Similarity Renormalization Group for Chiral Two- plus Three-Body Hamiltonians

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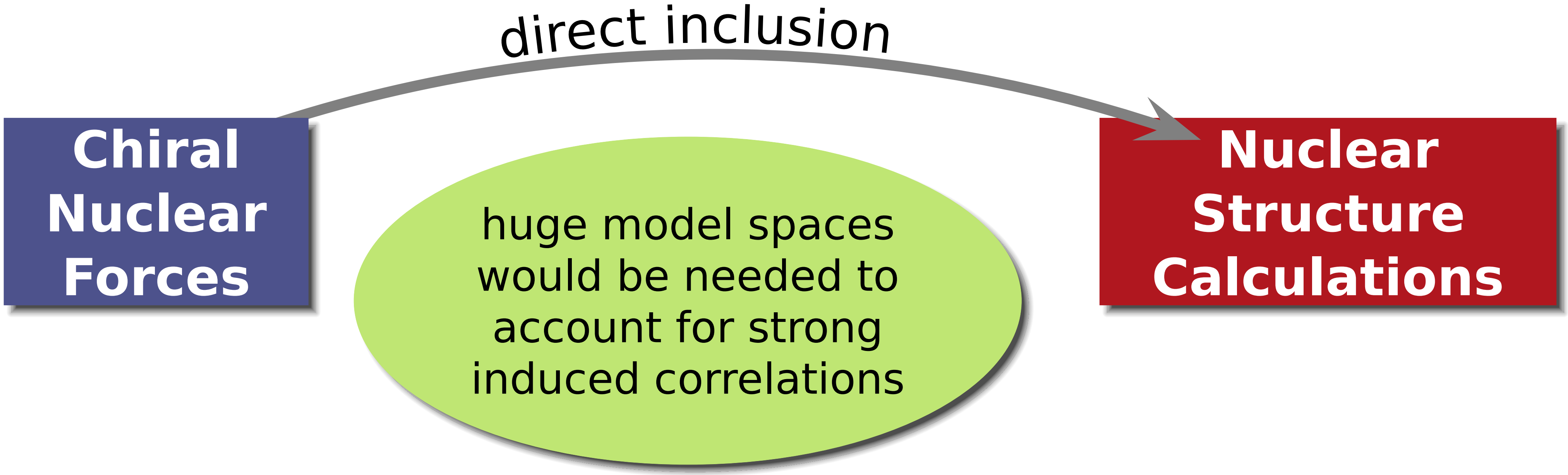
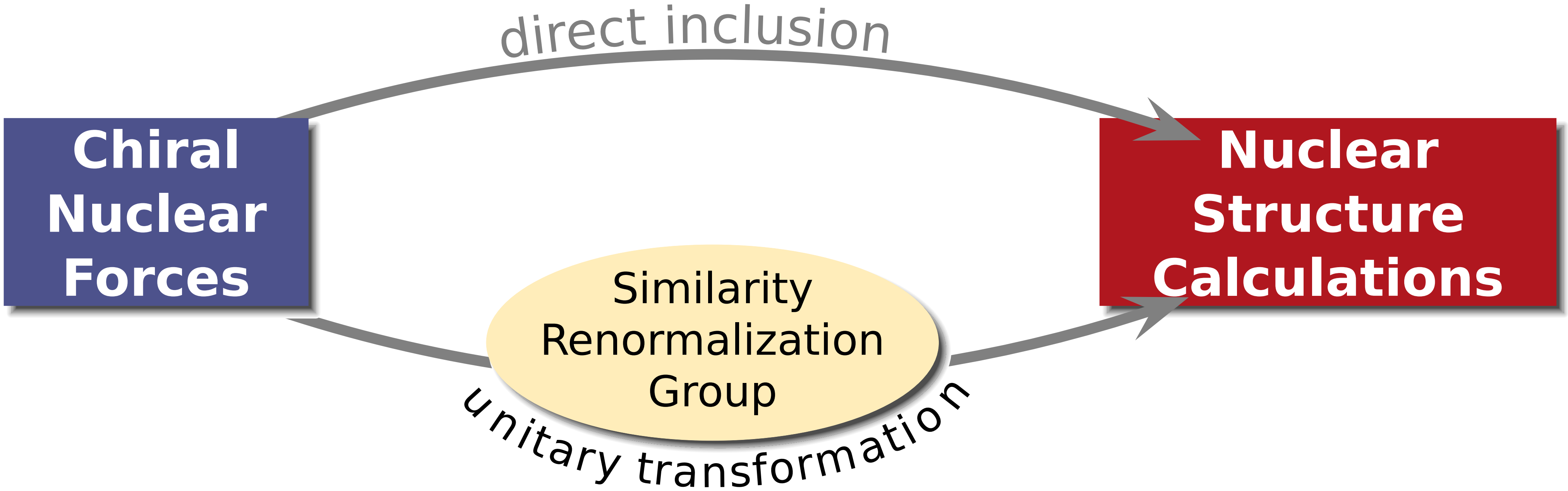
Chiral Hamiltonian



Chiral Hamiltonian



- NN interaction @ N^3LO [Entem, Machleidt, Phys.Rev C68, 041001(R) (2003)]
- 3N interaction @ N^2LO

- c_D & c_E fixed by binding energy and β-decay halflife of triton

  [Gazit et.al., Phys.Rev.Lett. 103, 102502 (2009)]

Similarity Renormalization Group (SRG)

evolution of the **Hamiltonian to band-diagonal form** with respect to a chosen many-body basis

simplicity and flexibility are great advantages of the SRG approach

- **unitary transformation** of Hamil…

$$\widetilde{H}_\alpha = U_\alpha^\dagger \, H \, U_\alpha$$

- **evolution equations** for $\widetilde{H}_\alpha$ depending on generator η_α

$$\frac{d}{d\alpha}\widetilde{H}_\alpha = \left[\eta_\alpha, \widetilde{H}_\alpha\right] \qquad \eta_\alpha = -U_\alpha^\dagger \frac{dU_\alpha}{d\alpha} = -\eta_\alpha^\dagger$$

