

# Ab-Initio Spectroscopy of p- and sd-Shell Nuclei with Chiral Two- plus Three-Body Interactions

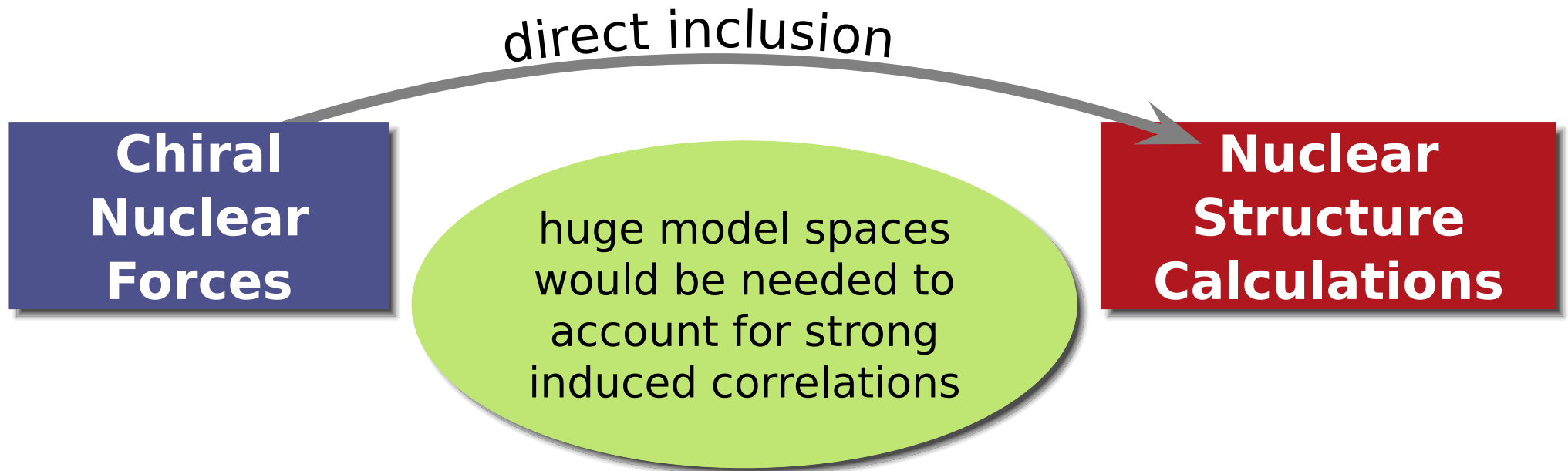
Joachim Langhammer

INSTITUT FÜR KERNPHYSIK

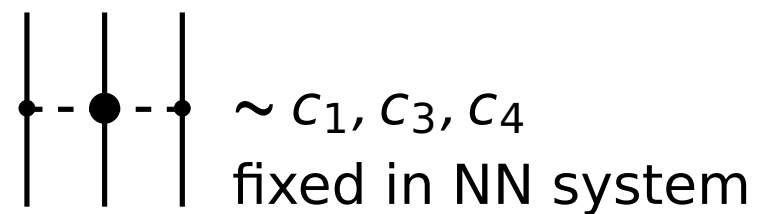
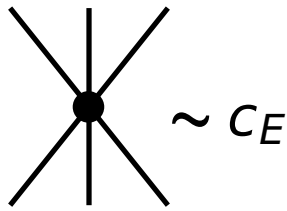
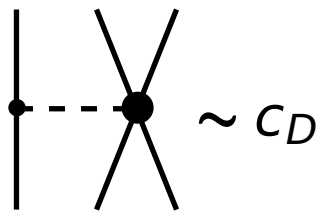


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# Chiral Hamiltonian

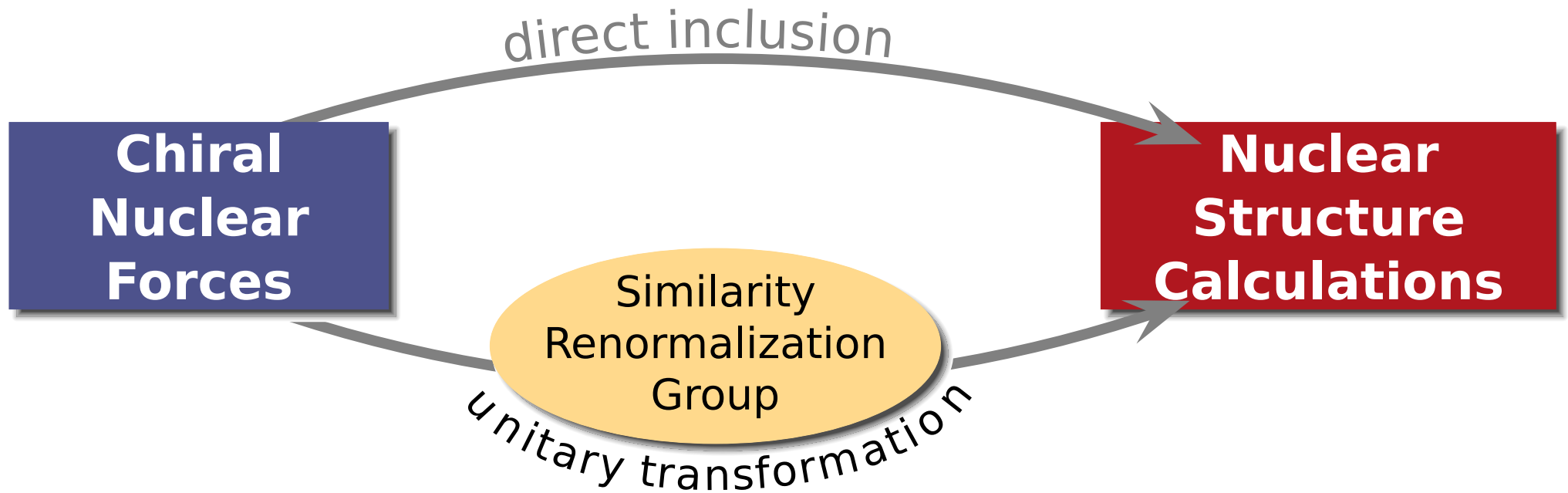


- NN interaction @  $N^3LO$  [Entem, Machleidt, Phys.Rev C68, 041001(R) (2003)]
- 3N interaction @  $N^2LO$  without approximations



