Ab Initio Nuclear Structure and Reactions with Chiral Three-Body Forces

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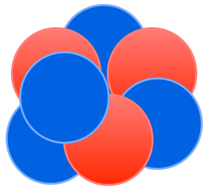
TECHNISCHE UNIVERSITÄT DARMSTADT

Outline

- What we are aiming for...
- Ingredients from Three-Body Technology
- 3N Forces in the NCSM/RGM and NCSMC
  - Nucleon-^{4}He scattering
  - Continuum effects on the ^{9}Be energy levels
- Conclusions

What we are aiming for...

Realistic ab-initio description of light nuclei



Bound states
& spectroscopy

(IT-)NCSM
Ab-initio description of
nuclear clusters

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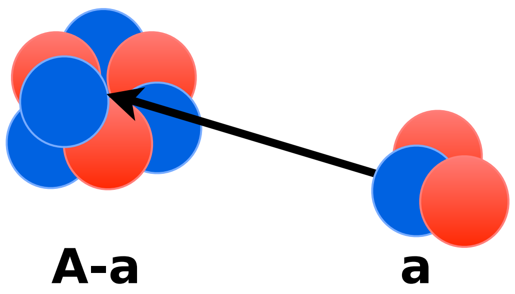

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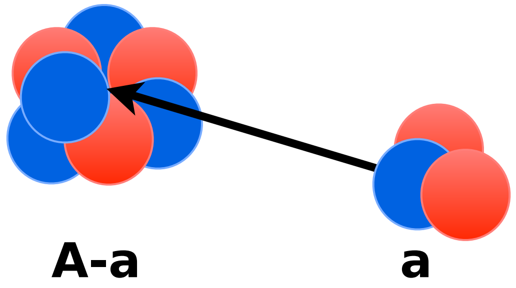

Resonances
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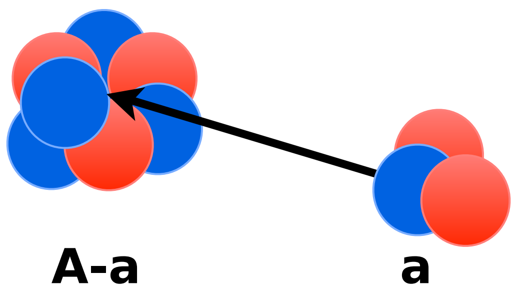

Resonances & scattering states

(IT-)NCSM

Ab-initio description of nuclear clusters

RGM

Describing relative motion of clusters

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Realistic ab-initio description of light nuclei

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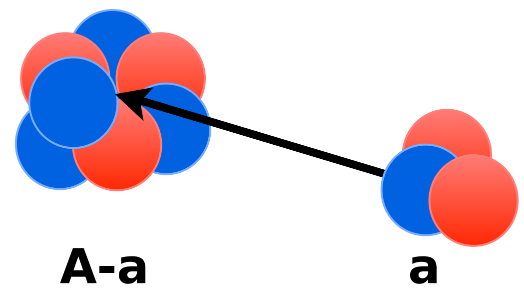


Resonances
& scattering states

(IT-)NCSM
Ab-initio description
nuclear clusters

(IT-)NCSM/RGM
& NCSMC approaches

RGM
Describing relative
motion of clusters

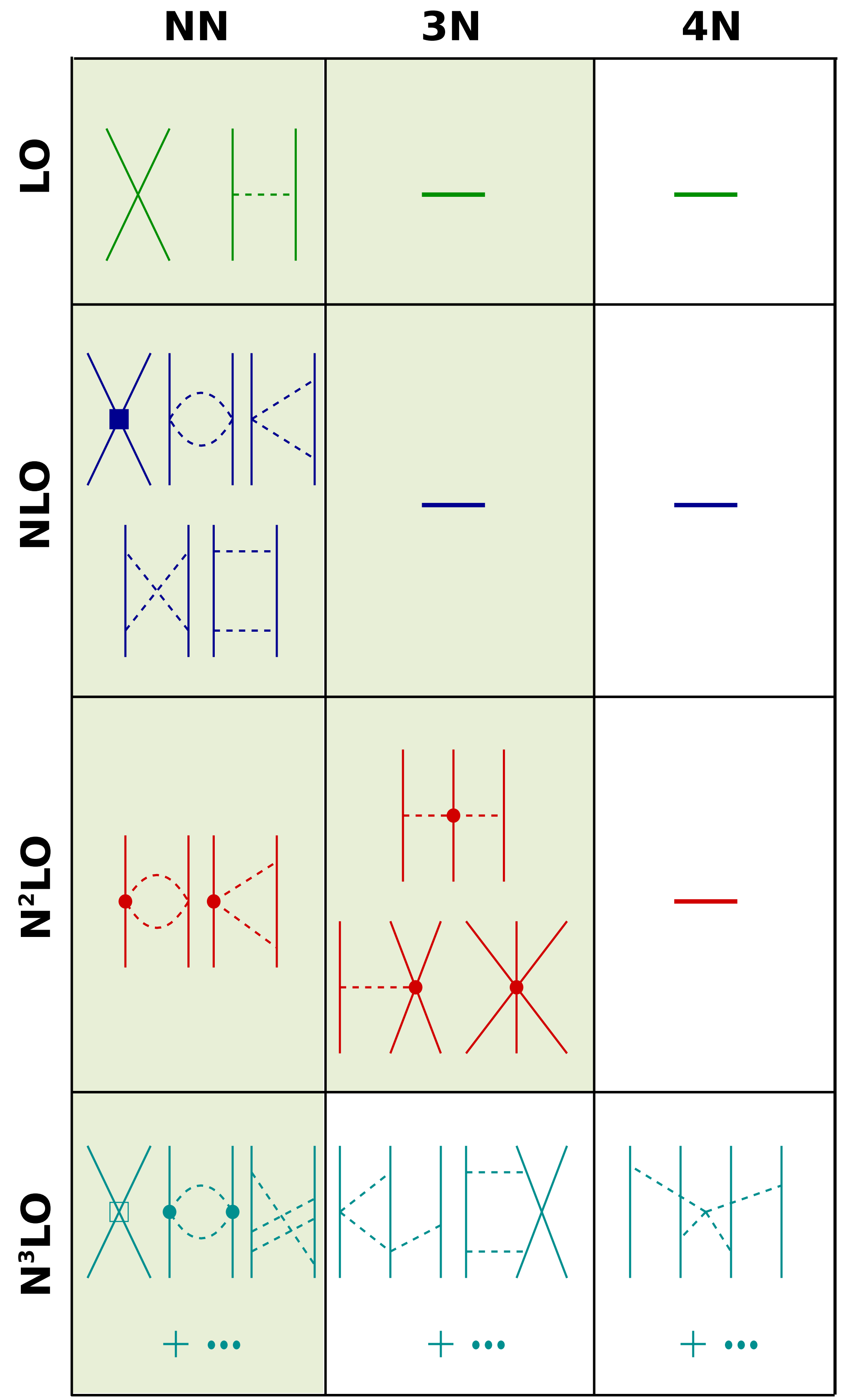
Successfully applied with NN interactions
Now: Inclusion of 3N Forces

Ingredients from Three-Body Technology

The Chiral NN+3N Hamiltonian

Weinberg, van Kolck, Machleidt, Entem, Meißner, Epelbaum, Krebs, Bernard, Skibinski, Golak...

- Hierarchy of consistent nuclear NN, 3N,... forces (and currents)
- NN interaction @ N^3LO (Λ=500MeV)
  [Entem, Machleidt, Phys.Rev C **68**, 041001(R) (2003)]
- Standard Hamiltonian
  - 3N interaction @ N^2LO(Λ_{3N}=500MeV)
    - LECs c_D, c_E fitted to β-decay halflife & binding energy of ^3H
      [Gazit et.al., Phys.Rev.Lett. **103**, 102502 (2009)]
- Reduced-Cutoff Hamiltonian
  - 3N interaction @ N^2LO(Λ_{3N}=400MeV)
    - c_D=-0.2, c_E fitted to ^4He



The Similarity Renormalization Group

Wegner, Glazek, Wilson, Perry, Bogner, Furnstahl, Hergert, Calci, Langhammer, Roth, Jurgenson, Navrátil,...

...yields an evolved Hamiltonian with improved convergence properties in many-body calculations

- Unitary transformation of Hamiltonian $H_\alpha = U_\alpha^\dagger H U_\alpha$

Different SRG-Evolved Hamiltonians

- **NN+3N-induced**: start with NN initial Hamiltonian and keep two- and three-body terms
- **NN+3N-full**: start with NN+3N initial Hamiltonian and keep two- and three-body terms

3N Forces
in the
NCSM/RGM and NCSMC

G. Hupin, J. Langhammer et al. ----- Phys. Rev C **88** 054622 (2013)

S. Quaglioni and P. Navrátil ----- Phys. Rev. Lett. **101**, 092501 (2008)

P. Navrátil, R. Roth and S. Quaglioni ----- Phys. Rev. C **82**, 034609 (2010)

S. Quaglioni, P. Navrátil, G. Hupin, J. Langhammer et al. ----- Few-Body Syst. DOI 10.1007/s00601-012-0505-0 (2012)

S. Quaglioni, P. Navrátil, R. Roth, W. Horiuchi ----- J.Phys.Conf.Ser. 402 (2012)

General Approach of NCSM/RGM

Wildermuth,Thompson, Tang, ..., Navrátil, Quaglioni, Roth, Hupin, Langhammer,...

- Represent $H|\psi^{J\pi T}\rangle = E|\psi^{J\pi T}\rangle$ using the **over-complete basis**

$$|\psi^{J\pi T}\rangle = \sum_{\nu} \int \mathrm{d}r r^2 \frac{g_{\nu}^{J\pi T}(r)}{r} \mathcal{A}_{\nu} |\phi_{\nu r}^{J\pi T}\rangle$$

$g_{\nu}^{J\pi T}(r)$ unknown

with the binary-cluster channel states

$$|\phi^{J\pi T}\rangle = \left\{ |\Phi^{(A-a)}\rangle \otimes |\Phi^{(a)}\rangle \otimes |rl\rangle \right\}^{J\pi T}$$

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NCSM delivers $|\Phi^{(A-a)}\rangle$ and $|\Phi^{(a)}\rangle$

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- Solve **generalized eigenvalue** problem

$$\sum_{\nu} \int \mathrm{d}r r^2 \left[\mathcal{H}_{\nu,\nu'}^{J\pi T}(r',r) - E\mathcal{N}_{\nu,\nu'}^{J\pi T}(r,r')\right] \frac{g_{\nu r}^{J\pi T}}{r} = 0$$

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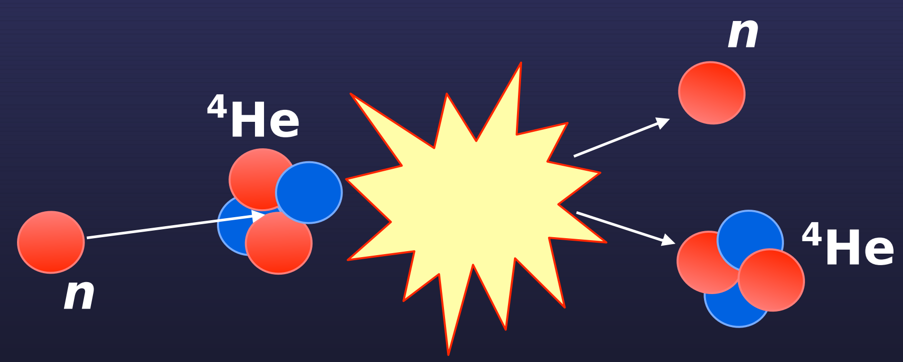
Hamiltonian kernel $\langle\phi_{\nu' r'}^{J\pi T}|\,\mathcal{A}_{\nu'} H \mathcal{A}_{\nu}\,|\phi_{\nu r}^{J\pi T}\rangle \propto \langle\Phi^{(A-1)}|\,a^\dagger a^\dagger a^\dagger a a a\,|\Phi^{(A-1)}\rangle$

for single-nucleon projectiles and including 3N forces

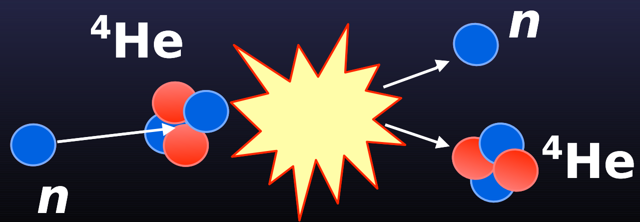


Nucleon-^{4}He Scattering

In collaboration with

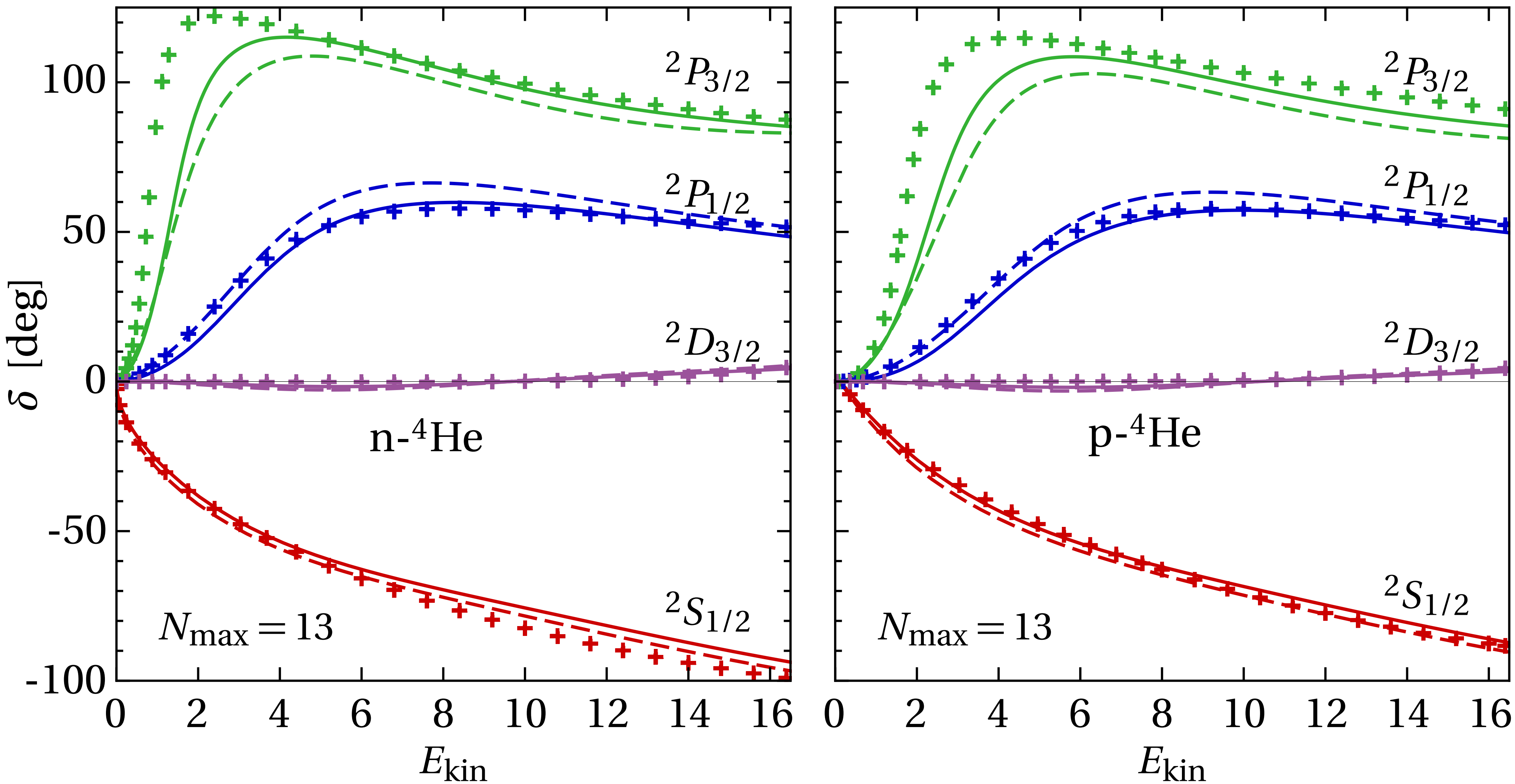
G. Hupin, S. Quaglioni, P. Navrátil & R. Roth

