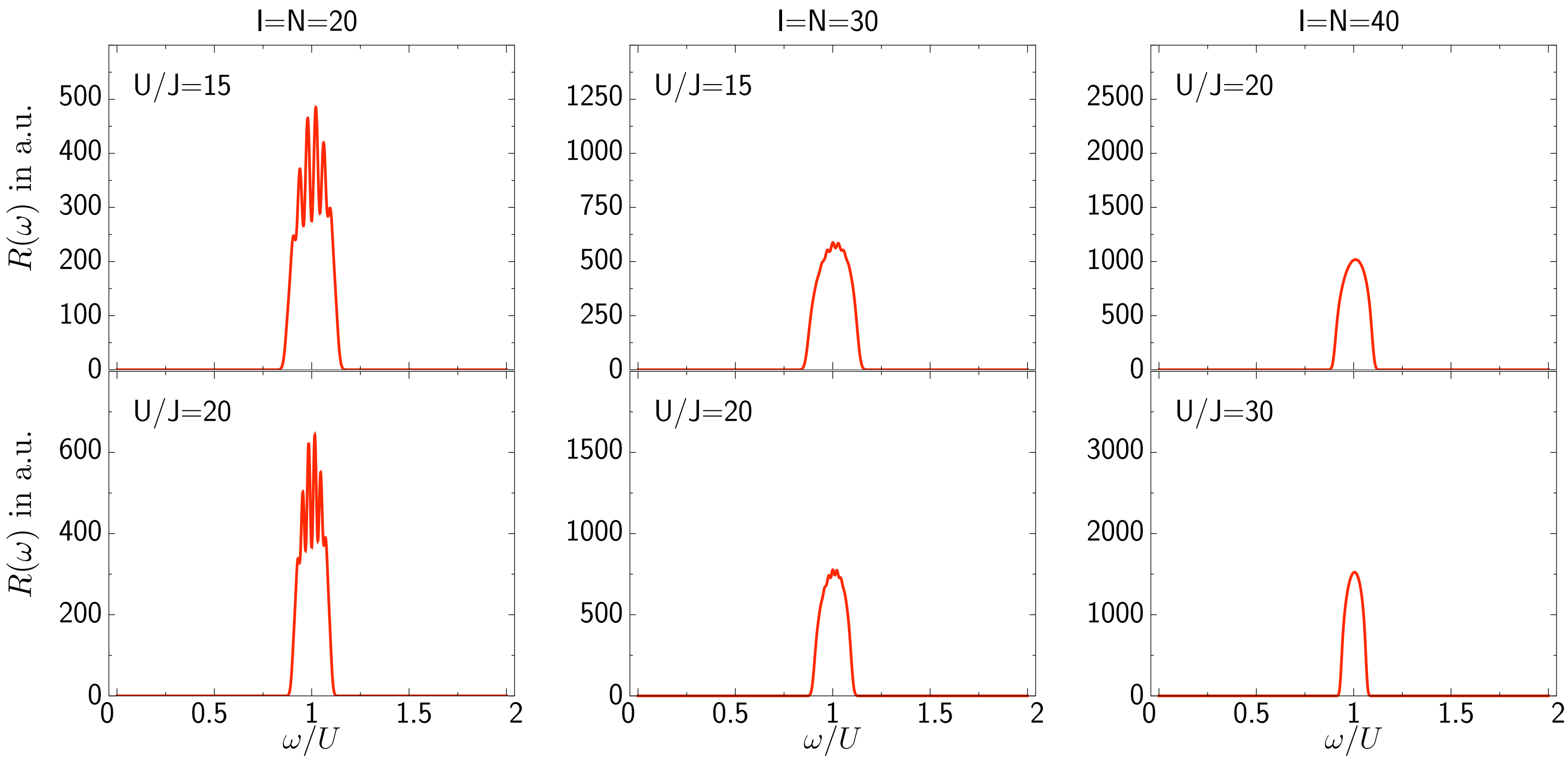
Results I: 1U Resonance Varying the System Size