- $c_D$  &  $c_E$  fixed by binding energy and  $\beta$ -decay halflife of triton  
[Gazit et.al., Phys.Rev.Lett. 103, 102502 (2009)]

# Chiral Hamiltonian



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- 3N interaction @  $N^2\text{LO}$  without approximations

A Feynman diagram representing the  $C_D$  counterterm. It shows two vertical lines on the left representing nucleons. A dashed line connects two vertices on these lines. From the right vertex, two lines cross each other, representing a two-body interaction.

$\sim C_D$

A Feynman diagram representing the  $C_E$  counterterm. It shows a central vertex where four lines meet: two vertical lines on the left and two diagonal lines on the right.

$\sim C_E$

A Feynman diagram representing the  $C_1, C_3, C_4$  counterterms. It shows three vertical lines. The middle line has a vertex connected to the other two lines by dashed lines.

$\sim C_1, C_3, C_4$   
fixed in NN system

- $C_D$  &  $C_E$  fixed by binding energy and  $\beta$ -decay halflife of triton  
[Gazit et.al., Phys.Rev.Lett. 103, 102502 (2009)]

# Similarity Renormalization Group

...yields an evolved Hamiltonian with **improved convergence properties** in many-body calculations

- **unitary transformation** of Hamiltonian driven by

$$\frac{d}{d\alpha} \tilde{H}_\alpha = [\eta_\alpha, \tilde{H}_\alpha] \quad \eta_\alpha = (2\mu)^2 [T_{\text{int}}, \tilde{H}_\alpha]$$

- SRG induces irreducible  $n$ -body contributions  $\tilde{H}_\alpha^{[n]}$

$$\tilde{H}_\alpha = \tilde{H}_\alpha^{[1]} + \tilde{H}_\alpha^{[2]} + \tilde{H}_\alpha^{[3]} + \dots + \tilde{H}_\alpha^{[n]} + \dots$$

- we can keep cluster-orders up to  $n = 3$  from evolution in 2B or 3B space

for details

→ HK 10.5 A. Calci

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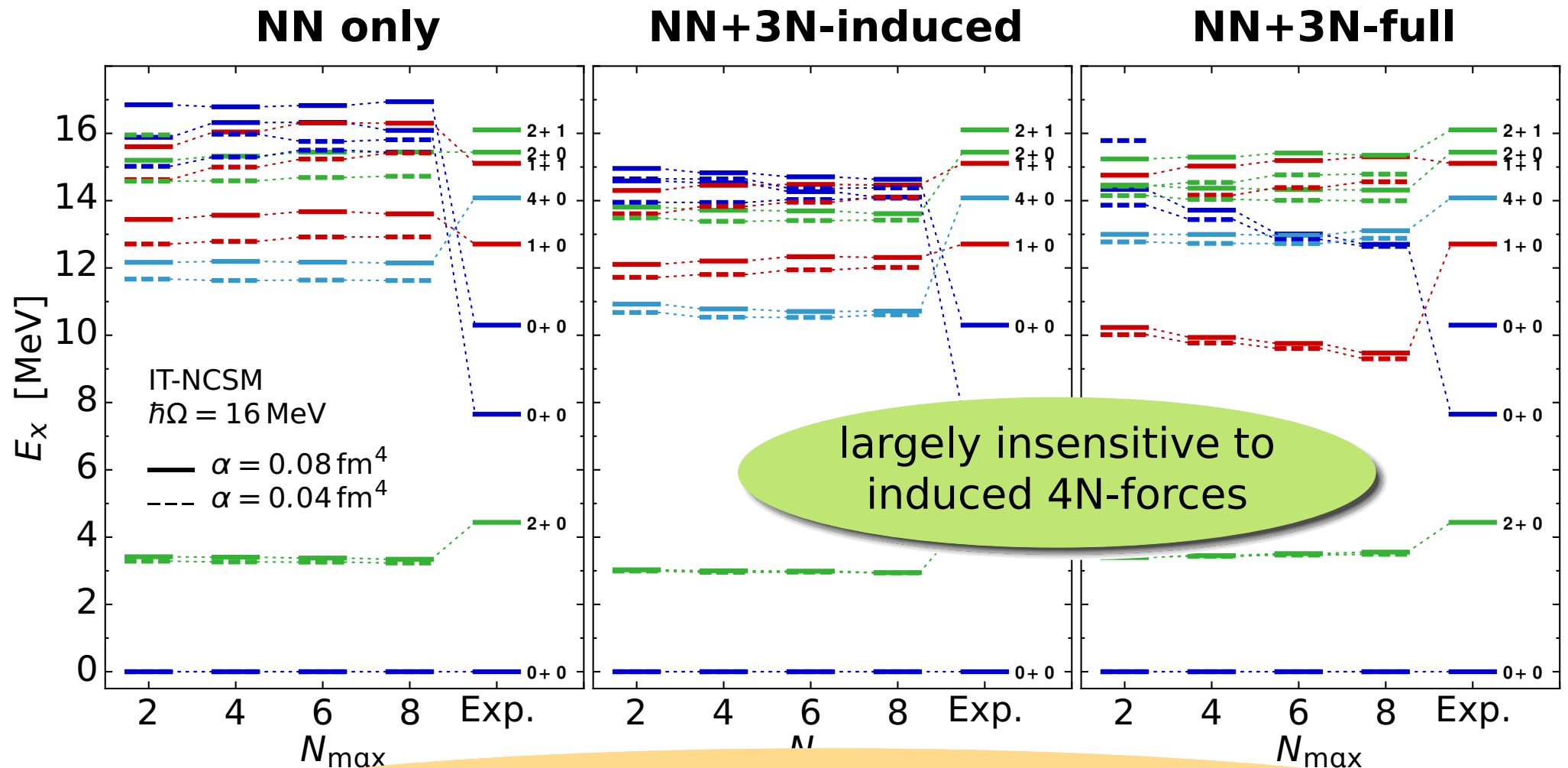
$$\frac{d}{d\alpha} \tilde{H}_\alpha = [\eta_\alpha, \tilde{H}_\alpha] \quad \eta_\alpha = (2\mu)^2 [T_{\text{int}}, \tilde{H}_\alpha]$$