G. Hupin, J. Langhammer et al. ----- Phys. Rev C **88** 054622 (2013)



3N Force Effects on Phase Shifts

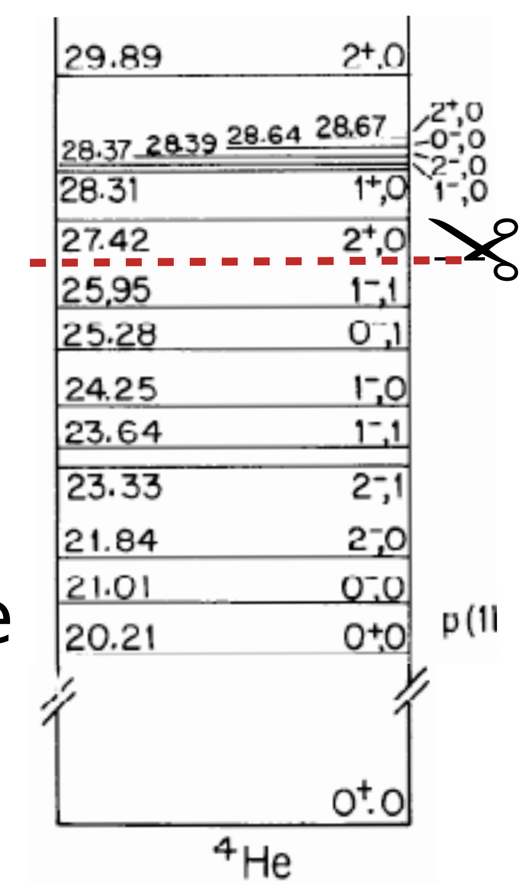
G. Hupin, J. Langhammer et al. - Phys. Rev C **88** 054622 (2013)

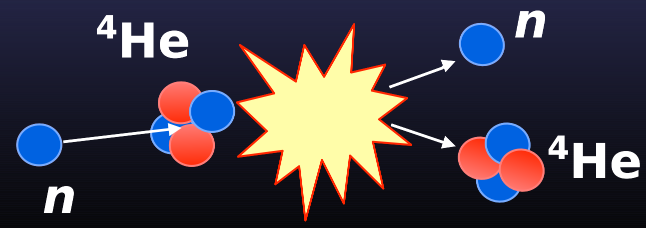


$N_{max} = 13$
$E_{3max} = 14$
$\hbar\Omega = 20\,\mathrm{MeV}$
$\alpha = 0.0625\,\mathrm{fm}^4$
$\lambda = 2.0\,\mathrm{fm}^{-1}$

+ Experiment
- - - 3N-induced
— 3N-full

Including seven eigenstates of ^{4}He





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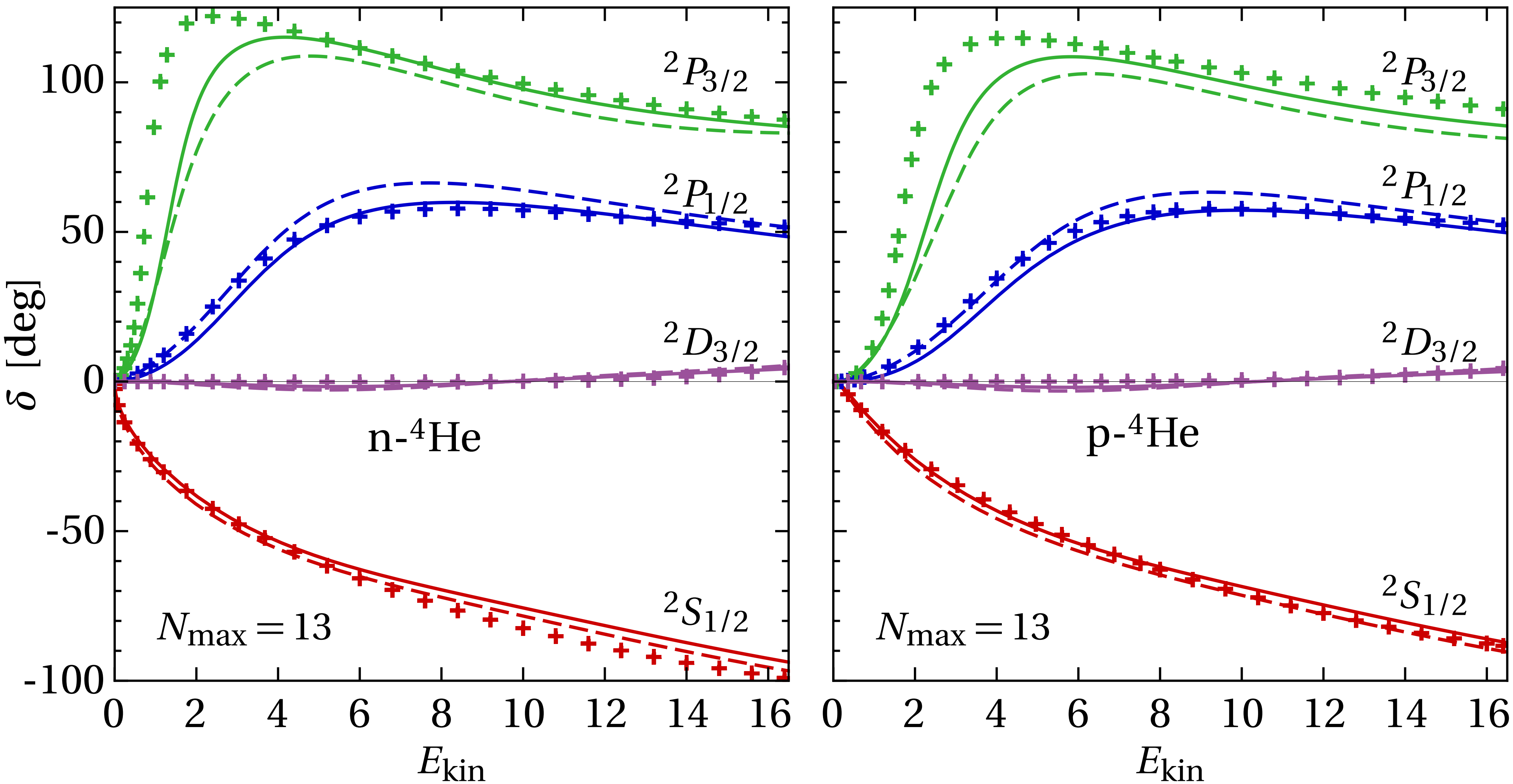

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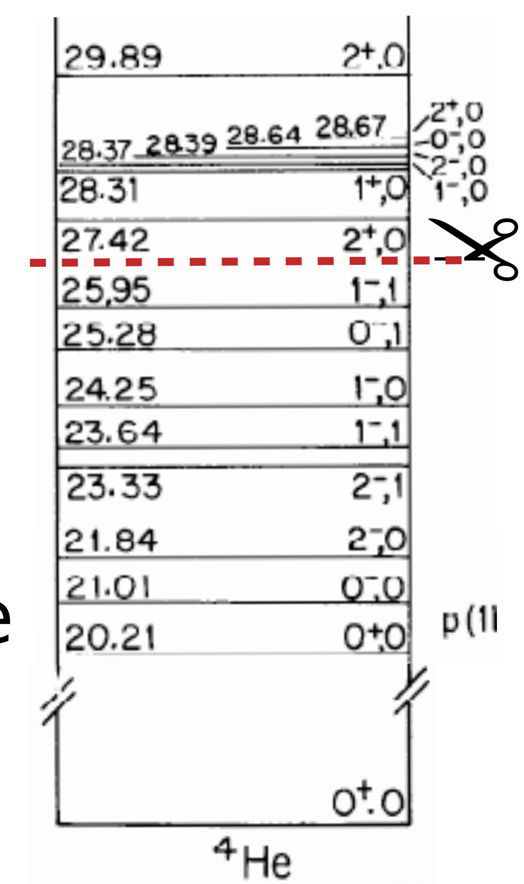

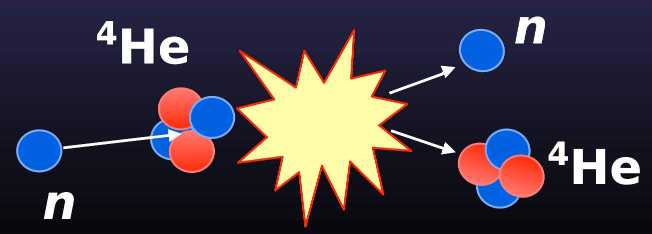

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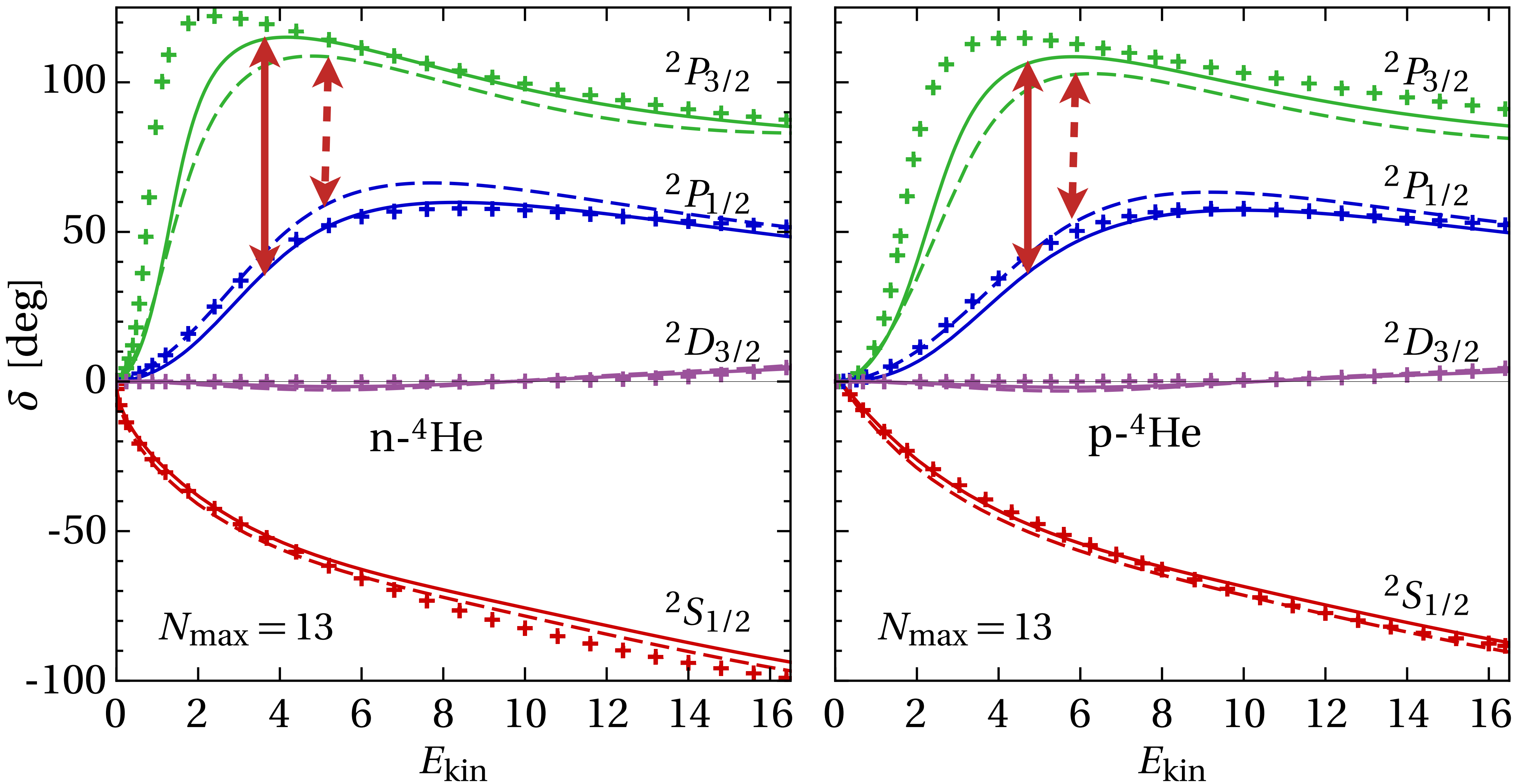