- **dynamic generator**: commutator with the operator in whose eigenbasis H shall be diagonalized

$$\eta_\alpha = (2\mu)^2 \left[T_{\text{int}}, \widetilde{H}_\alpha\right]$$

SRG Evolution of Matrix Elements

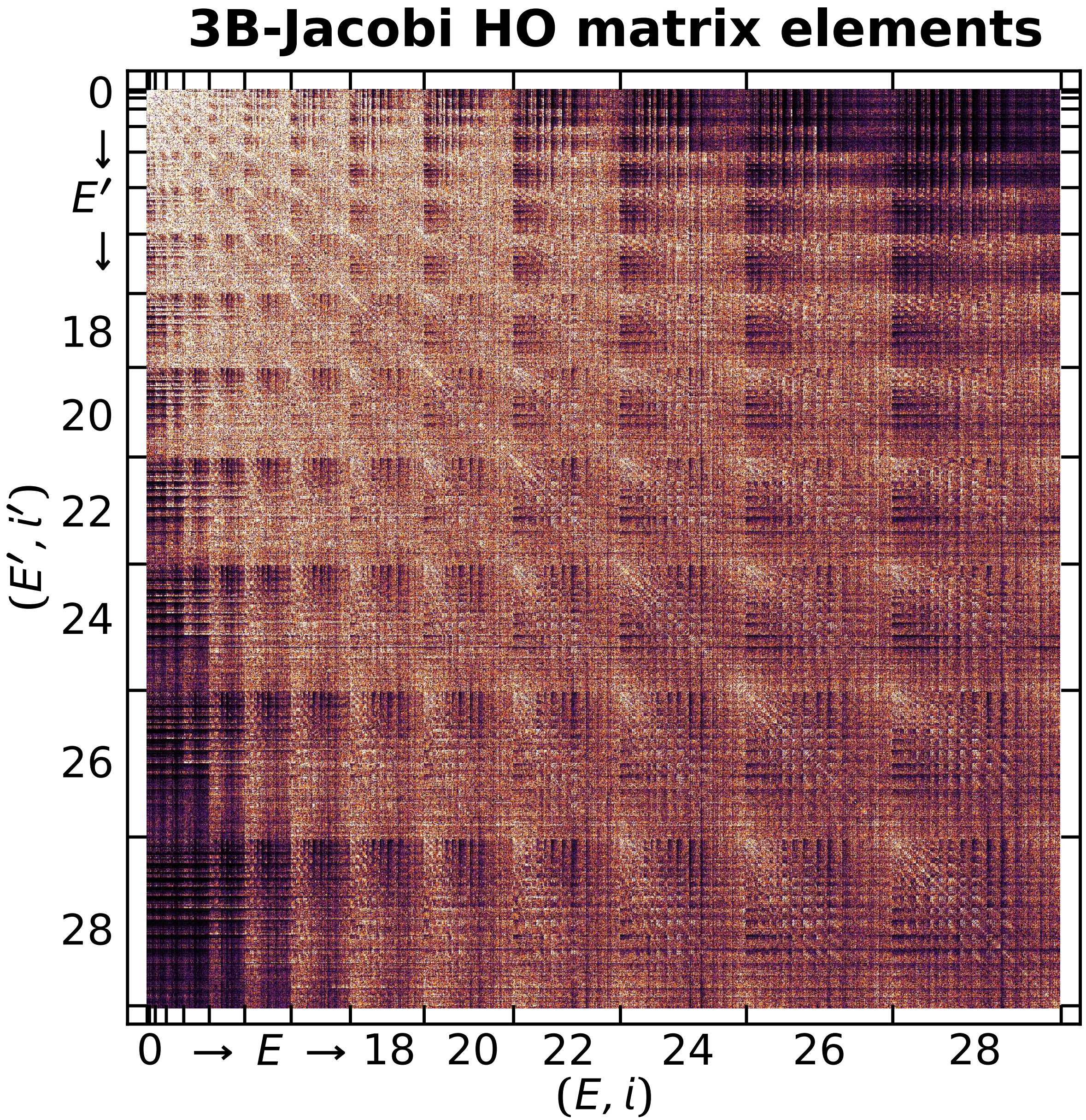
- represent operator equation in ***n*-body Jacobi HO basis** $\left|E i J^\pi T\right\rangle$
  - $n = 2$: relative LS-coupled HO states: $\left|E (LS) J^\pi T\right\rangle$
  - $n = 3$: antisymmetrized Jacobi-coordinate HO states: $\left|E i J^\pi T\right\rangle$

- system of **coupled evolution equations** for each $(J^\pi T)$-block

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\alpha}\left\langle E i J^\pi T\right| \widetilde{\mathrm{H}}_\alpha \left|E' i' J^\pi T\right\rangle = (2\mu)^2 \sum_{E'',i''}^{E_{\mathrm{SRG}}} \sum_{E''',i'''}^{E_{\mathrm{SRG}}} \Big[\\
\left\langle E i J^\pi T\right| \mathrm{T}_{\mathrm{int}} \left|E'' i'' J^\pi T\right\rangle \left\langle E'' i'' J^\pi T\right| \widetilde{\mathrm{H}}_\alpha \left|E''' i''' J^\pi T\right\rangle \left\langle E''' i''' J^\pi T\right| \widetilde{\mathrm{H}}_\alpha \left|E' i' J^\pi T\right\rangle \\
-2\left\langle E i J^\pi T\right| \widetilde{\mathrm{H}}_\alpha \left|E'' i'' J^\pi T\right\rangle \left\langle E'' i'' J^\pi T\right| \mathrm{T}_{\mathrm{int}} \left|E''' i''' J^\pi T\right\rangle \left\langle E''' i''' J^\pi T\right| \widetilde{\mathrm{H}}_\alpha \left|E' i' J^\pi T\right\rangle \\
+\left\langle E i J^\pi T\right| \widetilde{\mathrm{H}}_\alpha \left|E'' i'' J^\pi T\right\rangle \left\langle E'' i'' J^\pi T\right| \widetilde{\mathrm{H}}_\alpha \left|E''' i''' J^\pi T\right\rangle \left\langle E''' i''' J^\pi T\right| \mathrm{T}_{\mathrm{int}} \left|E' i' J^\pi T\right\rangle \Big]
\end{aligned}
$$