## Different SRG-Evolved Hamiltonians

- **NN only**: start with NN initial Hamiltonian and keep two-body terms only
- **NN+3N-induced**: start with NN initial Hamiltonian and keep two- and three-body terms
- **NN+3N-full**: start with NN+3N initial Hamiltonian and keep two- and three-body terms

$\alpha$ -variation provides a **diagnostic tool** to assess the contributions of omitted many-body interactions

# Spectroscopy of $^{12}\text{C}$



⇒ **benchmark of chiral 3N forces possible by spectroscopic studies**

[R. Roth, J. Langhammer, A. Calci et al., Phys. Rev. Lett. 107, 072501]

# Spectra of p-Shell Nuclei – LEC Sensitivity –

R. Roth, A. Calci, J. Langhammer, S. Binder — in prep.

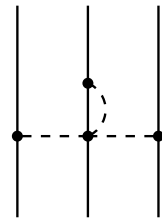
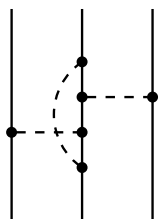
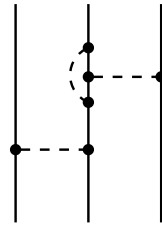
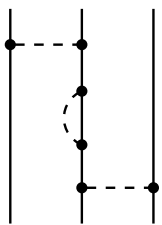
# Why Study the LEC Sensitivity?

- LECs from  $\pi N$  vertices have sizable uncertainties

	$c_1$ [GeV $^{-1}$ ]	$c_3$ [GeV $^{-1}$ ]	$c_4$ [GeV $^{-1}$ ]
Entem et al. – PRC 68,041001(R)	<b>-0.81</b>	<b>-3.20</b>	<b>5.40</b>
Rentmeester et al. – PRC 67, 044001	-0.76	-4.78	3.96
Büttiker et al. – NPA 668, 97	-0.81	-4.70	3.40
Fettes et al. – NPA 640, 199	-1.23	-5.94	3.47
Entem et al. – PRC 66,014002	-0.81	-3.40	3.40

- different  $c_i$  combinations have impact on 3NF contributions

- some N<sup>3</sup>LO corrections to TPE can be included by



$$\bar{c}_1 = c_1 - \frac{g_A^2 M_\pi}{64\pi F_\pi^2} = -0.81 - 0.13$$

$$\bar{c}_3 = c_3 + \frac{g_A^4 M_\pi}{16\pi F_\pi^2} = -3.2 + 0.89$$

$$\bar{c}_4 = c_4 - \frac{g_A^4 M_\pi}{16\pi F_\pi^2} = 5.4 - 0.89$$

change of 20-30%



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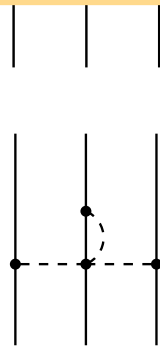
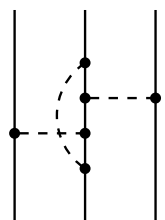
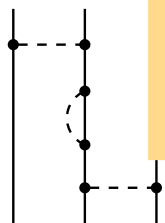
- Are spectra sensitive to different  $c_i$  combinations?

- Which LEC is most relevant for spectra?