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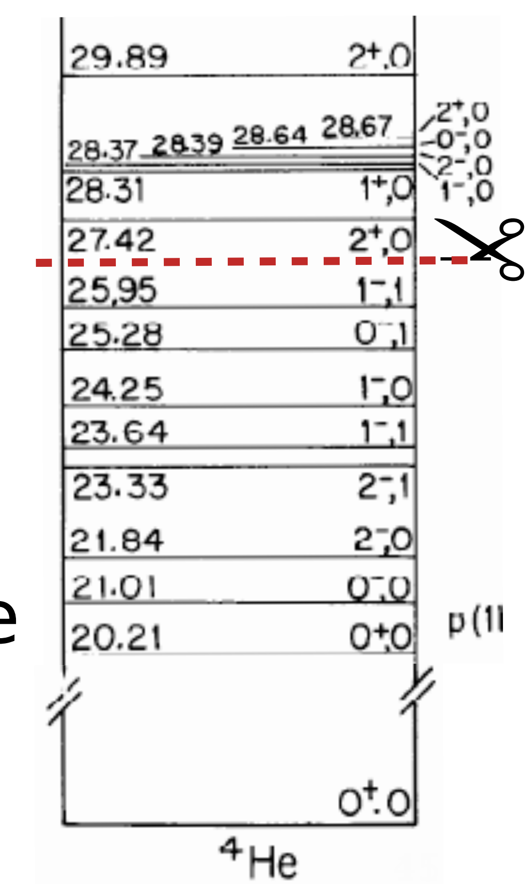
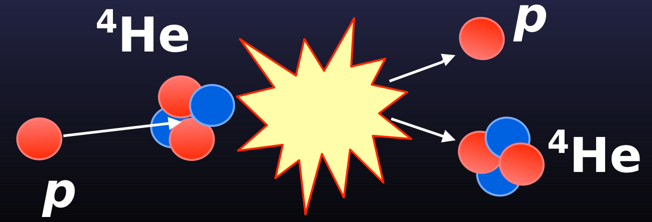
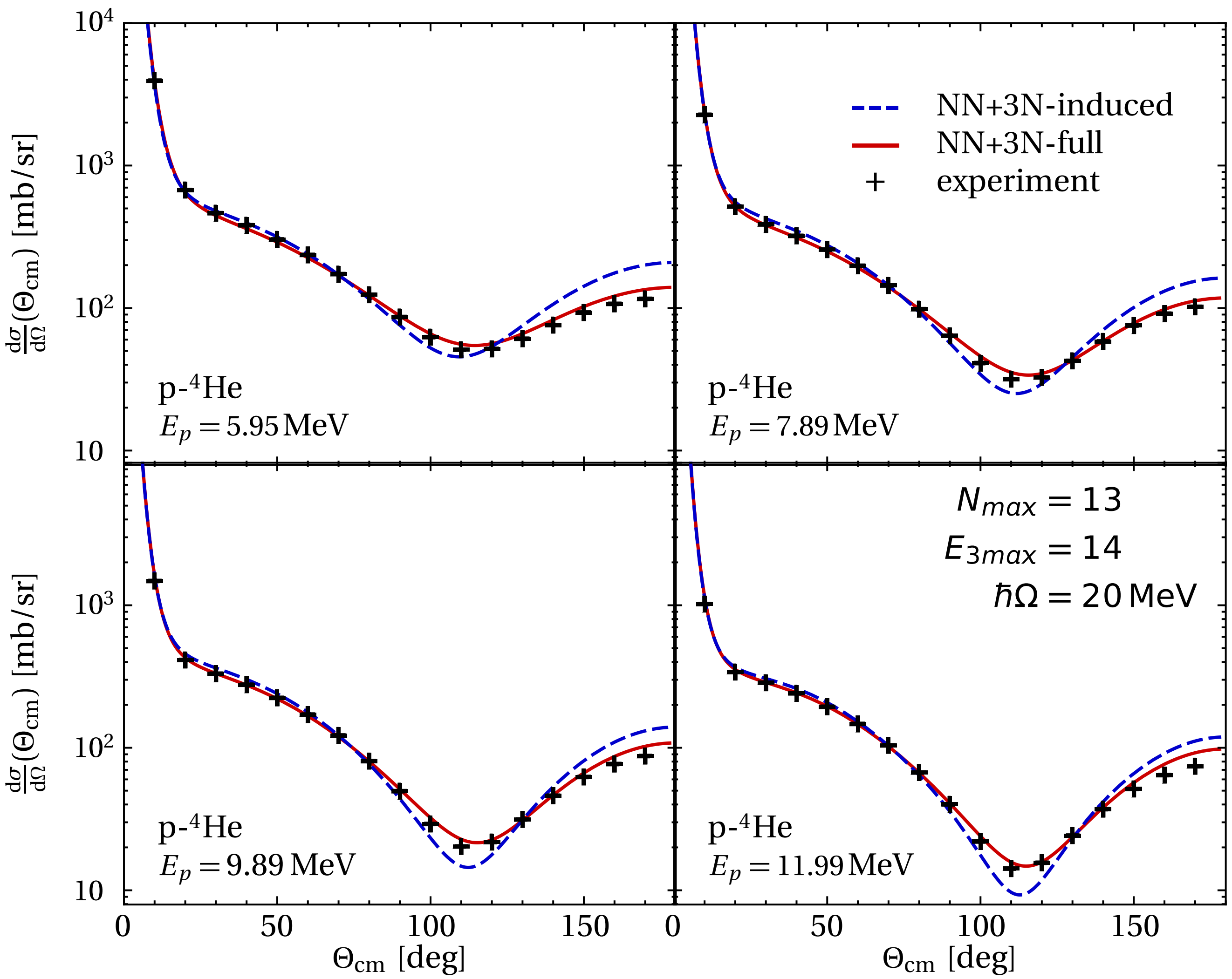
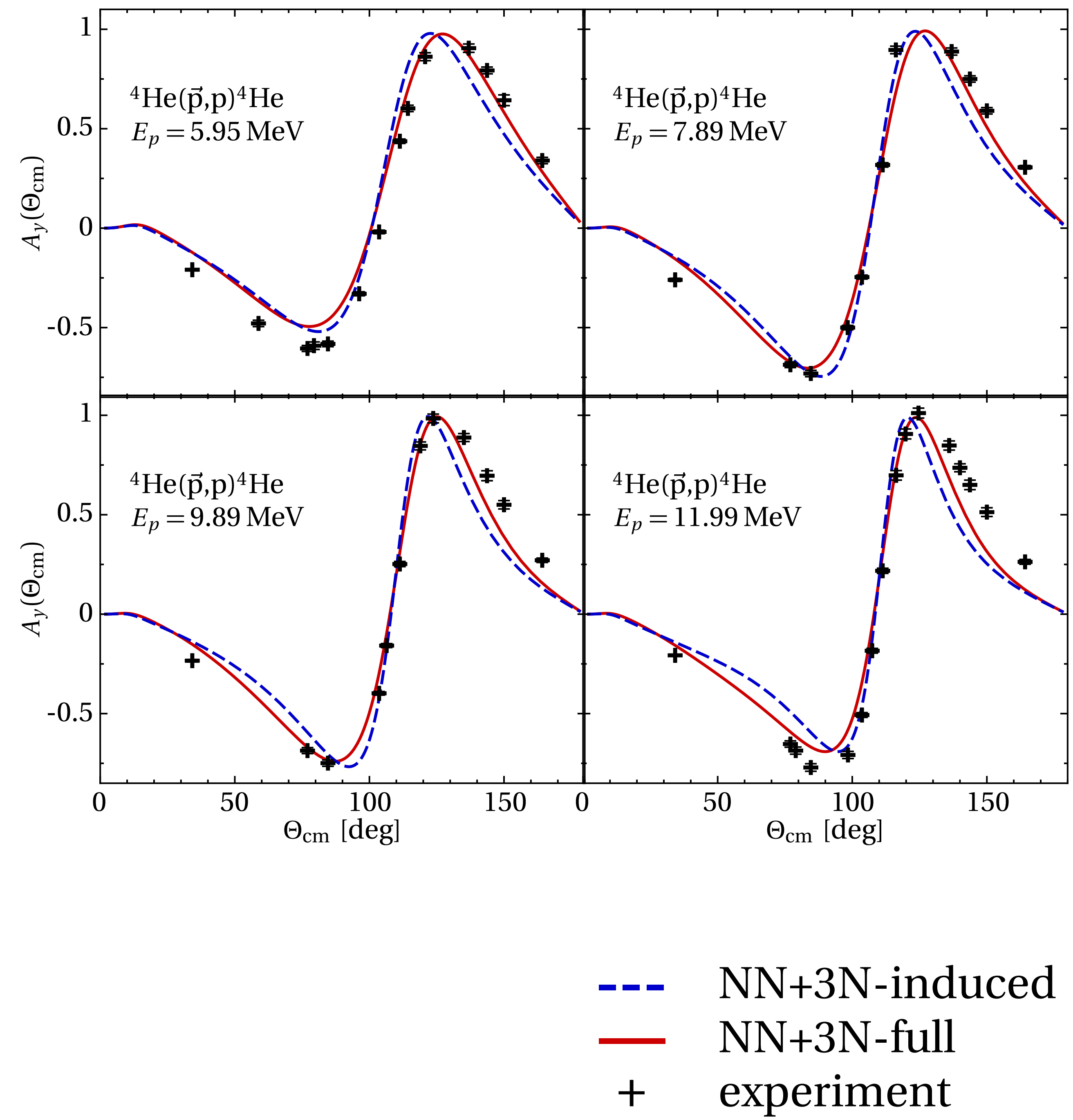


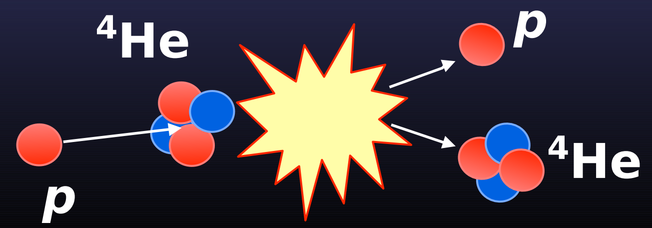


Cross Section & Analyzing

G. Hupin, J. Langhammer et al. - Phys. Rev C **88** 054622 (2013)







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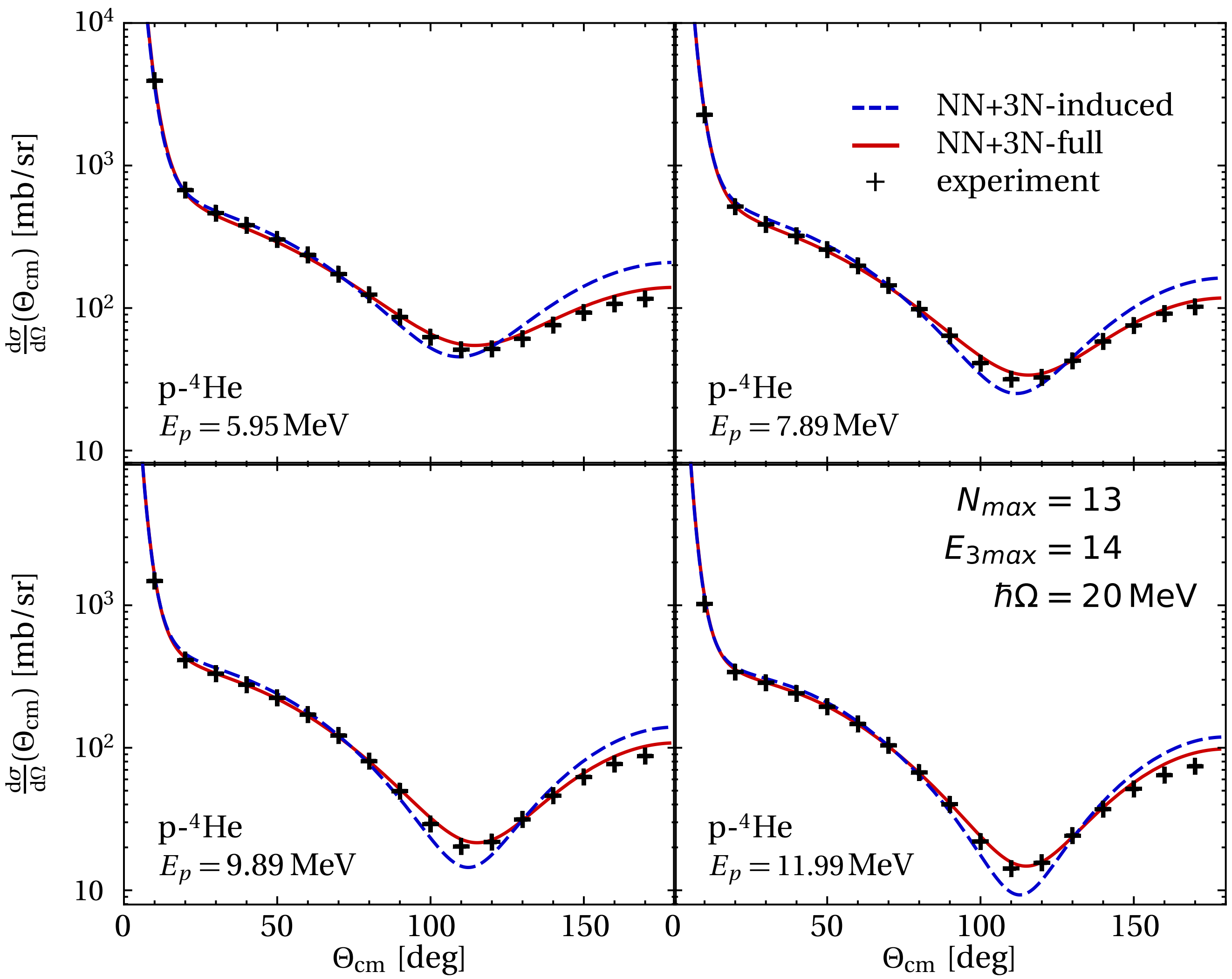
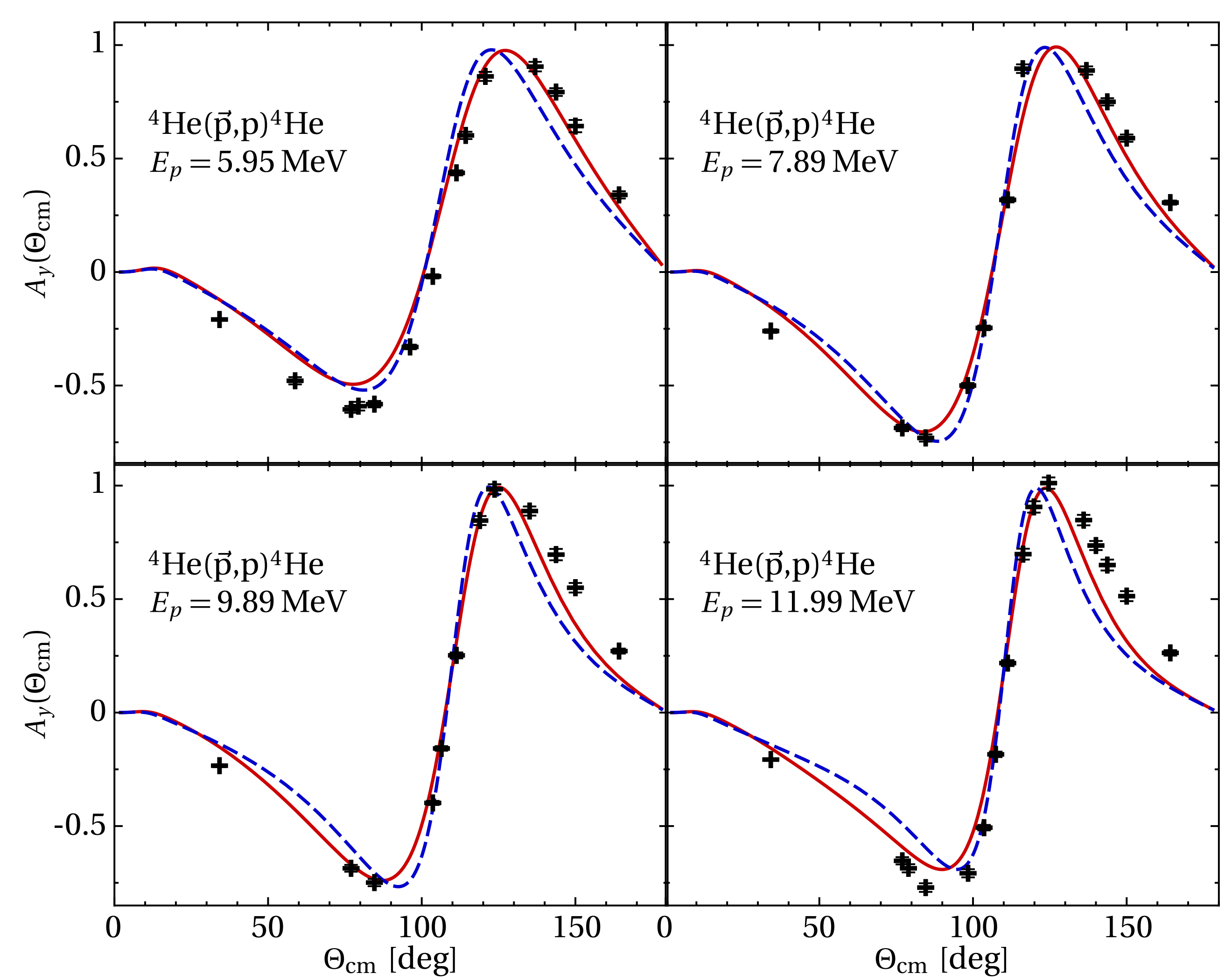




