Three-Nucleon Forces
in the
No-Core Shell Model with Continuum

Joachim Langhammer

TECHNISCHE UNIVERSITÄT DARMSTADT

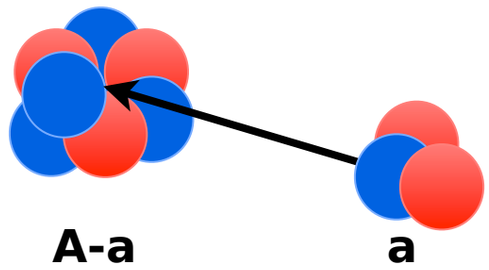
Outline

- What we are aiming for...
- Ingredients from Three-Body Technology
- 3N Forces in the NCSM/RGM and NCSMC
  - How to access targets heavier than ^{4}He
  - Continuum effects on the ^{9}Be energy levels
  - First results: p-^{10}C & n-^{16}C scattering
- Conclusions

What we are aiming for...

Realistic ab-initio description of light nuclei

Bound states
& spectroscopy



Resonances
& scattering states

(IT-)NCSM
Ab-initio description
nuclear clusters

(IT-)NCSM/RGM
& NCSMC approaches

RGM
Describing relative
motion of clusters

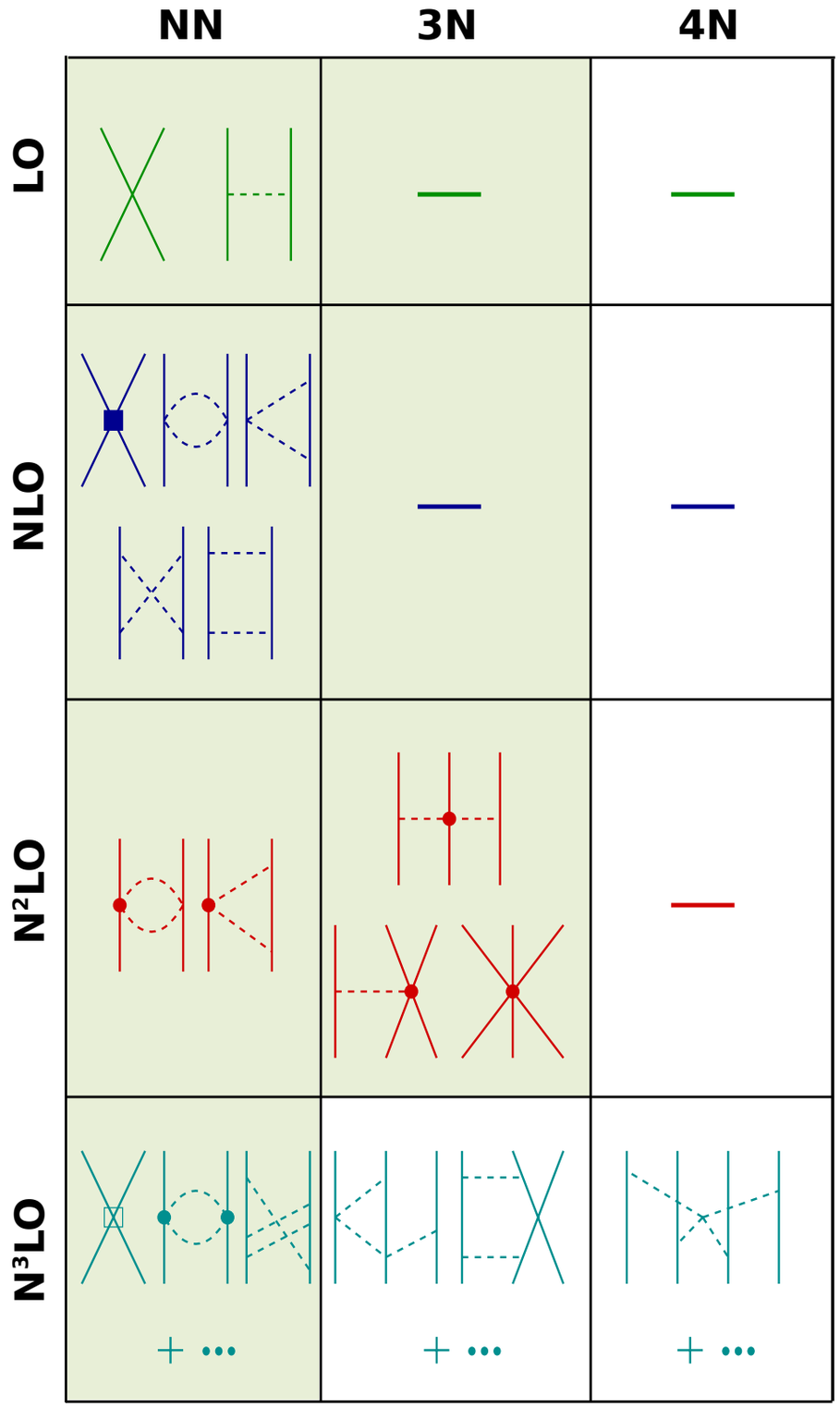
Successfully applied with NN interactions
Now: Inclusion of 3N Forces

Ingredients from Three-Body Technology

The Chiral NN+3N Hamiltonian

Weinberg, van Kolck, Machleidt, Entem, Meißner, Epelbaum, Krebs, Bernard, Skibinski, Golak...