- In the following

★ shift LECs  $c_{1/3/4}$

★ stick to  $c_D = -0.2$  and refit  $c_E$  to  ${}^4\text{He}$  binding energy

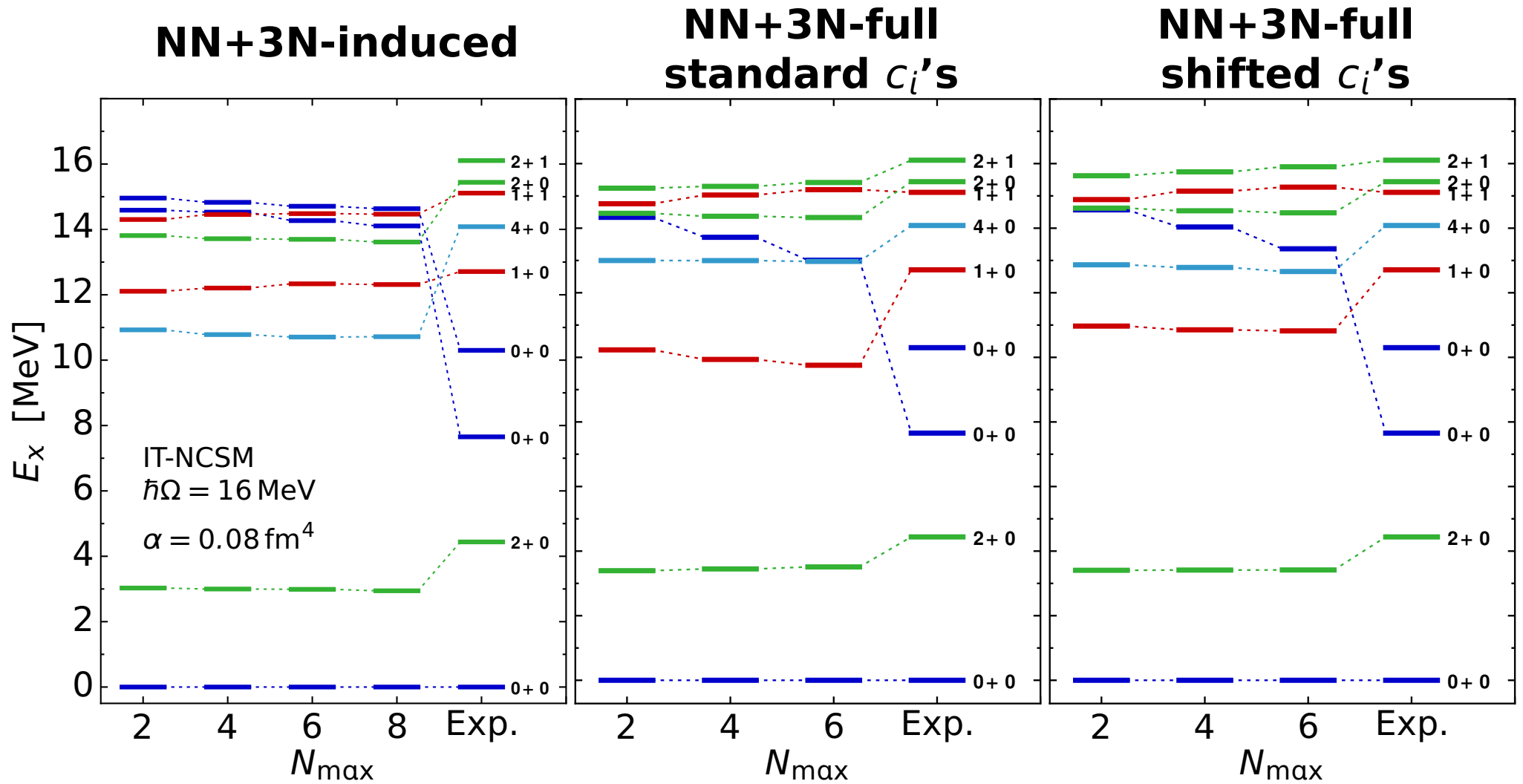


$$\bar{c}_3 = c_3 + \frac{g_A^4 M_\pi}{16\pi F_\pi^2} = -3.2 + 0.89$$

$$\bar{c}_4 = c_4 - \frac{g_A^4 M_\pi}{16\pi F_\pi^2} = 5.4 - 0.89$$