Chiral 3N improves agreement with experiment

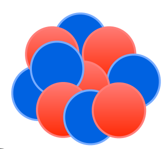
NN+3N-induced
NN+3N-full
+ experiment

NCSMC Formalism with 3N Forces

- Representing $H|\psi^{J\pi T}\rangle = E|\psi^{J\pi T}\rangle$ using the **over-complete basis**

$$|\Psi^{J\pi T}\rangle = \sum_{\lambda} c_{\lambda} |\Psi_A E_{\lambda} J^{\pi} T\rangle + \sum_{\nu} \int \mathrm{d}r\, r^2 \frac{\chi_{\nu}(r)}{r} |\xi^{J\pi T}_{\nu r}\rangle$$

Expansion in A-body (IT-)NCSM eigenstates



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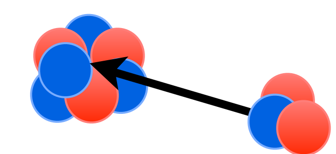

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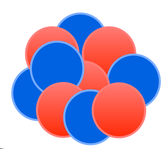

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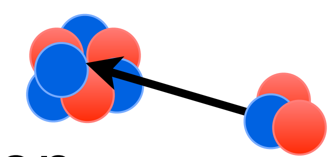

leads to the NCSMC equations

$$\begin{pmatrix} H_{\mathrm{NCSM}} & h \\ h & \mathscr{H} \end{pmatrix} \begin{pmatrix} c \\ \chi(r)/r \end{pmatrix} = E \begin{pmatrix} \mathbb{1} & g \\ g & \mathbb{1} \end{pmatrix} \begin{pmatrix} c \\ \chi(r)/r \end{pmatrix}$$

3N forces contribute in

H_{NCSM}: Covered by (IT-)NCSM

h: Given by $\langle \Psi_A E_{\lambda'} J^\pi T | \hat{H} | \xi_{\nu r}^{J\pi T} \rangle$

$\mathscr{H}$: Contains the NCSM/RGM Hamiltonian kernel

NCSMC Formalism with 3N Forces

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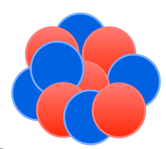

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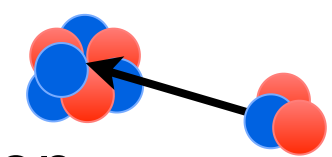

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Accessing targets beyond ^{4}He using uncoupled densities

3N forces contribute in

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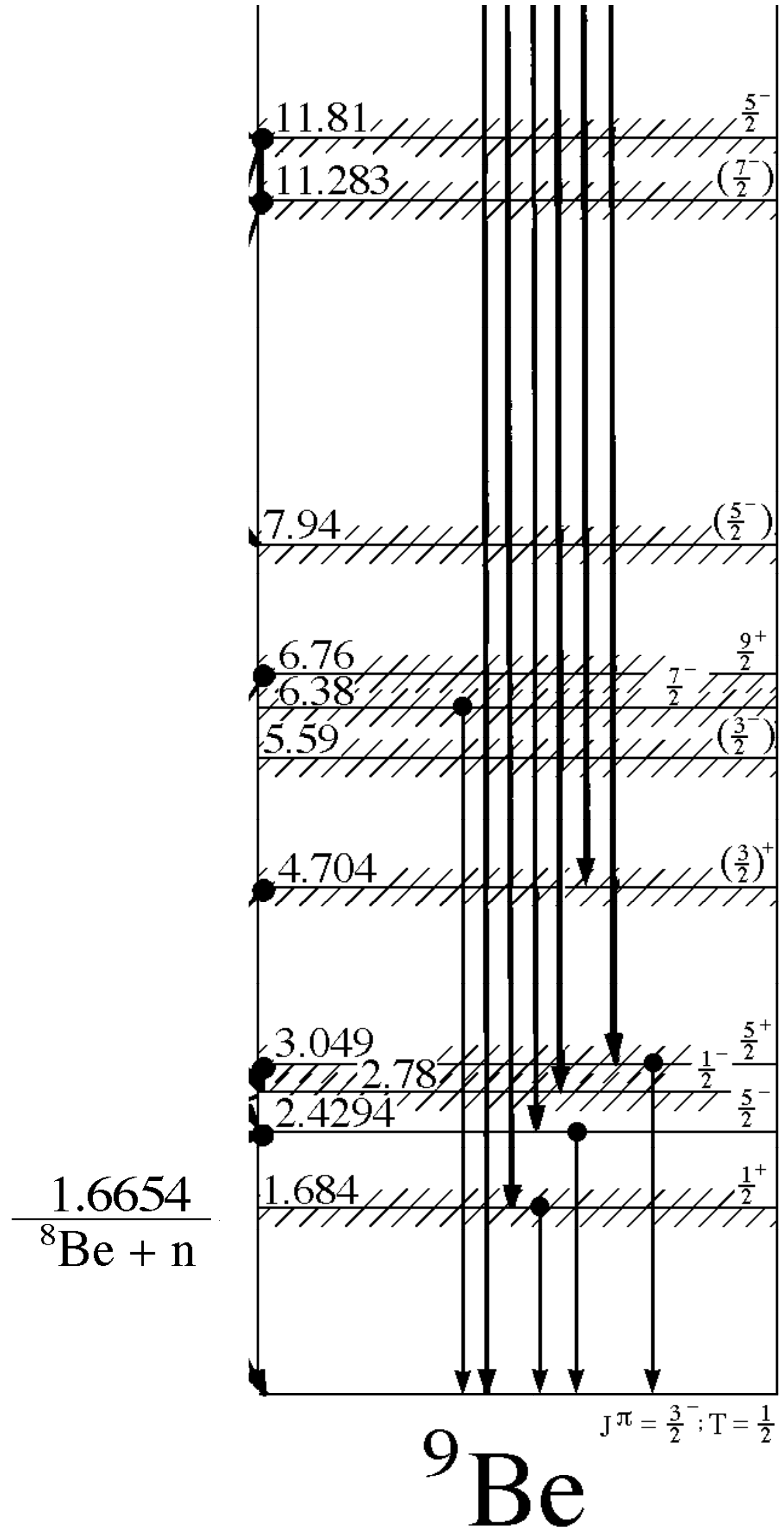
Ab-initio Description of ^{9}Be via NCSMC

Collaboration with Petr Navrátil

- All excited states are resonances
- Study the impact of the continuum by investigating neutron-^{8}Be scattering



Ab-initio Description of ^{9}Be via NCSMC

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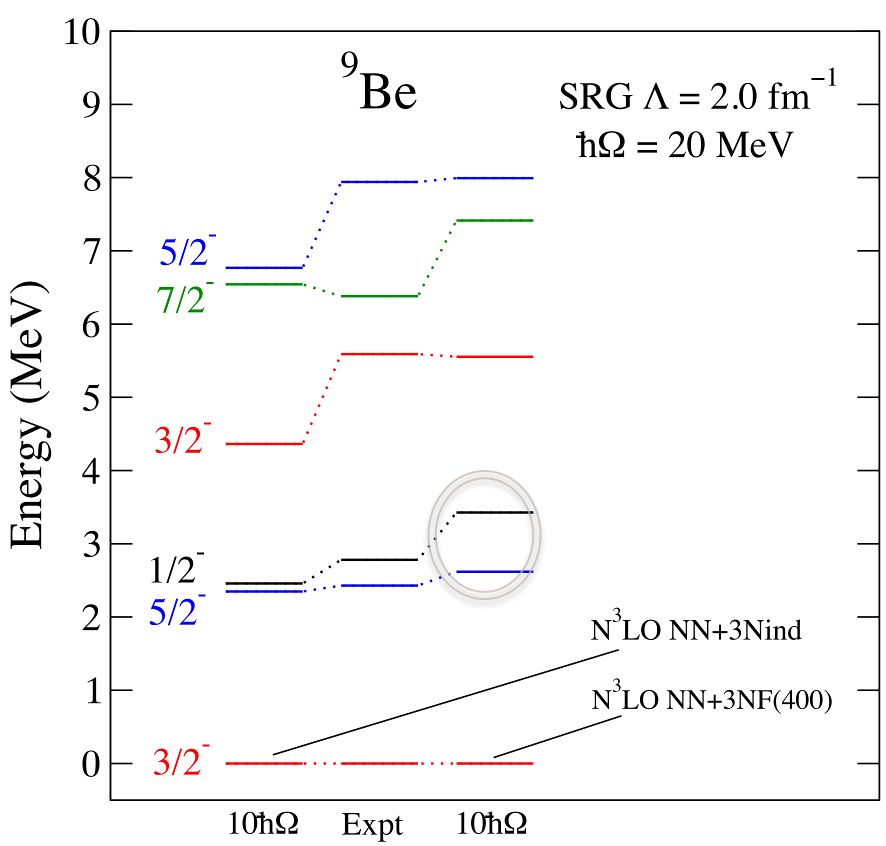
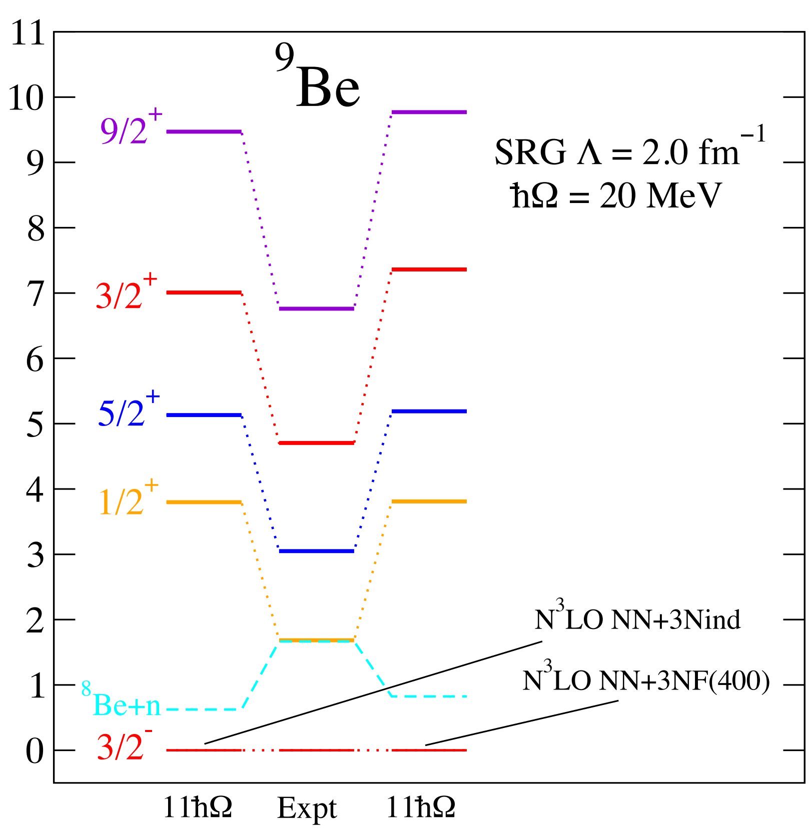
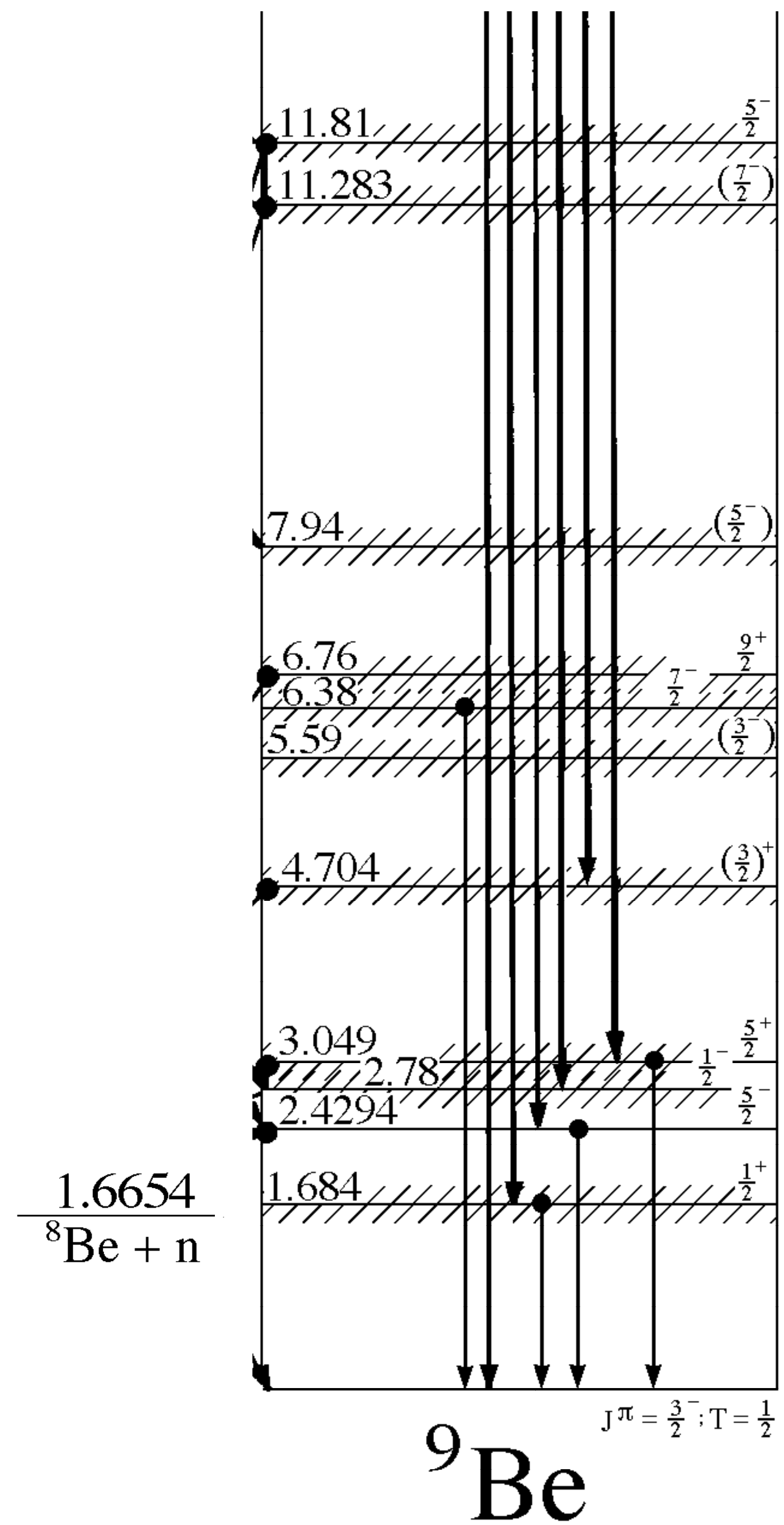
- NCSM with 3N forces reveals large discrepancies compared to experiment







