Consistent SRG Transformed Chiral Two- plus Three-Body Interactions

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Chiral Effective Field Theory

- based on the fundamental **symmetries of QCD**
- provides NN + 3N interactions **consistently**

**best available interaction from $\chi$EFT**

- NN at $N^3$LO (Entem & Machleidt, 500 MeV)
- 3N at $N^2$LO (low energy constants $c_D$ & $c_E$ from triton fit)

- initial chiral Hamiltonian causes strong **correlations**
- **slow convergence** in many-body calculations w.r.t. model space
- apply **unitary transformation**
Similarity Renormalization Group (SRG)

accelerate convergence by pre-diagonalizing the Hamiltonian with respect to the many-body basis

- unitary transformation leads to evolution equation

\[
\frac{d}{d\alpha} \tilde{H}_\alpha = [\eta_\alpha, \tilde{H}_\alpha] \quad \text{with} \quad \eta_\alpha = (2\mu)^2 [T_{\text{int}}, \tilde{H}_\alpha] = -\eta_\alpha^\dagger
\]

advantages of SRG: flexibility and simplicity

3B-Jacobi HO matrix elements

\[ \langle E'|ijT|\tilde{H}_\alpha - T_{\text{int}}|EjT\rangle \]

E-Block averages

\[ J^\pi = \frac{1}{2}^+, T = \frac{1}{2} \]

\[ \hbar \Omega = 20 \text{ MeV} \]
SRG Evolution in A-Body Space

- SRG induces **irreducible** many-body **contributions**

\[ U_\alpha^\dagger H U_\alpha = \tilde{H}_\alpha^{[2]} + \tilde{H}_\alpha^{[3]} + \cdots + \tilde{H}_\alpha^{[A]} \]

- restricted to a SRG evolution in 2B or 3B space

- formal **violation of unitarity**

**SRG-evolved Hamiltonians**

- **NN only**: start with NN initial Hamiltonian and keep two-body terms only

- **NN+3N-induced**: start with NN initial Hamiltonian and keep two- and three-body terms

- **NN+3N-full**: start with NN+3N initial Hamiltonian and keep all two- and three-body terms

\( \alpha \)-variation provides a **diagnostic tool** to assess the contributions of omitted many-body interactions
**4He: Ground-State Energies**

**NN only**
- Strong $\alpha$-dependence: induced 3N interactions
- IT-NCSM $\hbar\Omega = 20$ MeV

**NN+3N-induced**
- No $\alpha$-dependence: no induced 4N interactions

**NN+3N-full**
- No $\alpha$-dependence: no induced 4N interactions

\[
E \text{ [MeV]}
\]

\[
N_{\text{max}}
\]

\[
\alpha = 0.04 \text{ fm}^4
\]
\[
\Lambda = 2.24 \text{ fm}^{-1}
\]

\[
\alpha = 0.05 \text{ fm}^4
\]
\[
\Lambda = 2.11 \text{ fm}^{-1}
\]

\[
\alpha = 0.0625 \text{ fm}^4
\]
\[
\Lambda = 2.00 \text{ fm}^{-1}
\]

\[
\alpha = 0.08 \text{ fm}^4
\]
\[
\Lambda = 1.88 \text{ fm}^{-1}
\]

\[
\alpha = 0.16 \text{ fm}^4
\]
\[
\Lambda = 1.58 \text{ fm}^{-1}
\]
16O: Ground-State Energies

- **NN only**
  - IT-NCSM
  - \( \hbar \Omega = 20 \text{ MeV} \)

- **NN+3N-induced**
  - No \( \alpha \)-dependence: no induced 4N contrib.

- **NN+3N-full**
  - Sizable \( \alpha \)-dependence: induced 4N interactions

\begin{align*}
\alpha &= 0.04 \text{ fm}^4 \\
\Lambda &= 2.24 \text{ fm}^{-1} \\
\alpha &= 0.05 \text{ fm}^4 \\
\Lambda &= 2.11 \text{ fm}^{-1} \\
\alpha &= 0.0625 \text{ fm}^4 \\
\Lambda &= 2.00 \text{ fm}^{-1} \\
\alpha &= 0.08 \text{ fm}^4 \\
\Lambda &= 1.88 \text{ fm}^{-1} \\
\alpha &= 0.16 \text{ fm}^4 \\
\Lambda &= 1.58 \text{ fm}^{-1}
\end{align*}
switch off individual contributions of the 3N interaction

two-pion exchange term of the chiral 3N (in particular the $c_3$-term) is responsible for induced 4N

$\hbar\Omega = 20$ MeV

$\alpha = 0.04$ fm$^4$

$\Lambda = 2.24$ fm$^{-1}$

$\alpha = 0.08$ fm$^4$

$\Lambda = 1.88$ fm$^{-1}$

$\alpha = 0.16$ fm$^4$

$\Lambda = 1.58$ fm$^{-1}$
O: Lowering the Initial 3N Cutoff

**no initial 3N**

- NN+3N-induced

**500 MeV cutoff**

- NN+3N-full

**400 MeV cutoff**

- NN+3N-full

\( E [\text{MeV}] \) vs. \( N_{\text{max}} \)

- \( \hbar \Omega = 20 \text{ MeV} \)

**Parameters**

- \( \alpha = 0.04 \text{ fm}^4 \), \( \Lambda = 2.24 \text{ fm}^{-1} \)
- \( \alpha = 0.05 \text{ fm}^4 \), \( \Lambda = 2.11 \text{ fm}^{-1} \)
- \( \alpha = 0.0625 \text{ fm}^4 \), \( \Lambda = 2.00 \text{ fm}^{-1} \)
- \( \alpha = 0.08 \text{ fm}^4 \), \( \Lambda = 1.88 \text{ fm}^{-1} \)
- \( \alpha = 0.16 \text{ fm}^4 \), \( \Lambda = 1.58 \text{ fm}^{-1} \)
Conclusions

- **consistent SRG** evolution in 3B space
  - indispensable for converged IT-NCSM calculations
  - initial NN interaction under control

- **two-pion** exchange term of 3N **induces 4N** contributions
  - lowering the cutoff reduces induced contributions

Outlook: Alternative Generators

1. **exclude initial 3N**: include only NN+3N-induced in generator
2. **restrict range**: introduce explicit range in generator definition
3. search **trade-off** between induced 4N & convergence acceleration
thanks to my group & my collaborators

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- **P. Navrátil**
  TRIUMF Vancouver, Canada

- J. Vary, P. Maris
  Iowa State University, USA

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- P. Papakonstantinou
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- C. Forssén
  Chalmers University, Sweden

- **H. Feldmeier**, T. Neff
  GSI Helmholtzzentrum