- we use $E_{\mathrm{SRG}} = 40$ for $J \leq 5/2$ and ramp down to 24 in steps of 4 (sufficient to converge the intermediate sums for $\hbar\Omega \gtrsim 16\,\mathrm{MeV}$)

SRG Evolution in Three-Body Space

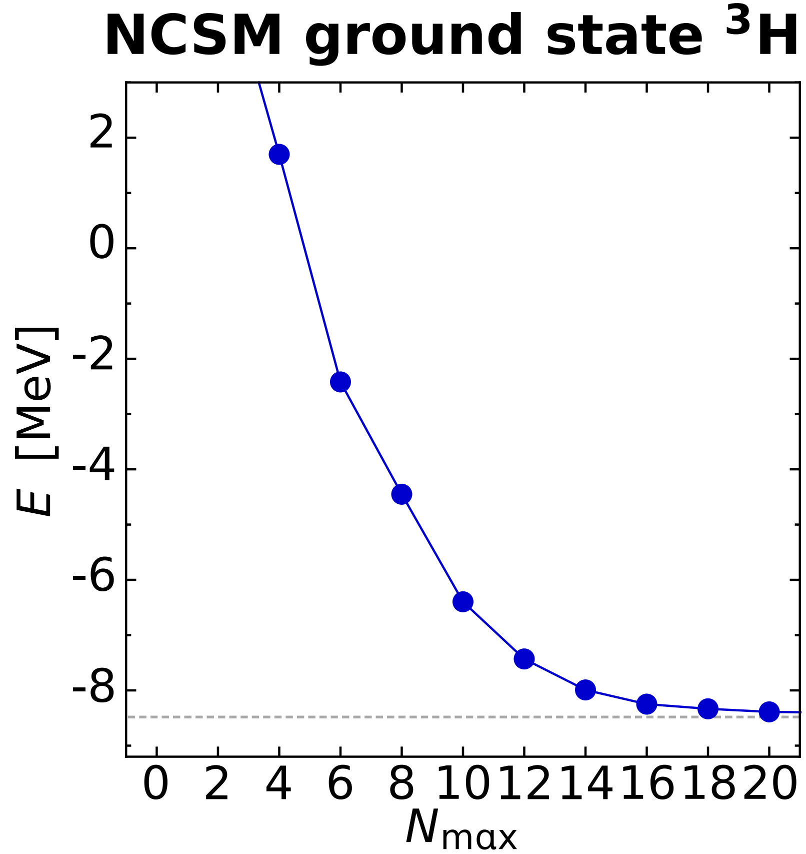


$\alpha = 0.00\,\mathrm{fm}^4$

$\Lambda = \infty\,\mathrm{fm}^{-1}$

$|\langle E' i' JT | \widetilde{\mathrm{H}}_\alpha | E i J T \rangle|$

$J^\pi = \frac{1}{2}^+, T = \frac{1}{2}, \hbar\Omega = 28\,\mathrm{MeV}$



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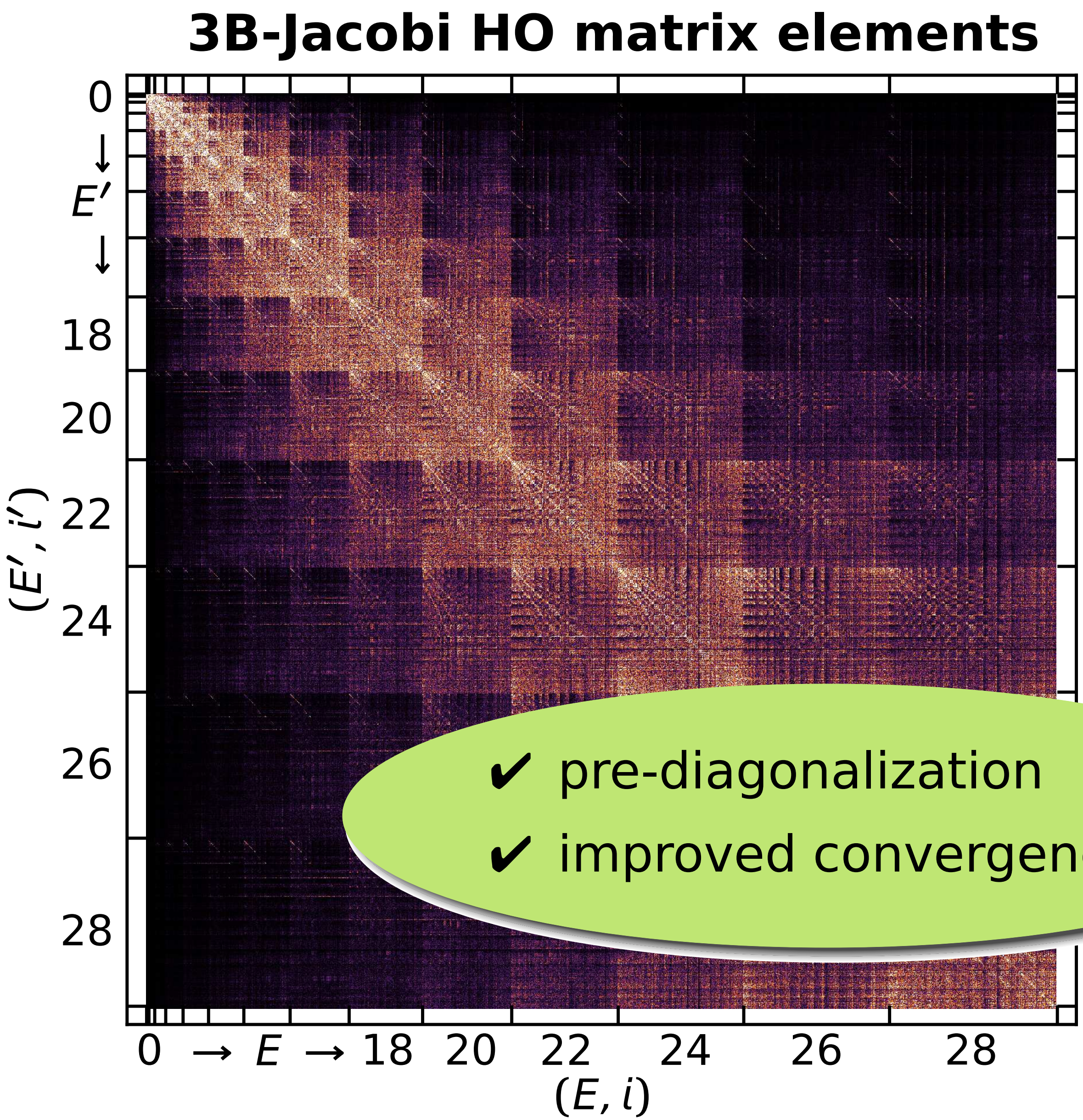
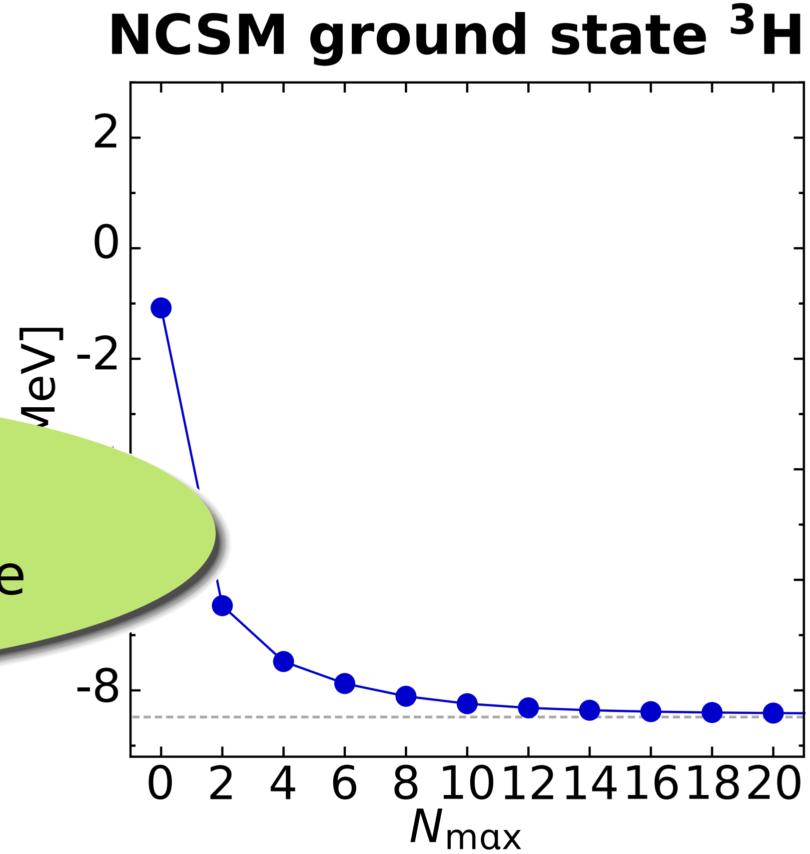


$\alpha = 0.32\,\mathrm{fm}^4$

$\Lambda = 1.33\,\mathrm{fm}^{-1}$

$|\langle E' i' J T | \widetilde{\mathrm{H}}_\alpha | E i J T \rangle|$

$J^\pi = \frac{1}{2}^+, T = \frac{1}{2}, \hbar\Omega = 28\,\mathrm{MeV}$



✔ pre-diagonalization

✔ improved convergence

Calculations in A-Body Space

- SRG transformation induces **irreducible *n*-body forces**
- we omit all contributions with $n > 3$
  ⇒ unitarity might be lost

Investigate induced and genuine 3N effects

- **NN only**:
  evolve NN-only intial Hamiltonian in two-body space
  ⇒ omit induced 3N forces
- **NN+3N-induced**:
  evolve NN-only initial Hamiltonian in three-body space
  ⇒ account for induced 3N forces
- **NN+3N-full**:
  evolve NN+3N intial Hamiltonian i
  ⇒ omit induced 4N contributions