3N Force Effects on Phase Shifts

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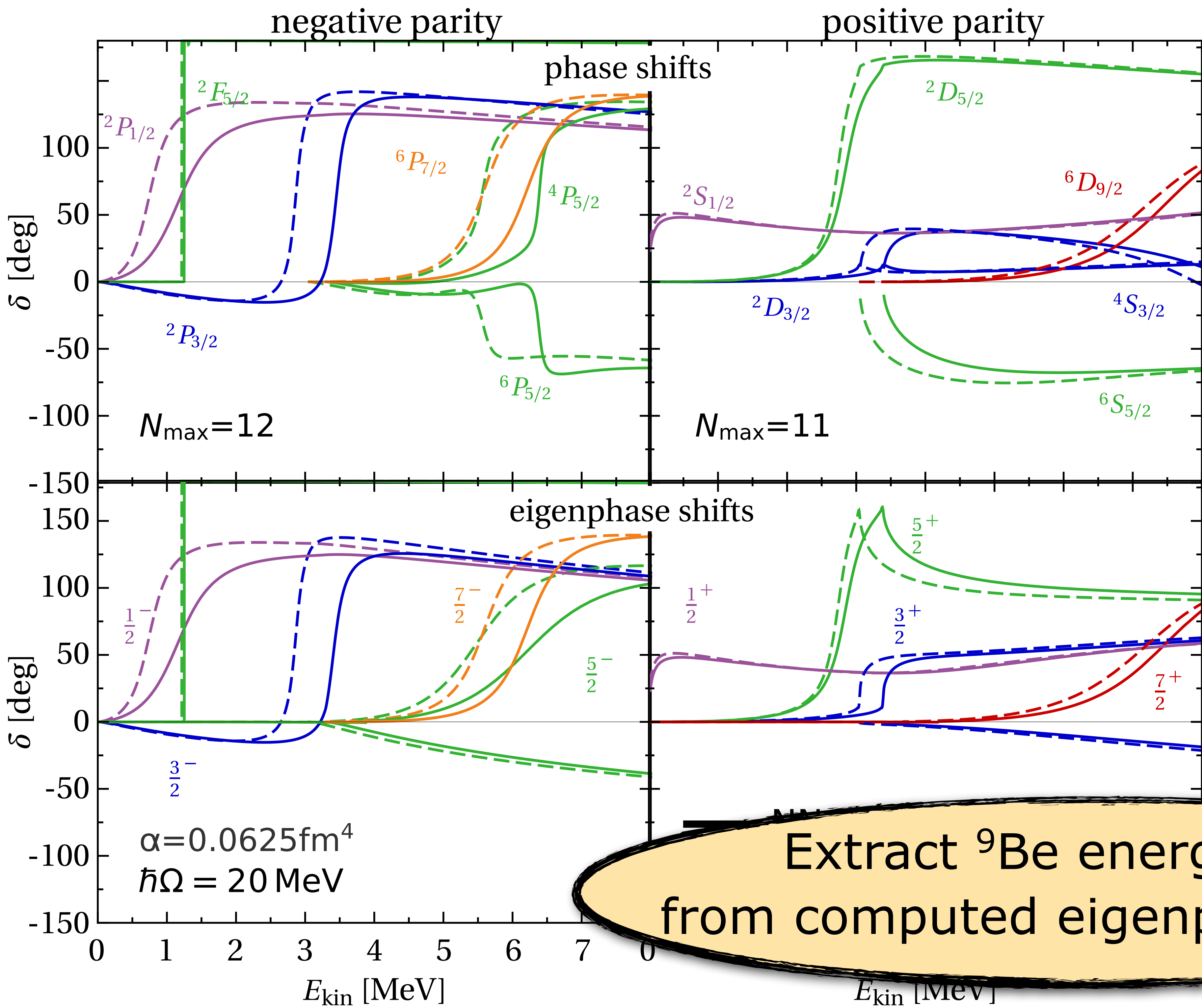

Extract ^{9}Be energy levels from computed eigenphase shifts…

^{9}Be Energy Levels: NCSM vs. NCSMC

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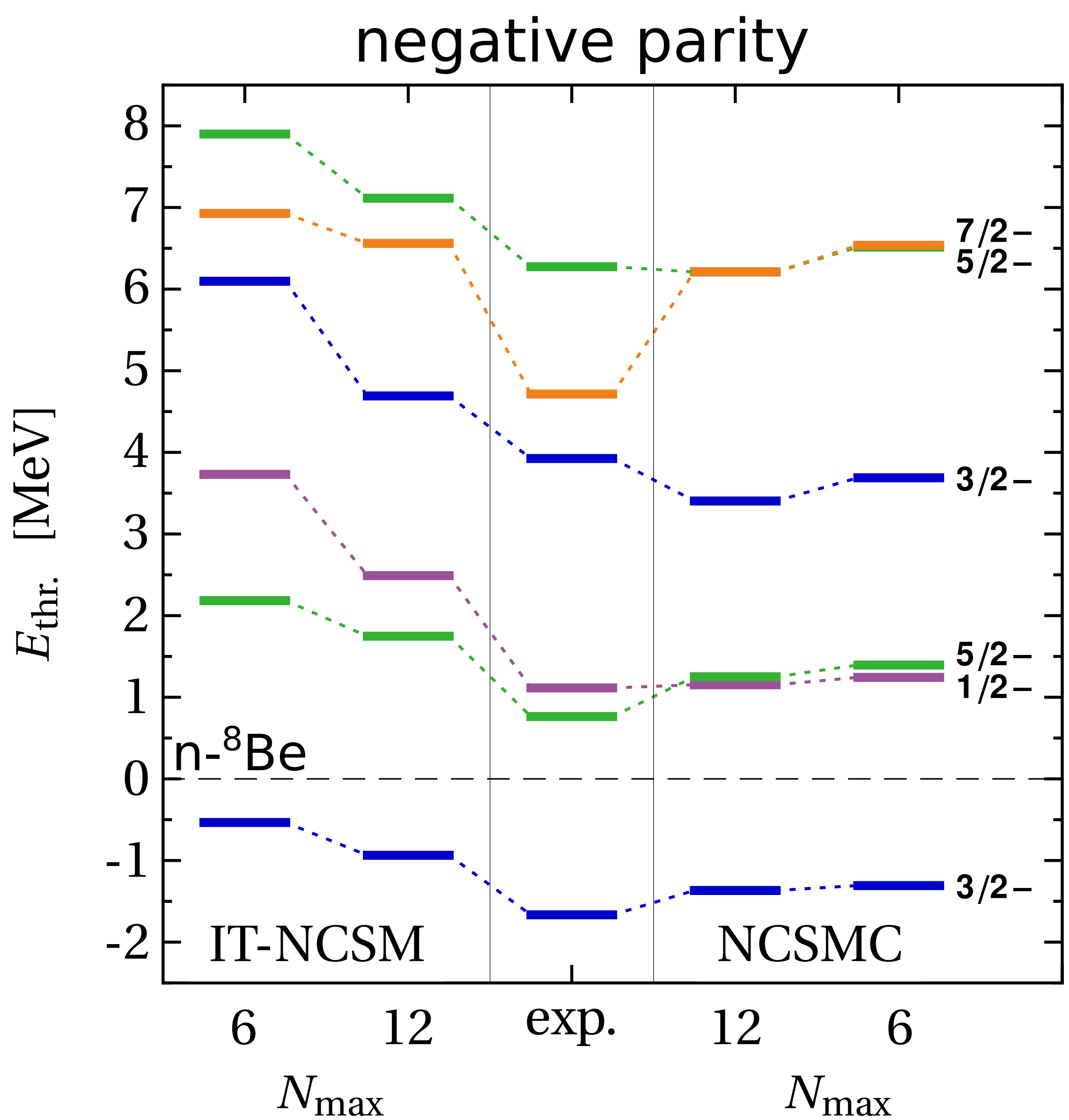

^{9}Be Energy Levels: NCSM vs. NCSMC

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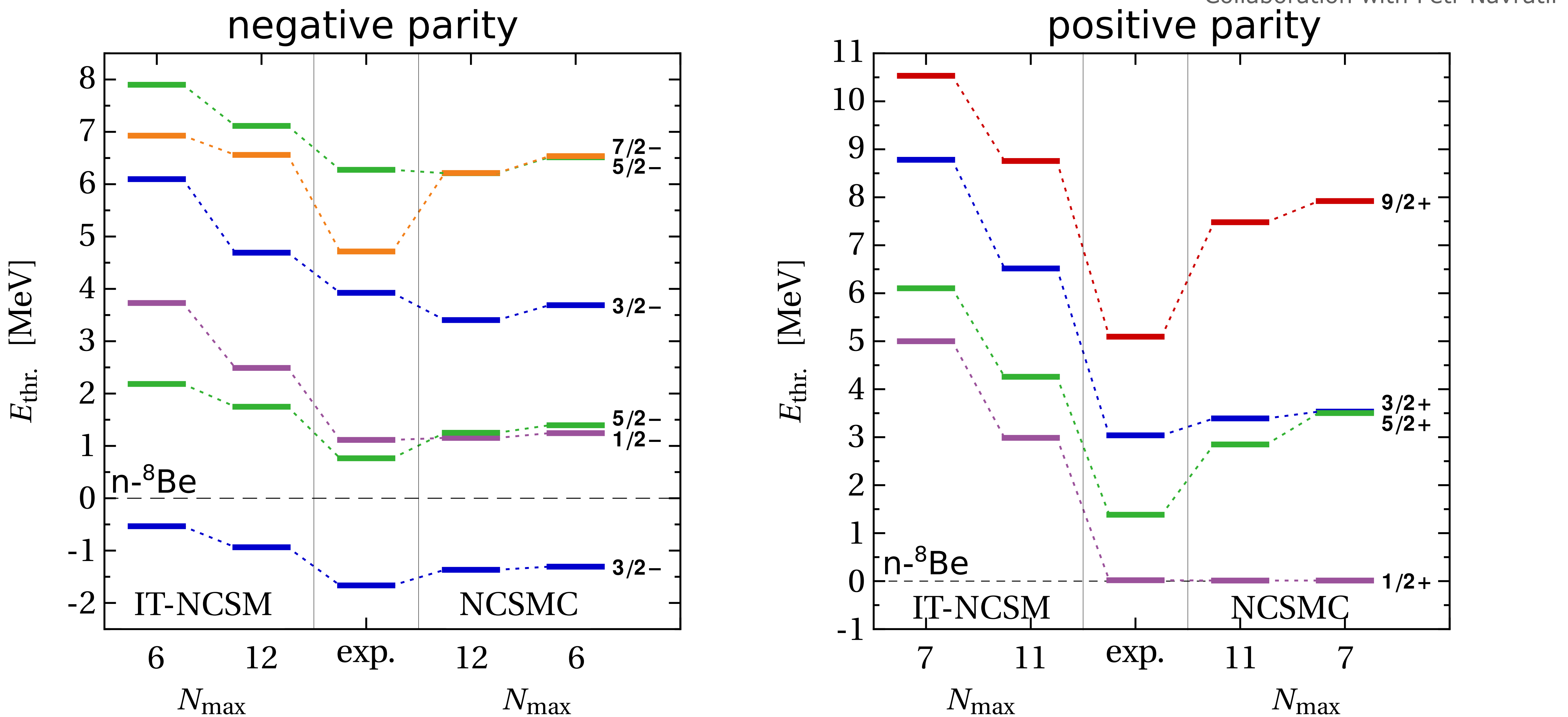