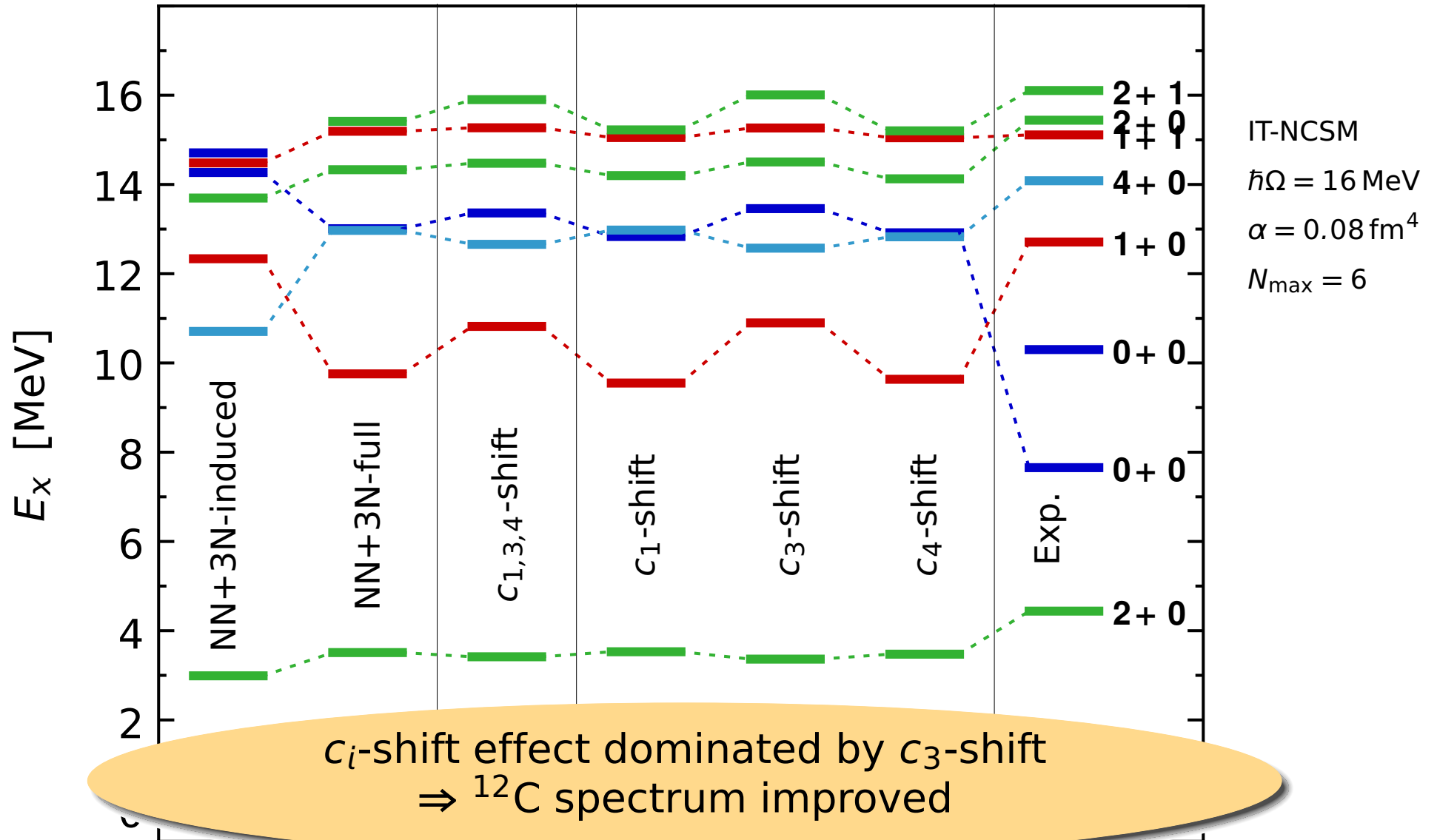
change of 20-30%

# $^{12}\text{C}$ – Shifted $c_i$ 's

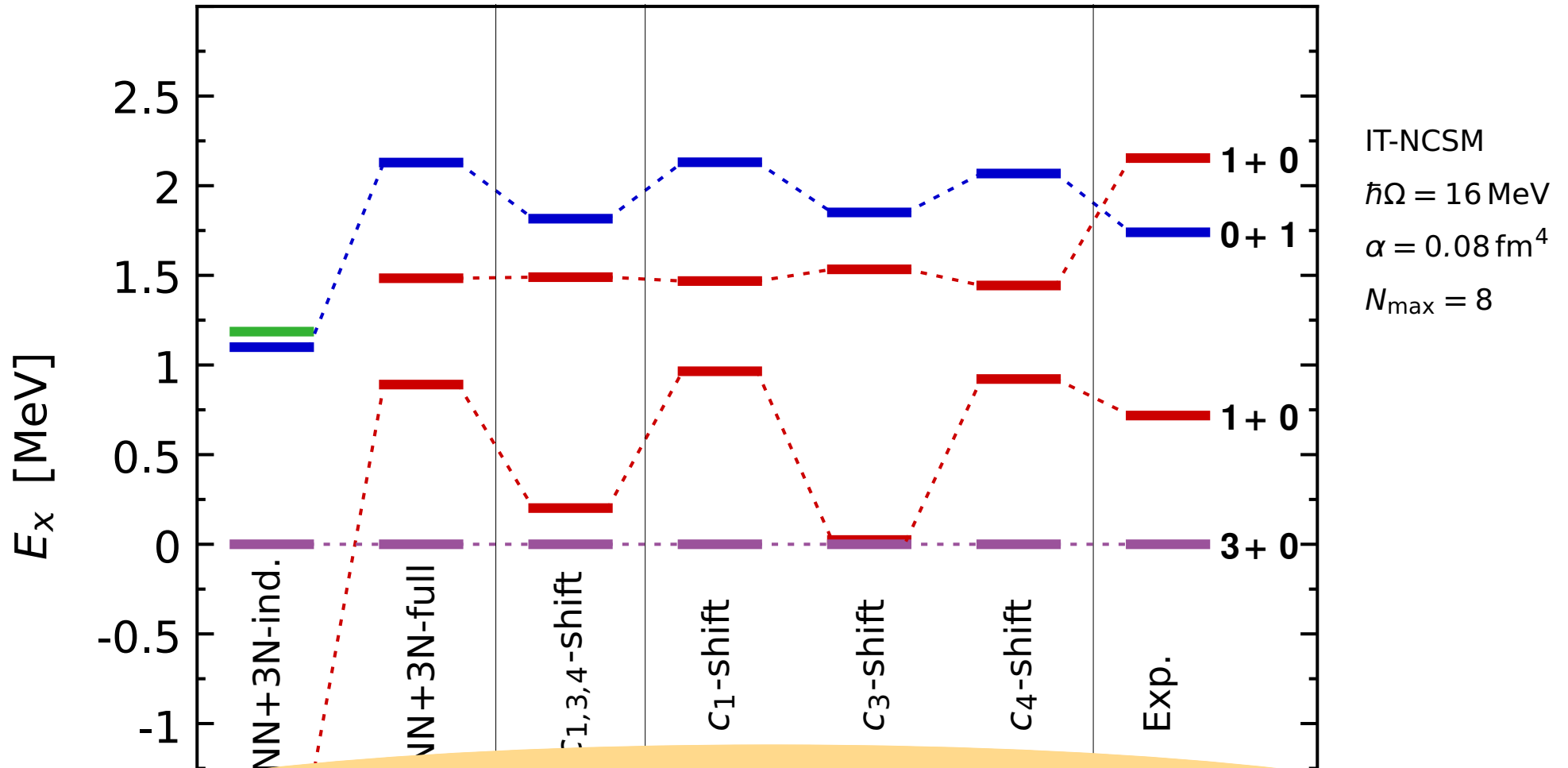


- $1^+$  state: very sensitive to  $c_i$ -shift
- other states: minor effects

# $^{12}\text{C}$ – LEC Sensitivity Analysis



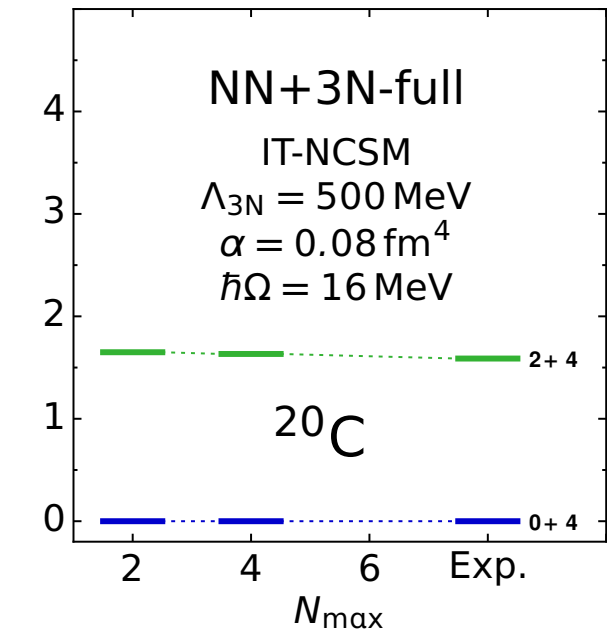