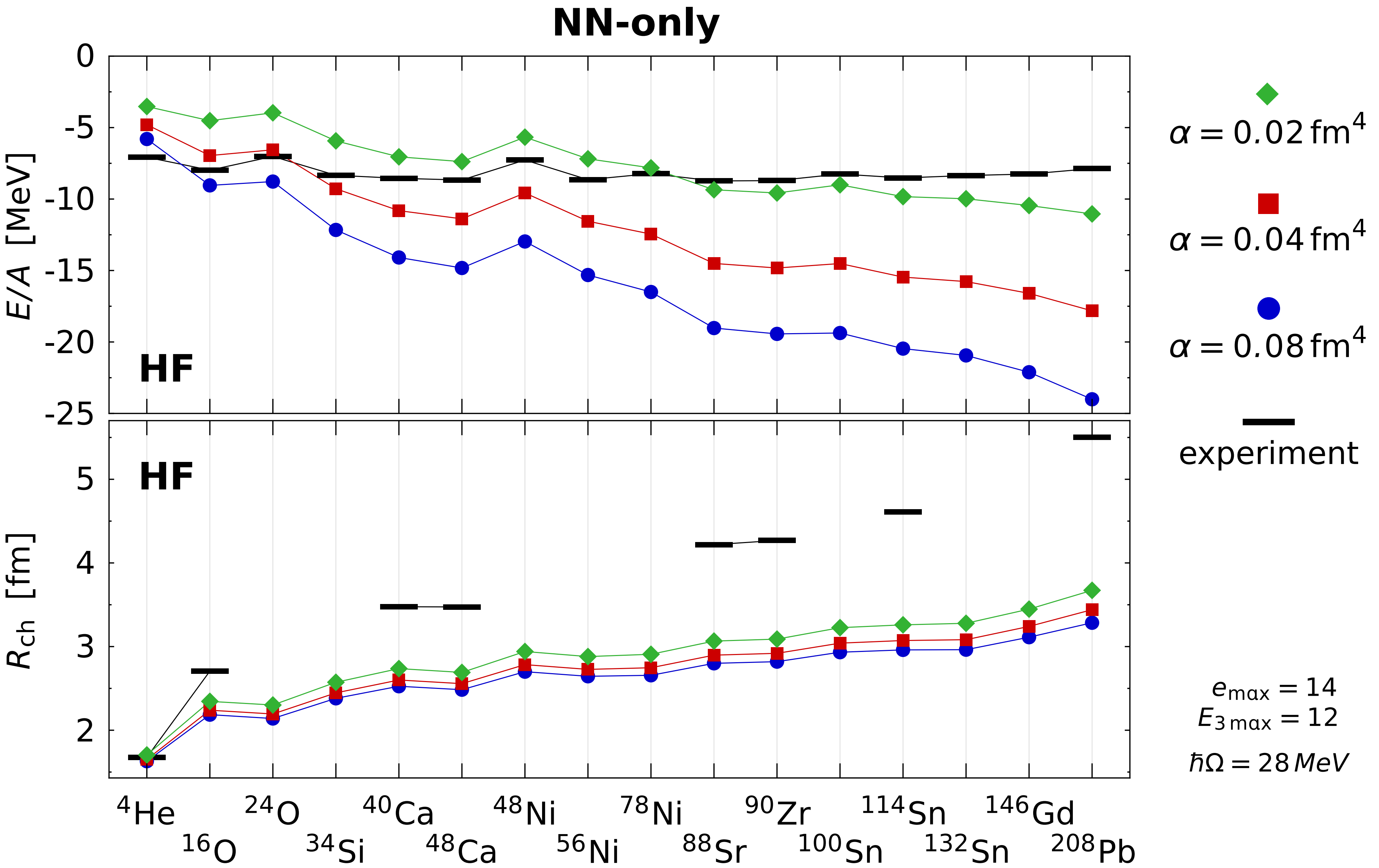
α-variation provides a **diagnostic tool** to assess the contributions of omitted many-body interactions

Hartree-Fock & Perturbation Theory

HF & PT provides information on the systematics of ground-state observables over a wide mass range

- solution of the HF equations with 3N interaction computationally simple
- second-order PT for energy $E_{HF}^{(2)} = \sum_{m \neq HF} \frac{|\langle m | \mathrm{H} | HF \rangle|^2}{E_{HF} - E_m}$ on top of HF results
- all following results preliminary with some limitations
  - 3N matrix elements only up to $E_{3\,\mathrm{max}} = 12$
  - fixed oscillator frequency $\hbar\Omega = 28$ MeV
  - second-order perturbative correction includes NN contribution only

Systematics: E/A and R_{ch}



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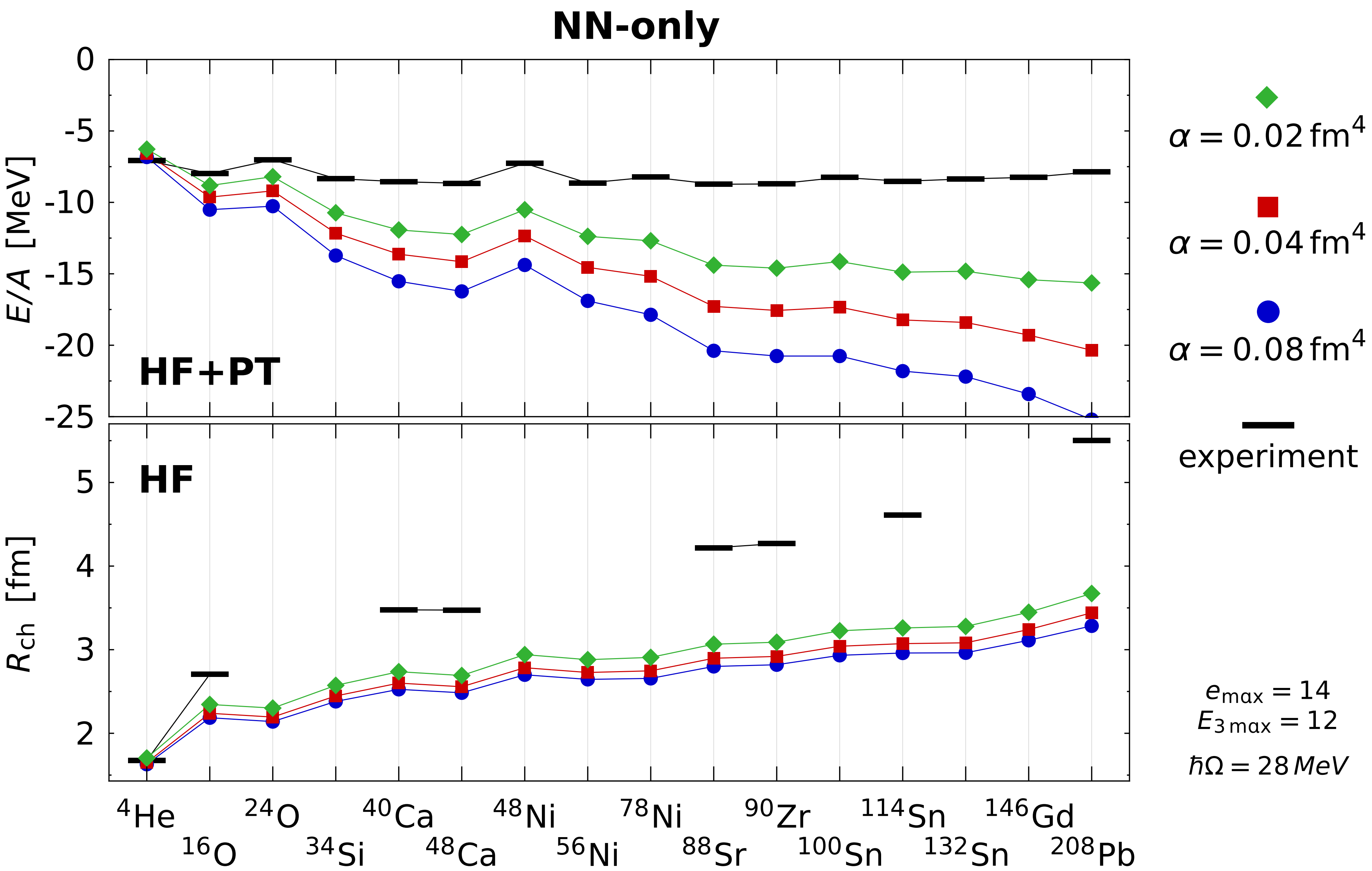