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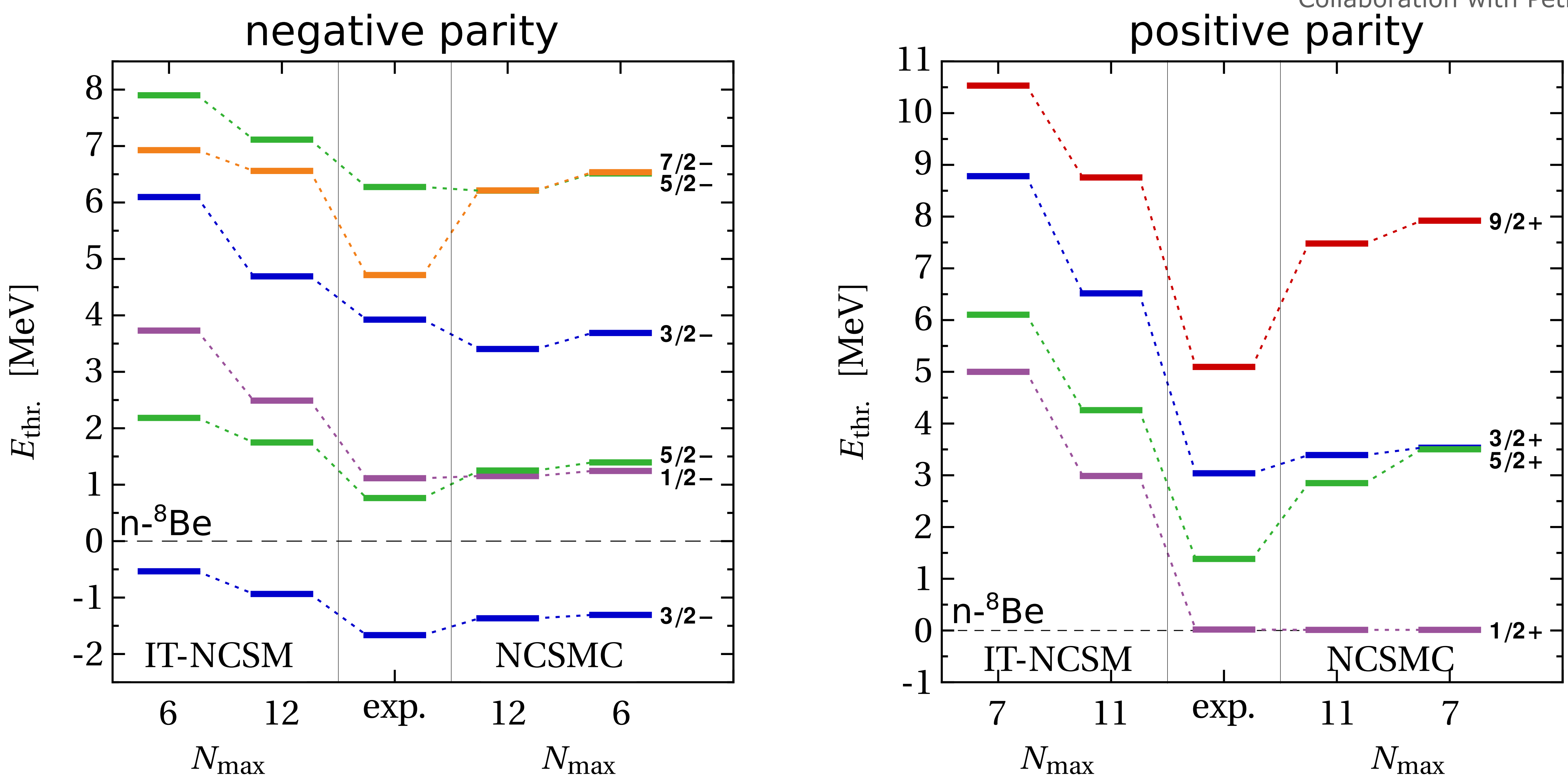
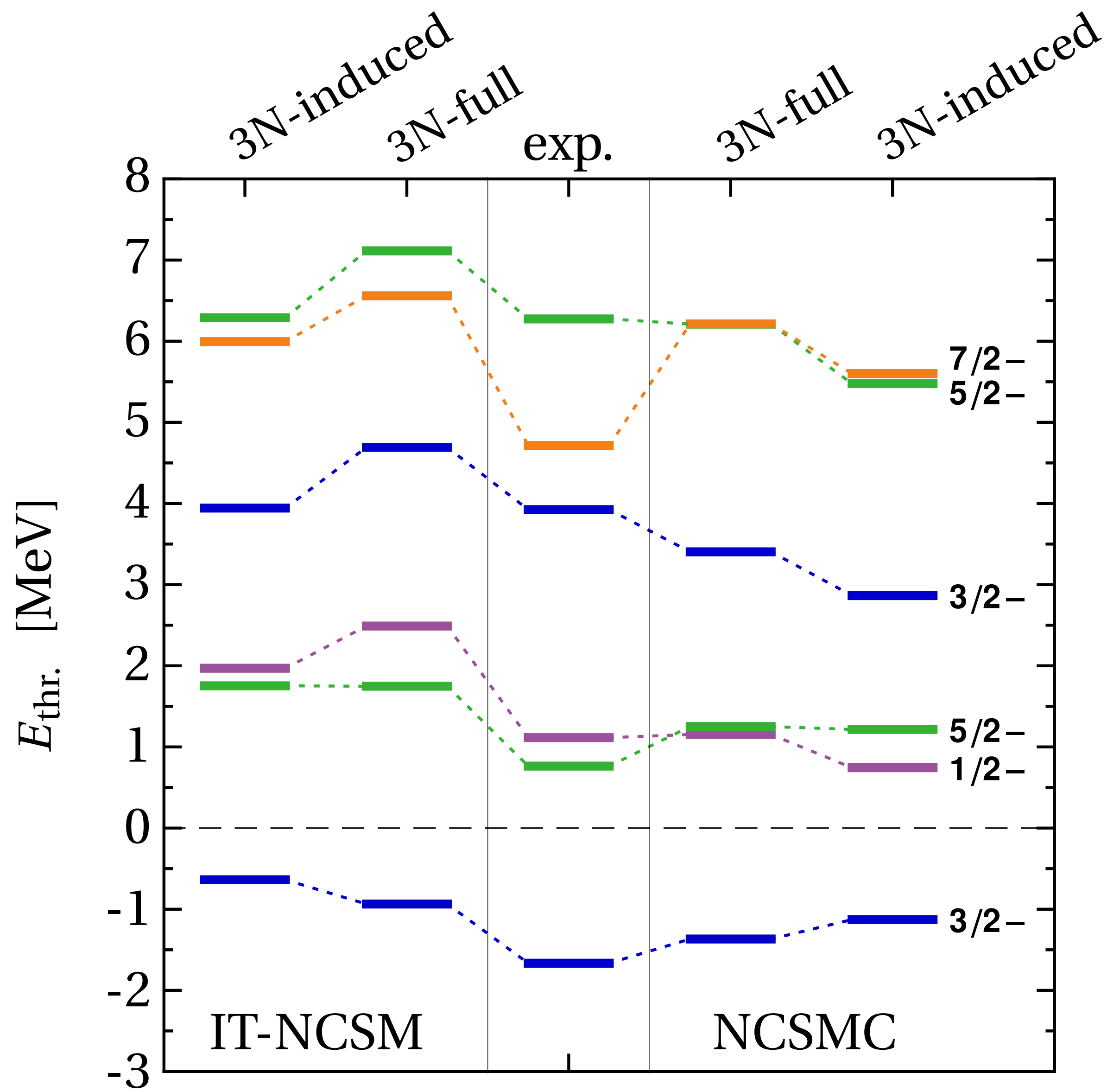


- Significant contributions from the continuum degrees of freedom
- Excellent agreement with experiment for 1/2$^-$ & second 5/2$^-$ as well as the 1/2$^+$ and 3/2$^+$ states
- NCSMC seems to be well-converged already at moderate N_{max}