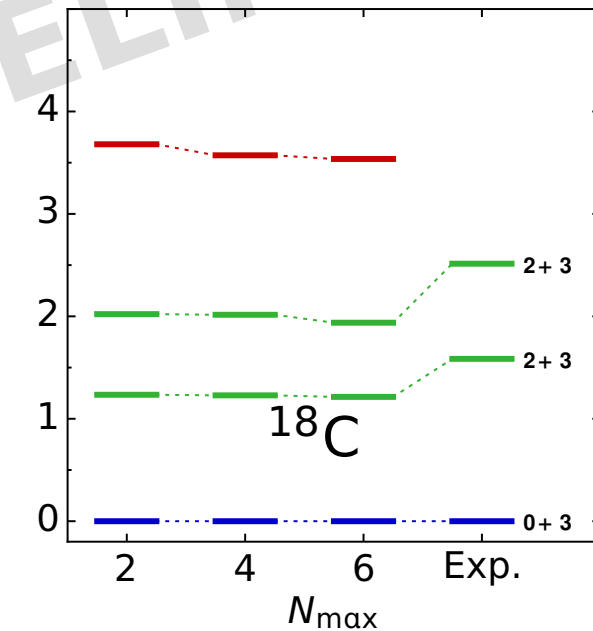
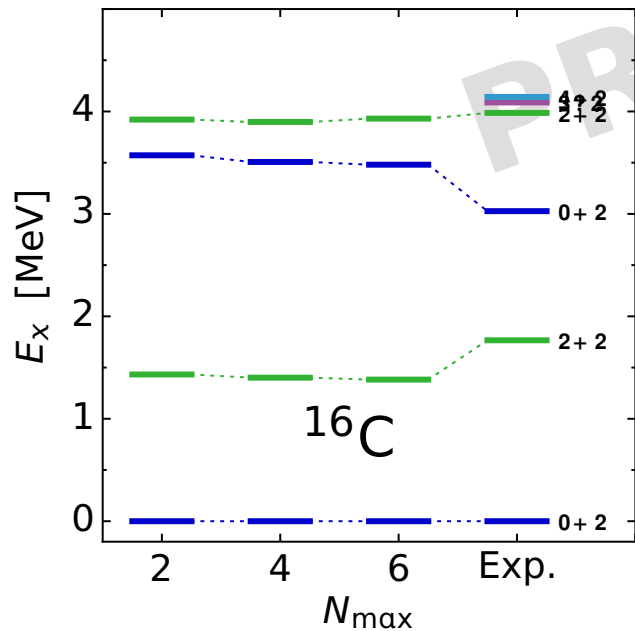
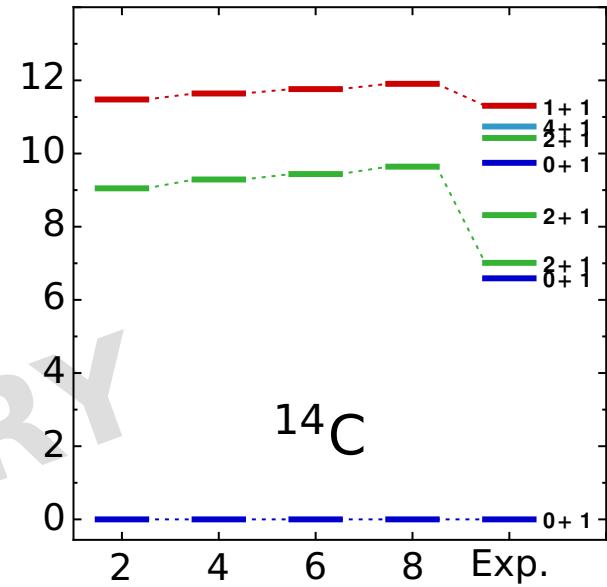
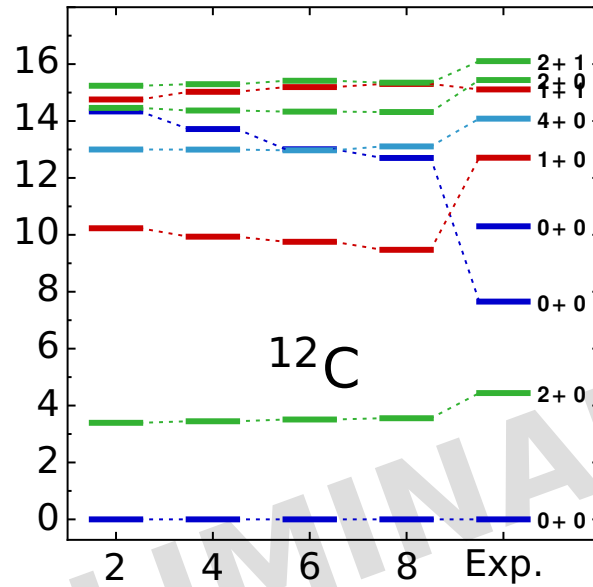
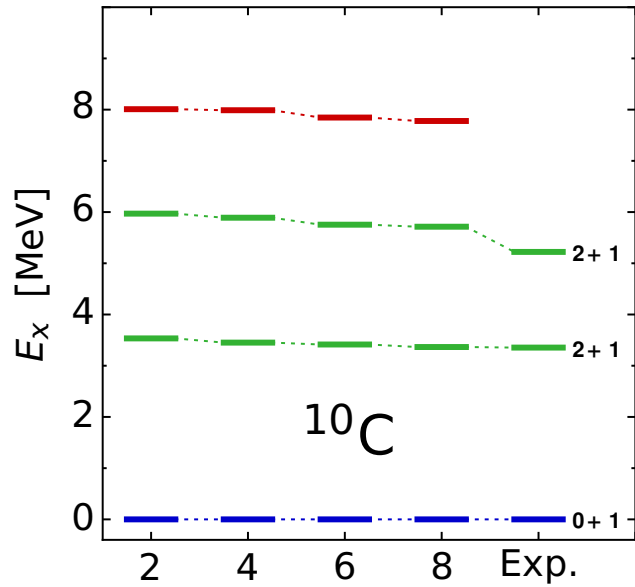
# $^{10}\text{B}$ – LEC Sensitivity Analysis