(——) Random Phase Approximation

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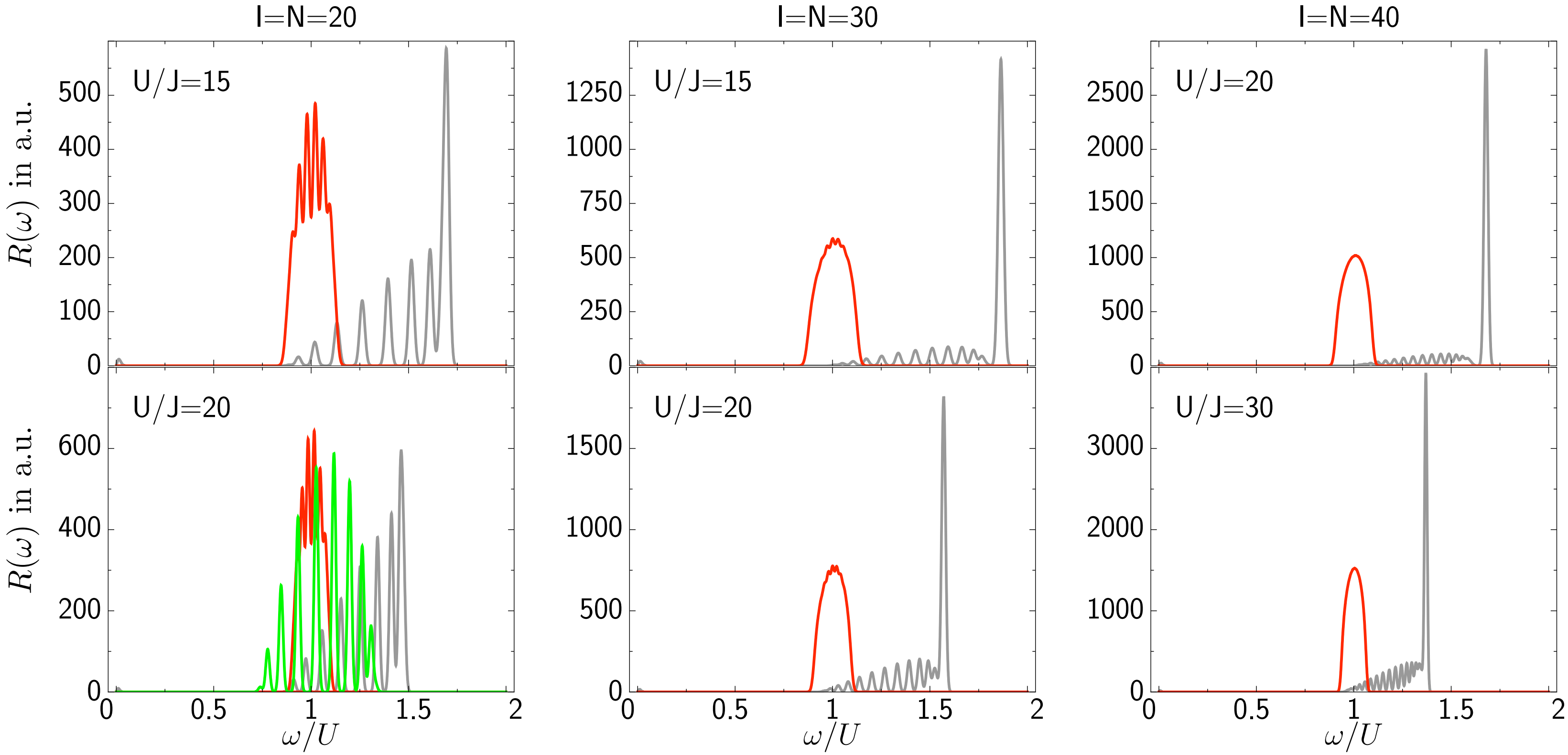
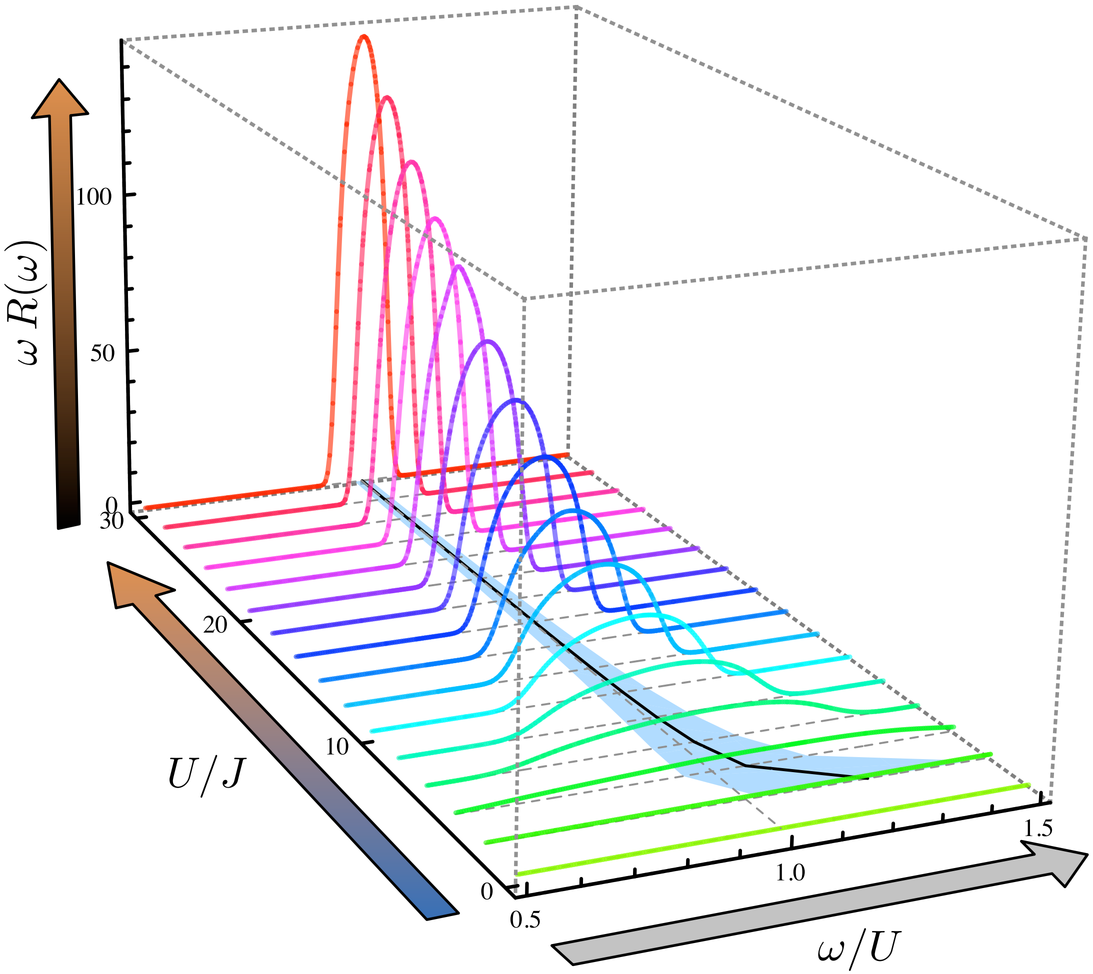
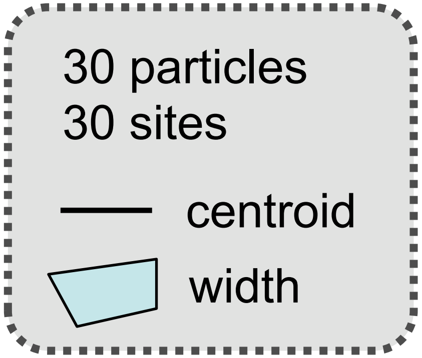


(——) Random Phase Approximation (——) Diagonalisation in ph-space (——) Diagonalisation in $3p3h$-space

Results II: 1U Resonance Varying U/J





- centroid slipping-off $\omega = U$ towards superfluid regime

Conclusion & Outlook

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- RPA offers numerically efficient access to the strength function (solving a single eigenproblem)
- can be easily extended via generalization of the phonon operators
  - adapt to other lattice topologies via additional matrix elements (e.g., superlattice potential)
  - improve on the resonance's width via correlated ground-state