^{9}Be Energy Levels: NCSM vs. NCSMC

Collaboration with Petr Navrátil



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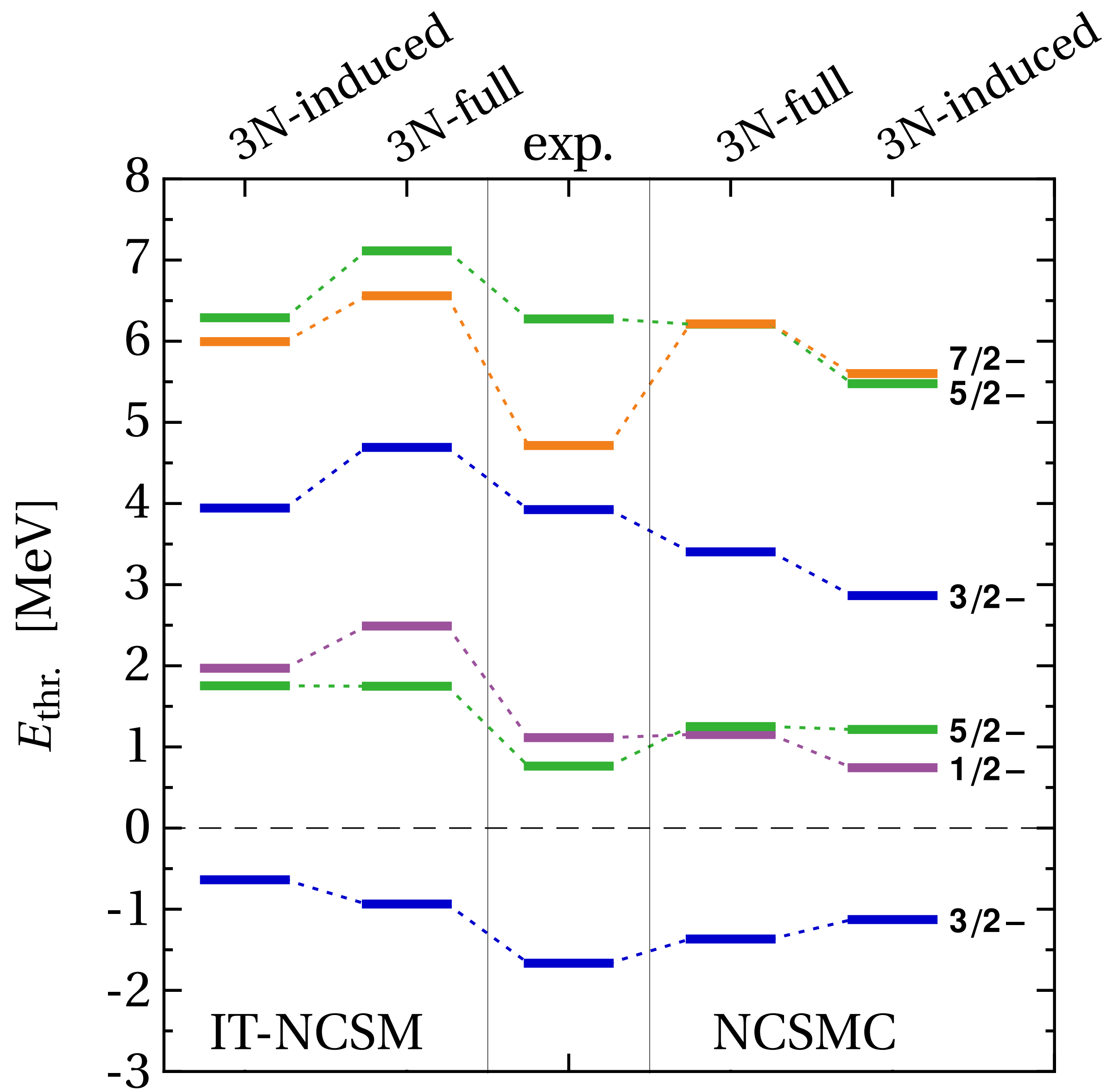
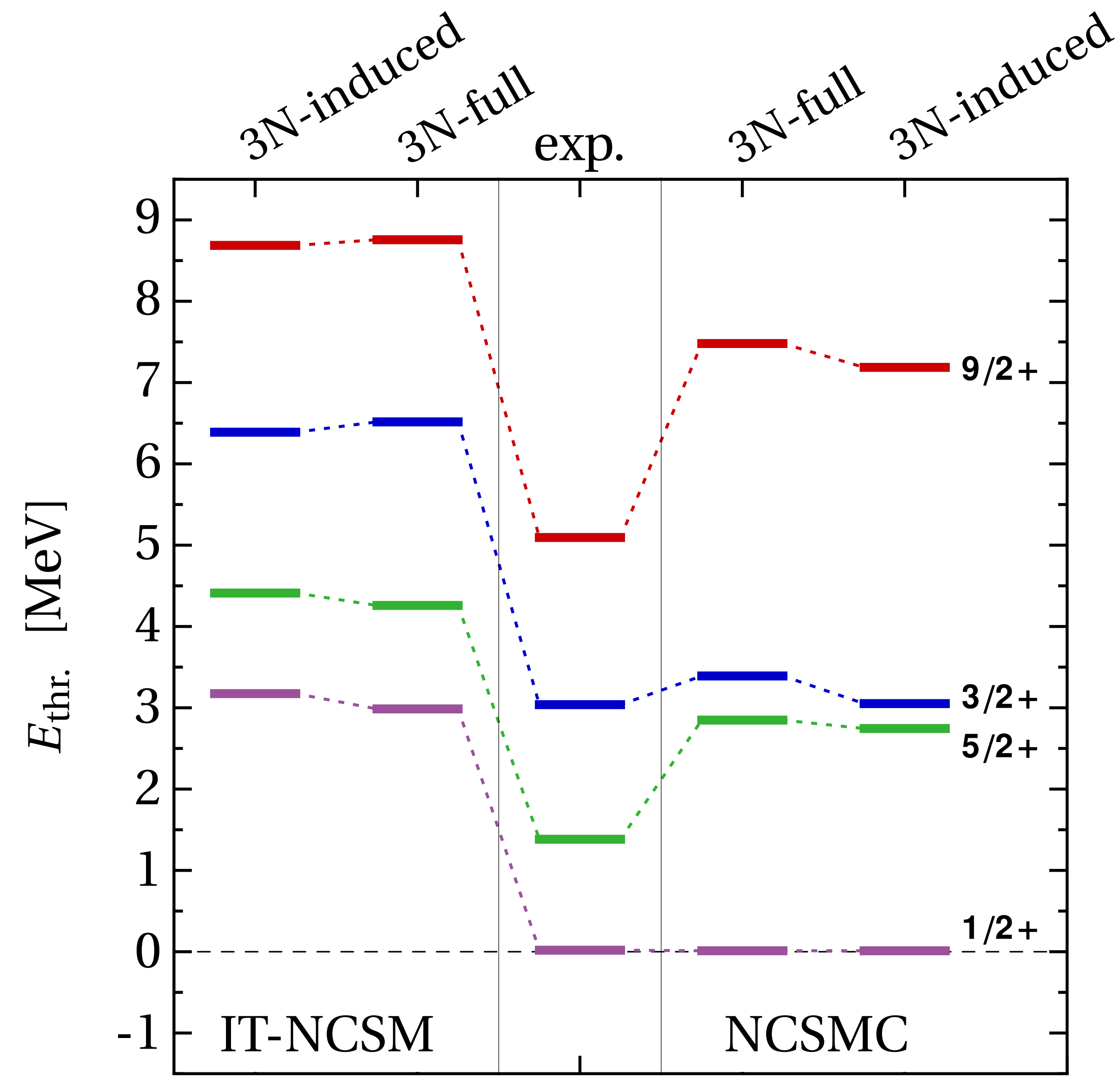
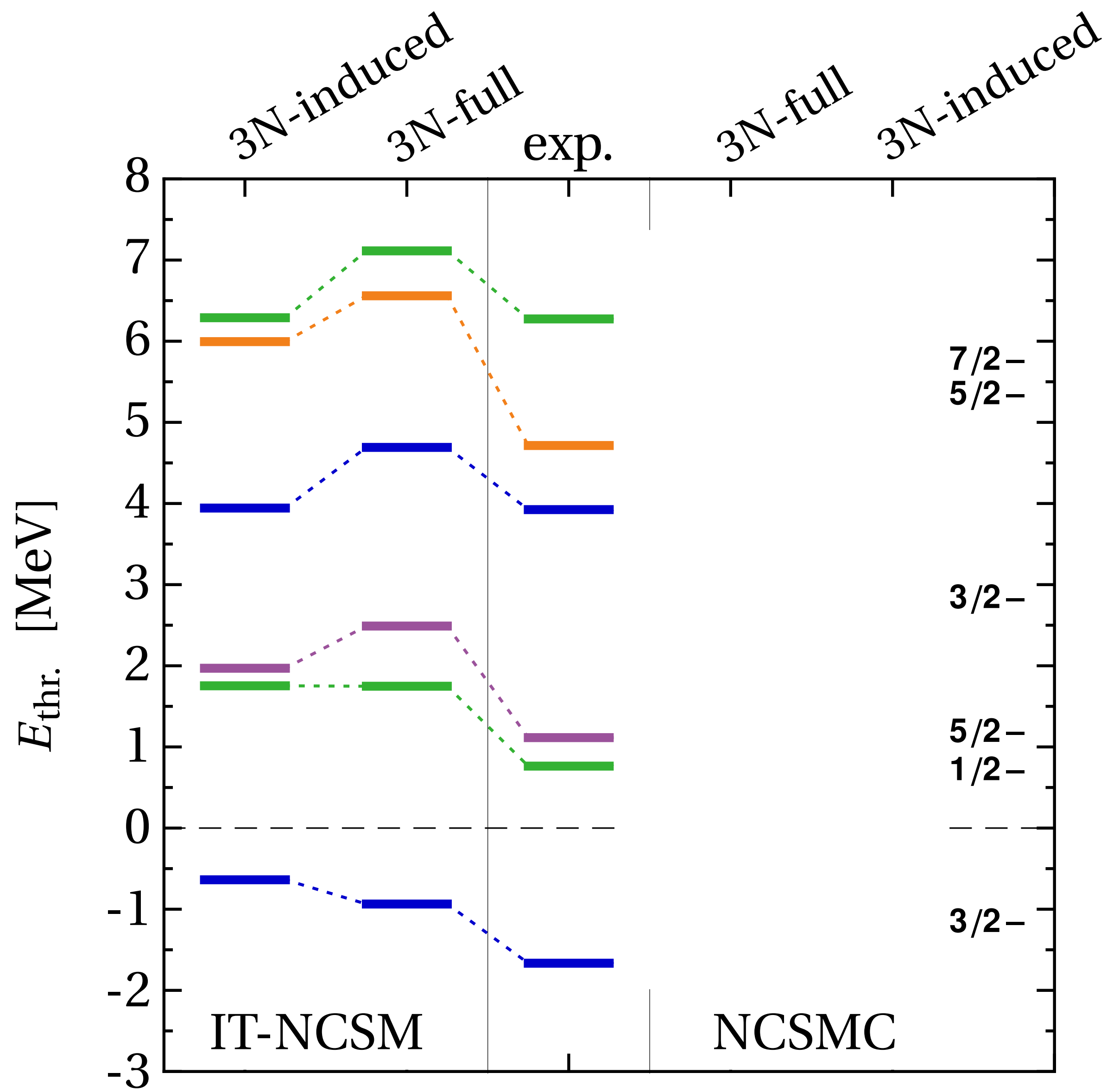
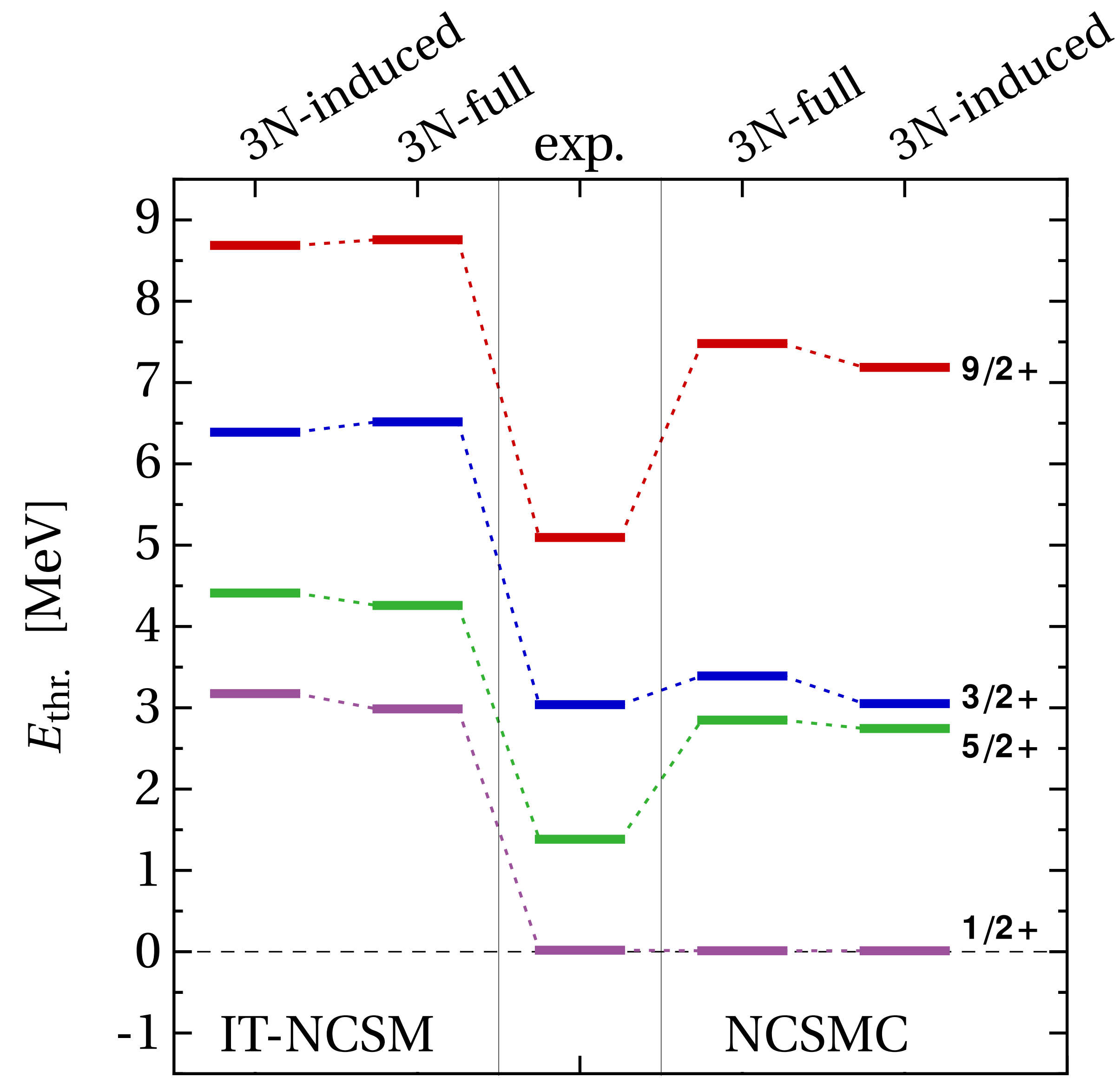




^{9}Be Energy Levels: NCSM vs. NCSMC

Collaboration with Petr Navrátil





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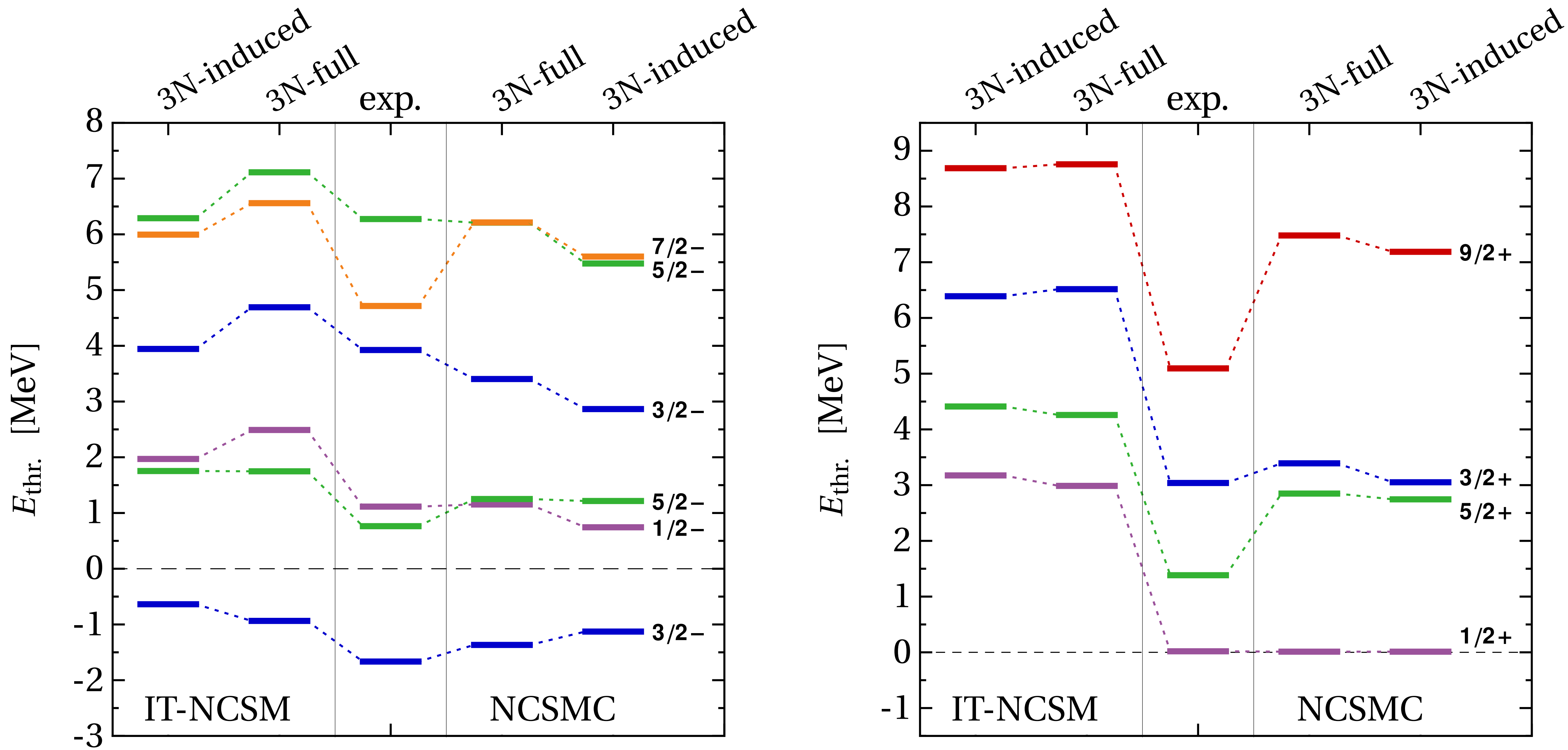