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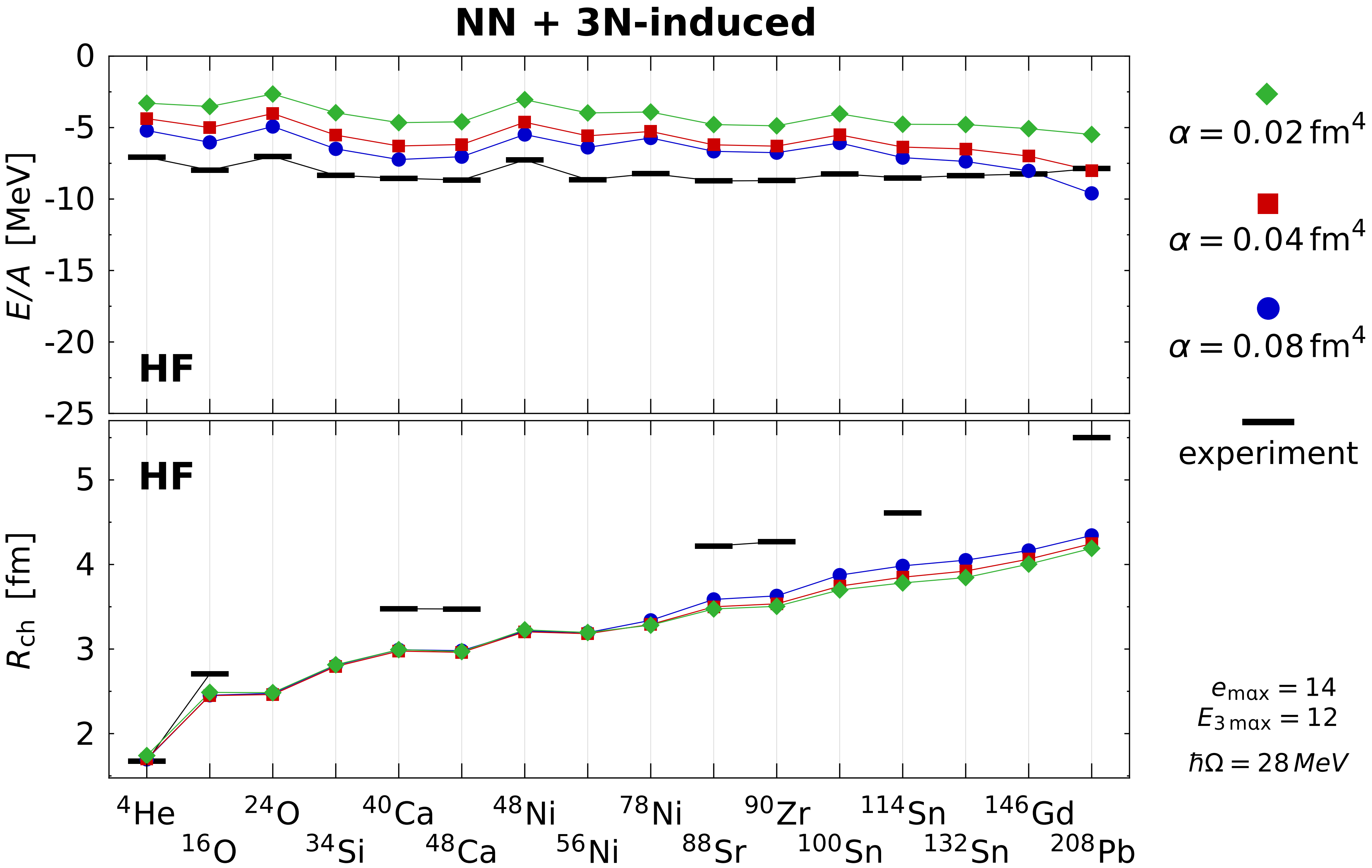