$c_i$ -shift effect again dominated by  $c_3$ -shift  
 **$1^+$ -state behaves contrarily than in  $^{12}\text{C}$**

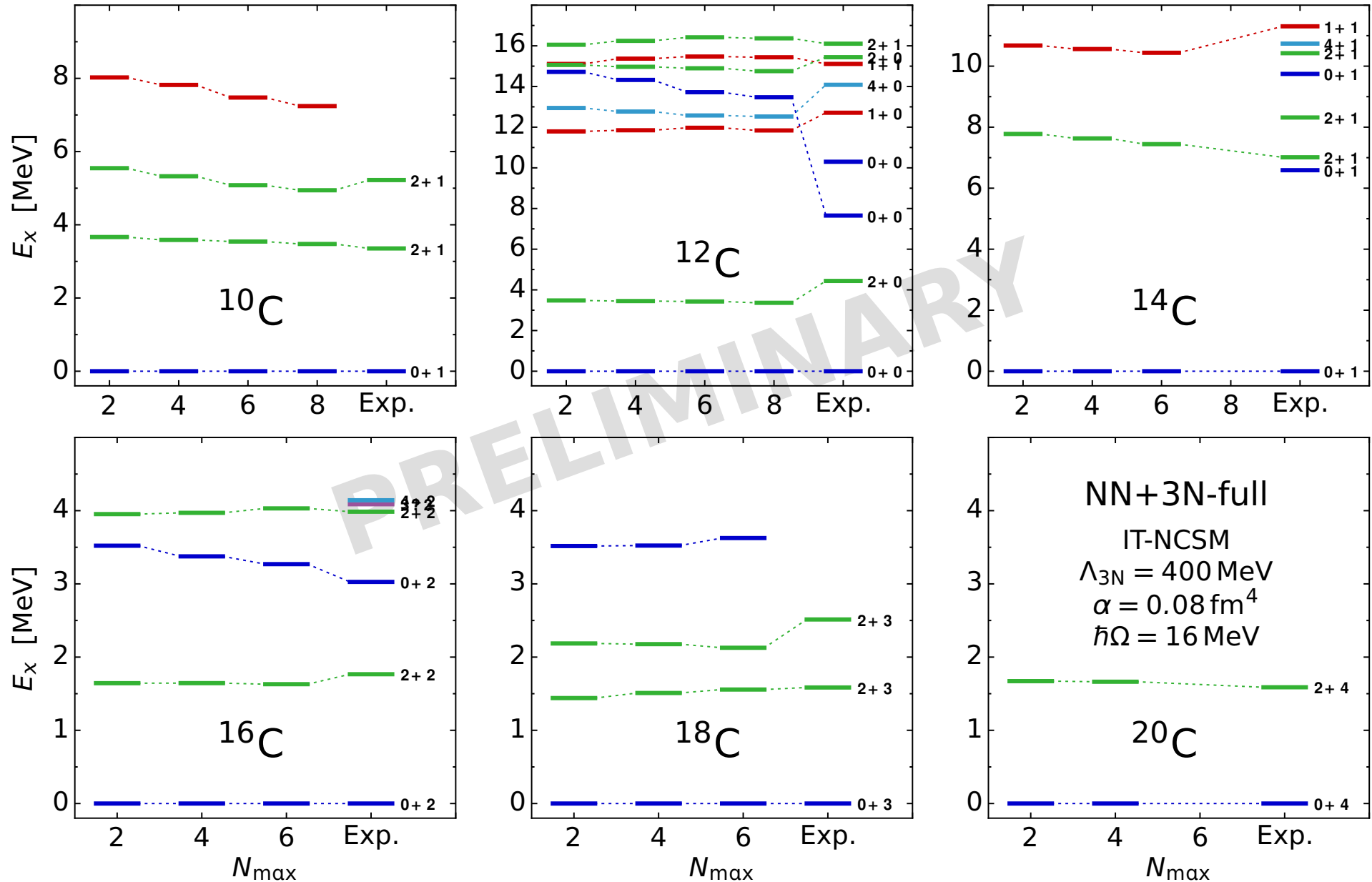
Outlook  
– The Carbon Isotopic Chain –

# Carbon Isotopic Chain

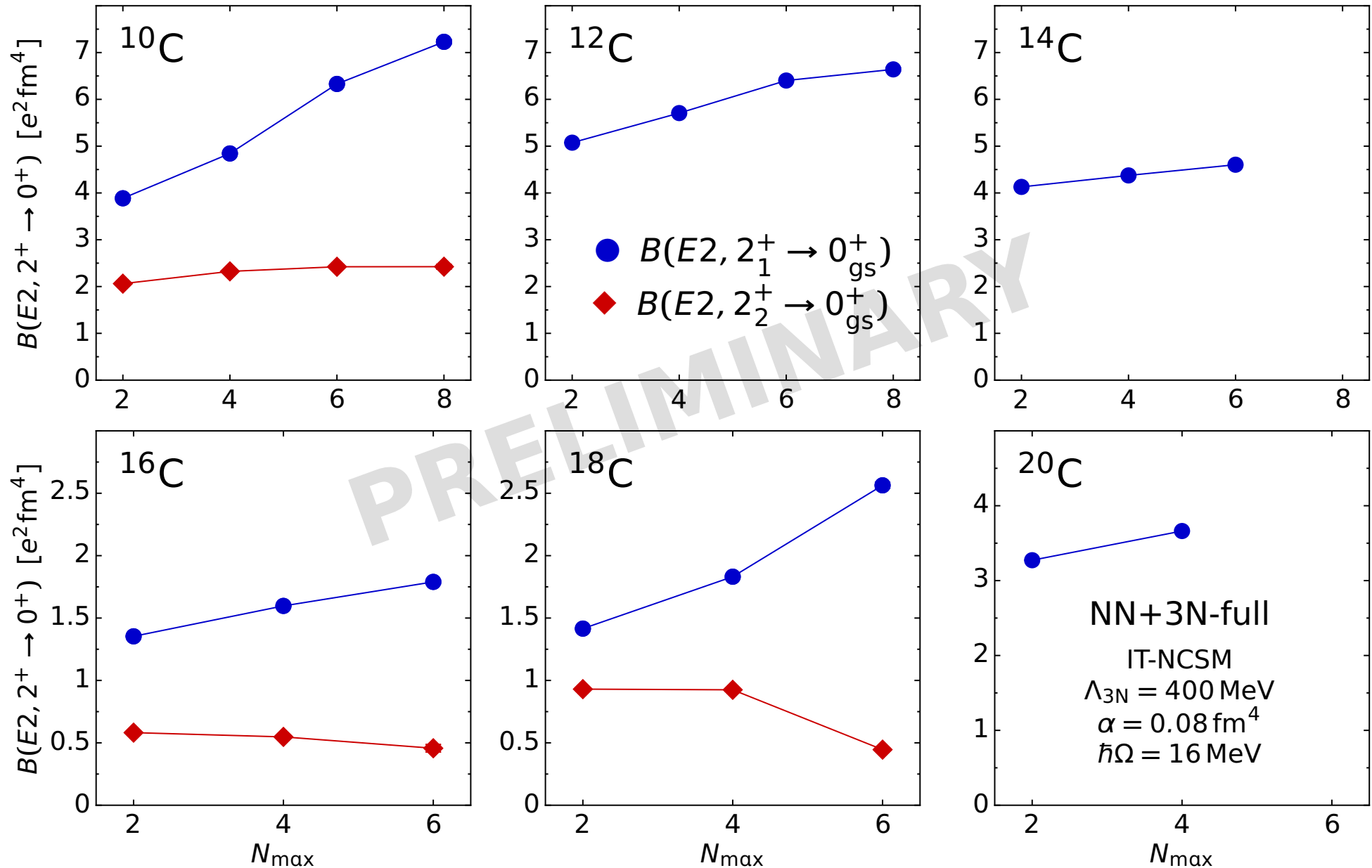


NN+3N-full  
 IT-NCSM  
 $\Lambda_{3N} = 500 \text{ MeV}$   
 $\alpha = 0.08 \text{ fm}^4$   
 $\hbar\Omega = 16 \text{ MeV}$

# Carbon Isotopic Chain



# Carbon Isotopic Chain





# Conclusions

- access to **complete spectroscopy** of p- and lower sd-shell nuclei **with full 3N forces**

- SRG-induced 4N forces negligible for converged spectra
- IT-NCSM is the key many-body method of these studies

⇒ powerful **benchmark for chiral forces**

- LEC  $c_3$  has a strong influence on spectra

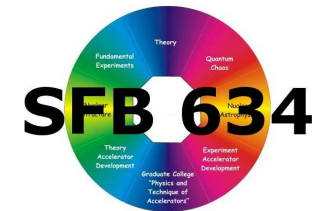
- with 3N @  $N^2$ LO not possible to describe the first  $1^+$ -states in  $^{10}\text{B}$  and  $^{12}\text{C}$  simultaneously
- inclusion of chiral 3N forces at  $N^3$ LO necessary

# Epilogue

## ■ thanks to our group & collaborators

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Thank you for your attention!



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