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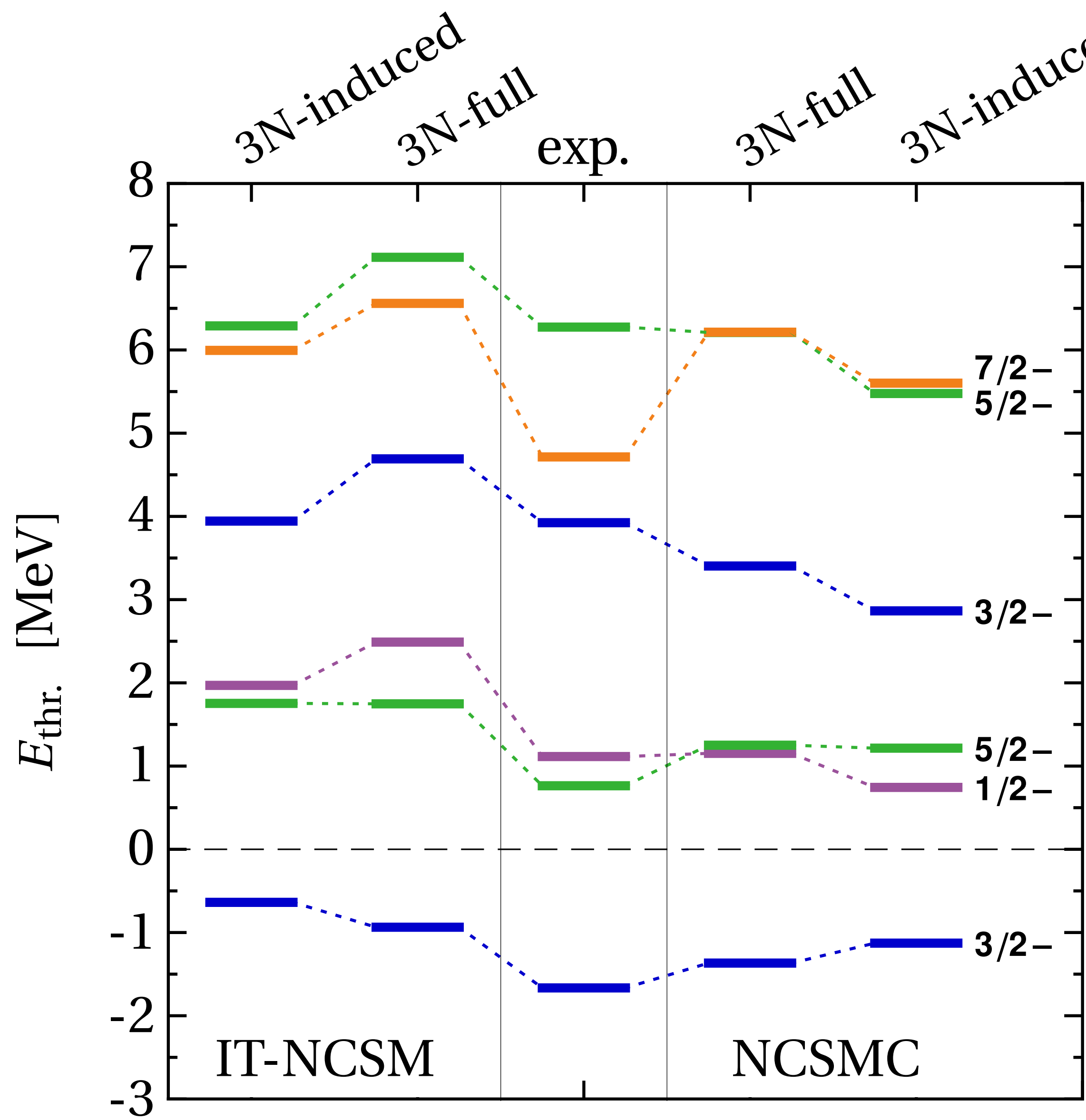

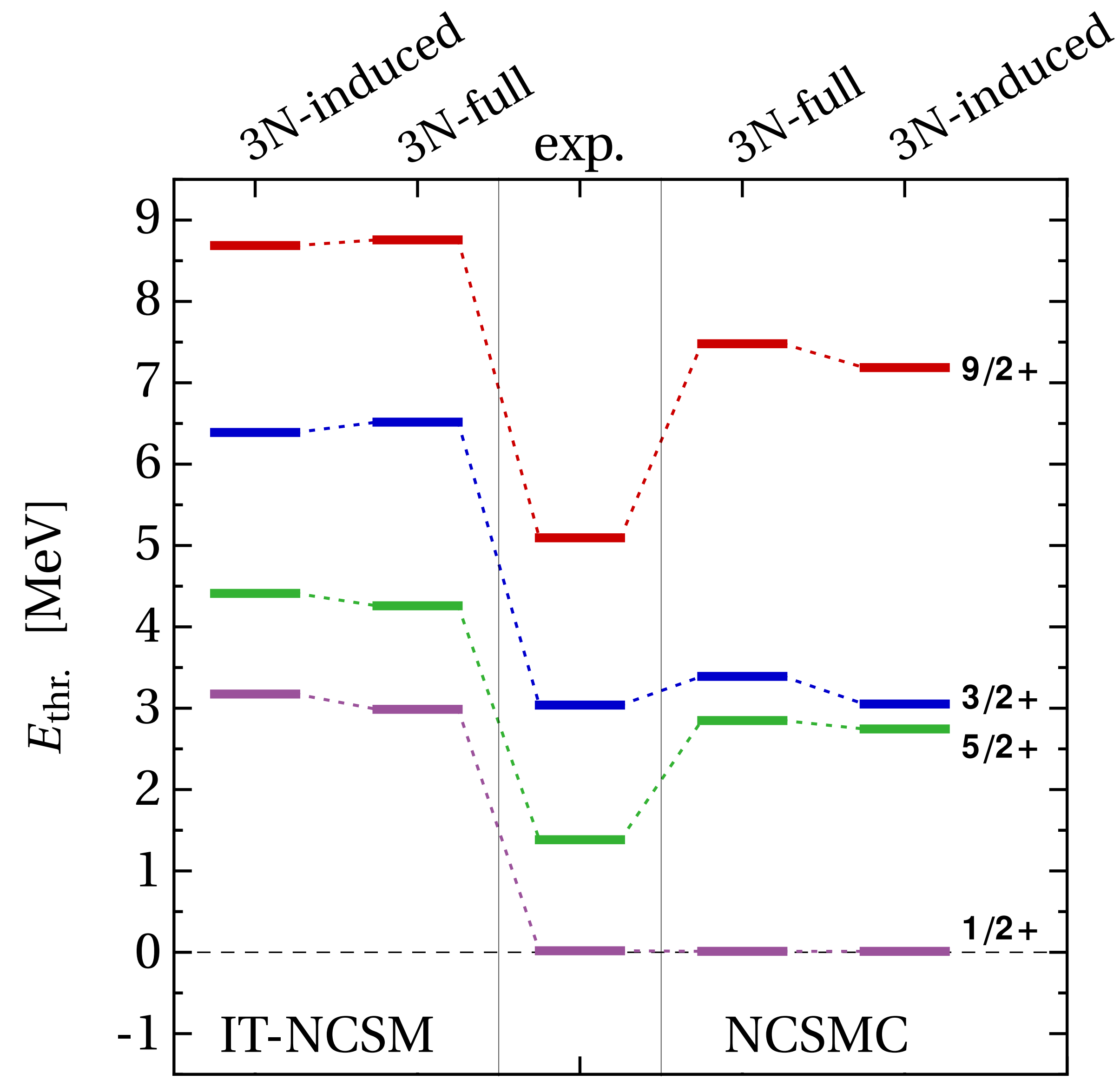

- Treatment of continuum important for conclusions about 3N interactions
- First 5/2$^-$ insensitive to the chiral 3N interaction
- 7/2$^-$ resonance → interaction problem?

Conclusions

Conclusions

Nuclear structure and reactions accessible
with full 3N treatment via the
No-Core Shell Model with Continuum

- **Inclusion of 3N forces** challenging but **completed** for single- and two-nucleon projectiles
- New computational scheme $\Longrightarrow$ **heavier targets accessible**
- Promising results for n-^{8}Be (and p-^{10}C and n-^{16}C)
- Proper **treatment of continuum vital** for validation of chiral 3N interactions

Epilogue

- **thanks to my group & collaborators**
  - S. Binder, A. Calci, E. Gebrerufael, S. Fischer, H. Krutsch, R. Roth, S. Schulz, C. Stumpf, A. Tichai, R. Trippel, R. Wirth

- P. Navrátil
  TRIUMF, Vancouver, Canada
- G. Hupin, S. Quaglioni
  LLNL, Livermore, USA
- J. Vary, P. Maris
  Iowa State University, USA
- H. Hergert
  The Ohio State University, USA
- P. Piecuch, S. Bogner
  Michigan State University, USA
- H. Feldmeier, T. Neff
  GSI Helmholtzzentrum
- P. Papakonstantinou
  IPN Orsay, France
- K. Hebeler
  TU Darmstadt

Computing Time

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Deutsche Forschungsgemeinschaft DFG

HIC for FAIR Helmholtz International Center

LOEWE Exzellente Forschung für Hessens Zukunft

HELMHOLTZ GEMEINSCHAFT

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Thanks for your attention!

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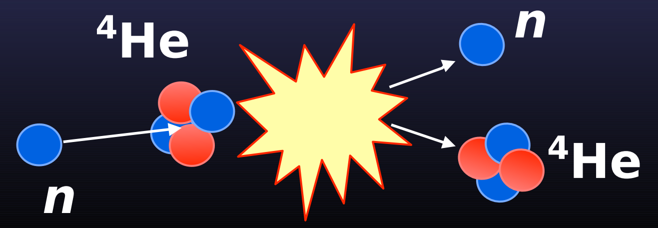
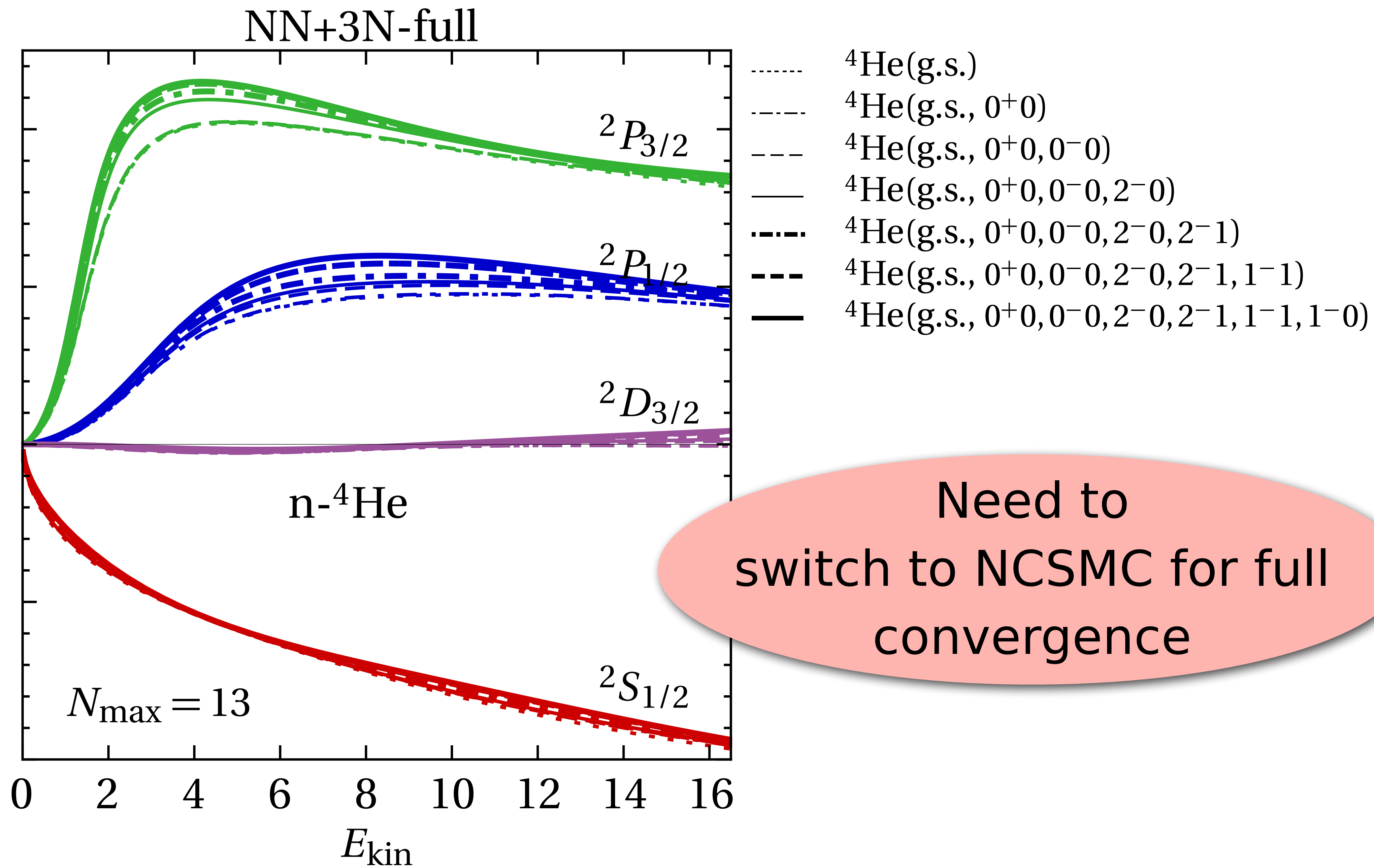


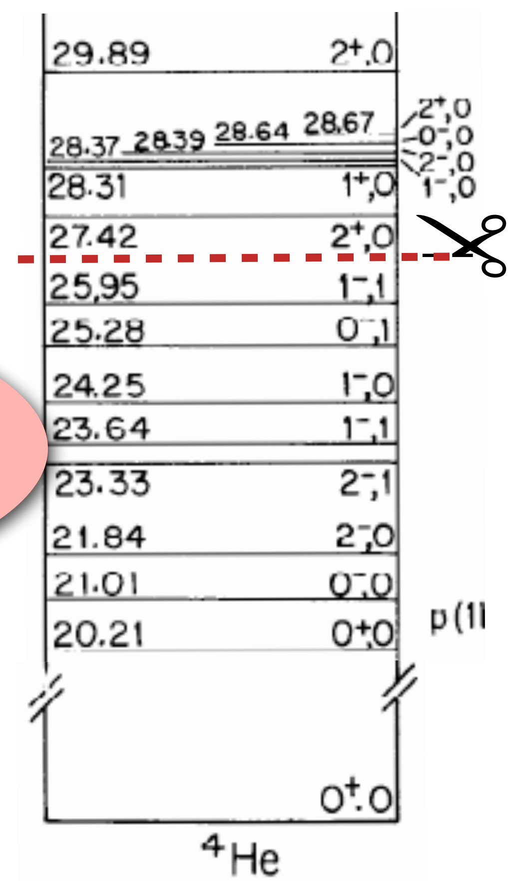
3N Force Effects on Phase Shifts

G. Hupin, J. Langhammer et al. - Phys. Rev C **88** 054622 (2013)

Inclusion of more excited states

$N_{max} = 13$
$E_{3max} = 14$
$\hbar\Omega = 20\,\mathrm{MeV}$
$\alpha = 0.0625\,\mathrm{fm}^4$
$\lambda = 2.0\,\mathrm{fm}^{-1}$





Need to switch to NCSMC for full convergence