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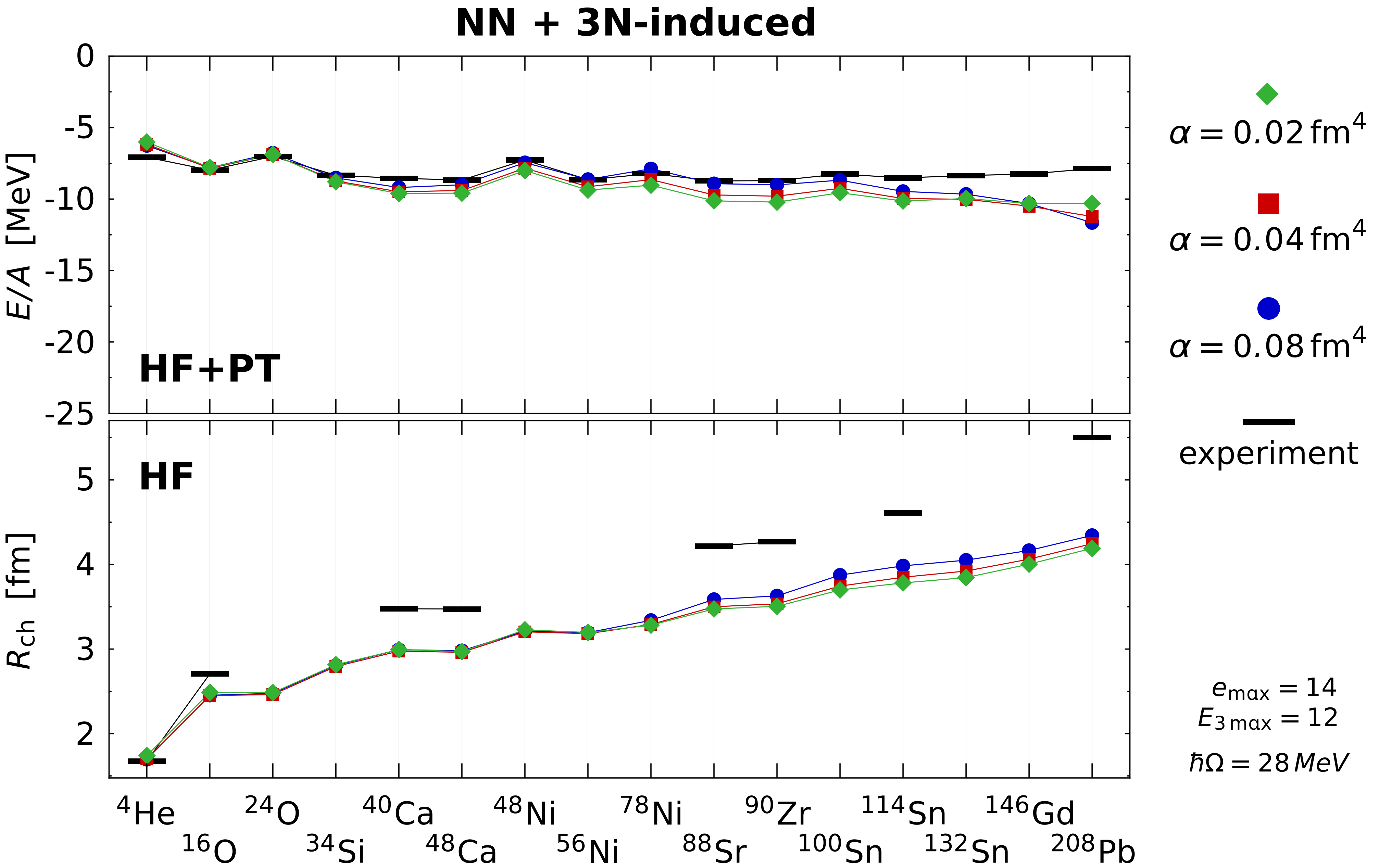

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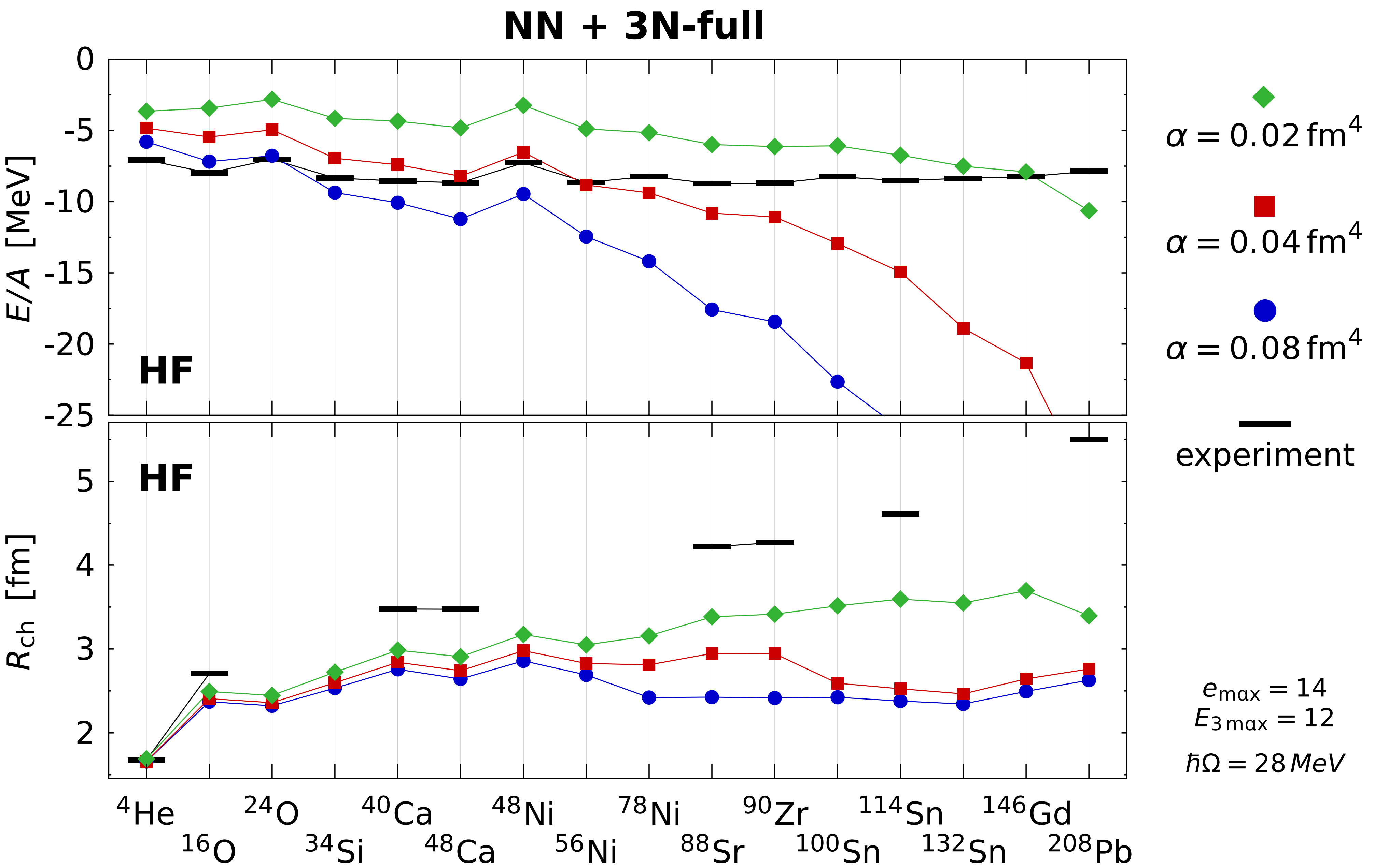

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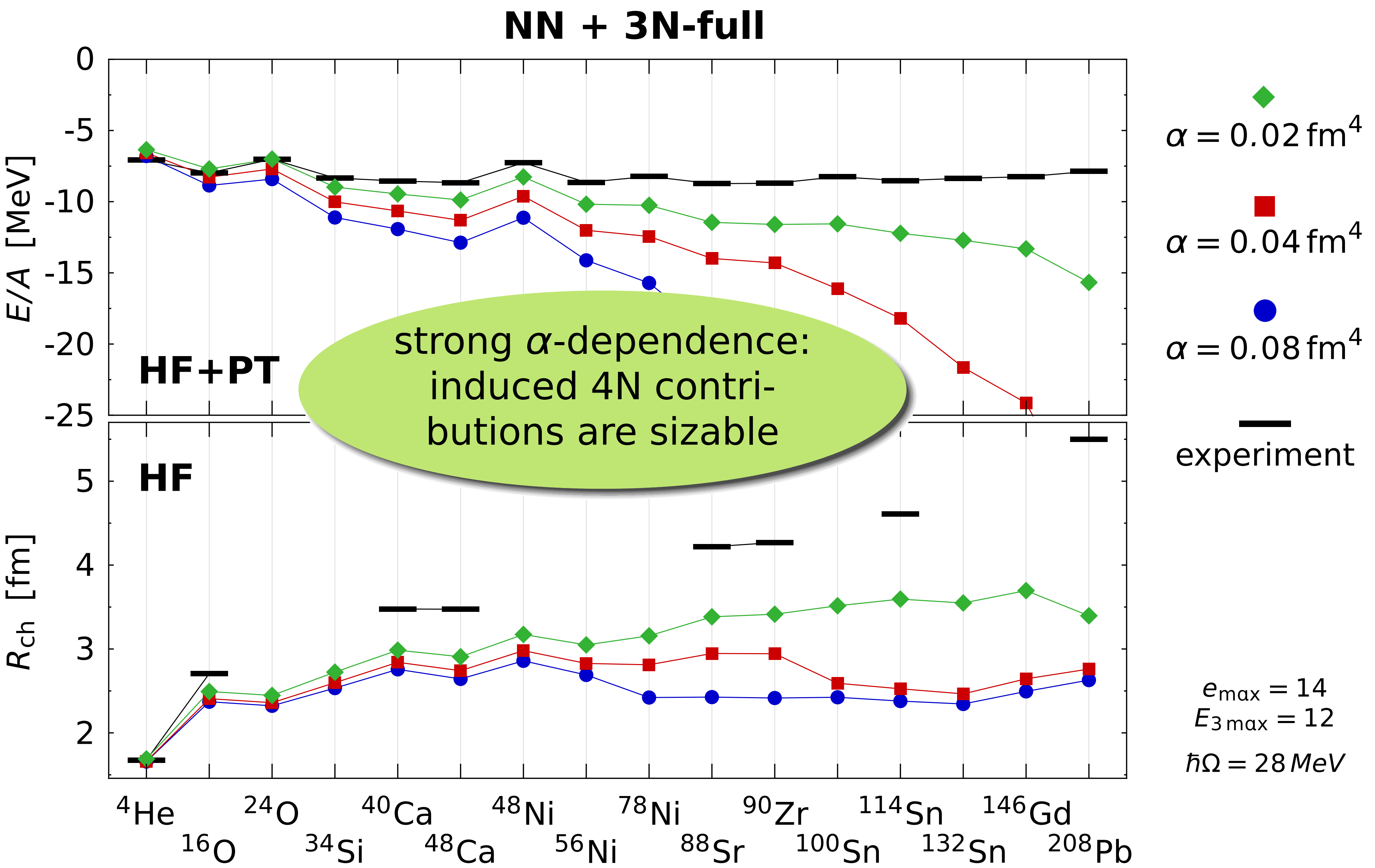

Conclusions

- **SRG transformation of chiral NN+3N interactions**
  - consistent SRG evolution in two- & three-body space
  - pre-diagonalization of Hamilton matrix leads to improved convergence in many-body calculations
  - effects of 3N-induced & genuine 3N forces distinguishable
- **Hartree-Fock and 2nd-Order Perturbation Theory**
  - efficient transformation and management of JT-coupled 3N matrix elements necessary

    → HK 23.4
    A. Calci
  - genuine 3N forces induce 4N contributions, which become important beyond the mid-p-shell
  - eliminate induced 4N contribution with help of alternative SRG generator from the beginning
- **many exciting applications ahead...**

Epilogue

- **thanks to our group & collaborators**
  - **S. Binder**, **A. Calci**, B. Erler, A. Günther, M. Hild, H. Krutsch, P. Papakonstantinou, S. Reinhardt, **R. Roth**, F. Schmitt, C. Stumpf, K. Vobig, R. Wirth

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  - **P. Navrátil**

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Thank you for your attention!

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