- Hierarchy of consistent nuclear NN, 3N,... forces (and currents)
- NN interaction @ N^{3}LO (Λ=500MeV)
  [Entem, Machleidt, Phys.Rev C **68**, 041001(R) (2003)]
- Standard Hamiltonian
  - 3N interaction @ N^{2}LO(Λ_{3N}=500MeV)
    - LECs c_D, c_E fitted to β-decay halflife & binding energy of ^{3}H
      [Gazit et.al., Phys.Rev.Lett. **103**, 102502 (2009)]
- Reduced-Cutoff Hamiltonian
  - 3N interaction @ N^{2}LO(Λ_{3N}=400MeV)
    - c_D=-0.2, c_E fitted to ^{4}He



The Similarity Renormalization Group

Wegner, Glazek, Wilson, Perry, Bogner, Furnstahl, Hergert, Calci, Langhammer, Roth, Jurgenson, Navrátil,...

...yields an evolved
Hamiltonian with improved convergence properties in
many-body calculations

- Unitary transformation of Hamiltonian $H_\alpha = U_\alpha^\dagger H U_\alpha$

Different SRG-Evolved Hamiltonians

- **NN+3N-induced**: start with NN initial Hamiltonian and keep two- and three-body terms
- **NN+3N-full**: start with NN+3N initial Hamiltonian and keep two- and three-body terms

3N Forces in the NCSM/RGM and NCSMC

How to Access Targets Heavier than ^{4}He

G. Hupin, J. Langhammer et al. ----- Phys. Rev C **88** 054622 (2013)

S. Quaglioni and P. Navrátil ----- Phys. Rev. Lett. **101**, 092501 (2008)

P. Navrátil, R. Roth and S. Quaglioni ----- Phys. Rev. C **82**, 034609 (2010)

S. Quaglioni, P. Navrátil, G. Hupin, J. Langhammer et al. ----- Few-Body Syst. DOI 10.1007/s00601-012-0505-0 (2012)

S. Quaglioni, P. Navrátil, R. Roth, W. Horiuchi ----- J.Phys.Conf.Ser. 402 (2012)

General Approach of NCSM/RGM

Wildermuth,Thompson, Tang, ..., Navrátil, Quaglioni, Roth, Hupin, Langhammer,...

- Represent $H|\psi^{J\pi T}\rangle = E|\psi^{J\pi T}\rangle$ using the **over-complete basis**

$$|\psi^{J\pi T}\rangle = \sum_\nu \int \mathrm{d}r r^2 \frac{g_\nu^{J\pi T}(r)}{r} \mathcal{A}_\nu |\phi_{\nu r}^{J\pi T}\rangle \qquad g_\nu^{J\pi T}(r) \text{ unknown}$$

with the binary-cluster channel states

$$|\phi^{J\pi T}\rangle = \left\{ |\Phi^{(A-a)}\rangle |\Phi^{(a)}\rangle \right\}^{J\pi T} \frac{\delta(r-r_{A-a,a})}{r r_{A-a,a}}$$

NCSM delivers $|\Phi^{(A-a)}\rangle$ and $|\Phi^{(a)}\rangle$

- Solve **generalized eigenvalue** problem

$$\sum_\nu \int \mathrm{d}r r^2 \left[\mathcal{H}_{\nu,\nu'}^{J\pi T}(r',r) - E\mathcal{N}_{\nu,\nu'}^{J\pi T}(r,r')\right] \frac{g_{\nu r}^{J\pi T}}{r} = 0$$

Hamiltonian kernel $\langle \phi_{\nu' r'}^{J\pi T} | \mathcal{A}_{\nu'} H \mathcal{A}_\nu | \phi_{\nu r}^{J\pi T} \rangle \propto \langle \Phi^{(A-1)} | a^\dagger a^\dagger a^\dagger a a a | \Phi^{(A-1)} \rangle$

for single-nucleon projectiles and including 3N forces

Handling of Three-Body Density

Langhammer, Roth, Navrátil

Computing uncoupled densities on-the-fly

⇒ Key to access heavier targets than ^{4}He

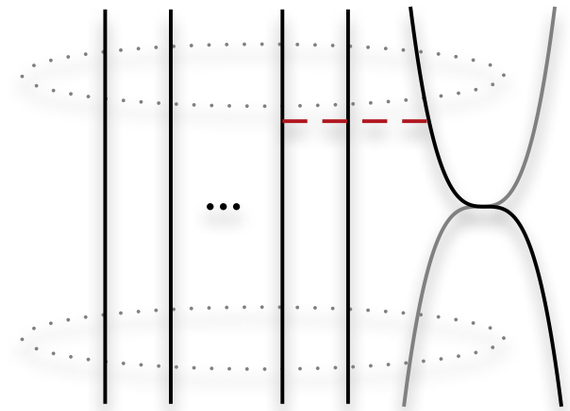
$$\sum_{jj'} \sum_{M_1 m_j} \sum_{M_{T_1} m_t} \sum_{M'_1 m'_j} \sum_{M'_{T_1} m'_t} \frac{1}{12}(-1)^{I_1+I'_1+2J+j+j'} \begin{Bmatrix} I_1 & \frac{1}{2} & s \\ l & J & j \end{Bmatrix} \begin{Bmatrix} I'_1 & \frac{1}{2} & s' \\ l' & J & j' \end{Bmatrix}$$

$$\begin{pmatrix} I_1 & j \\ M_1 & m_j \end{pmatrix} \begin{pmatrix} J \\ M_J \end{pmatrix} \begin{pmatrix} T_1 & \frac{1}{2} \\ M_{T_1} & m_t \end{pmatrix} \begin{pmatrix} T \\ M_T \end{pmatrix} \begin{pmatrix} I'_1 & j' \\ M'_1 & m'_j \end{pmatrix} \begin{pmatrix} J \\ M'_J \end{pmatrix} \begin{pmatrix} T'_1 & \frac{1}{2} \\ M'_{T_1} & m'_t \end{pmatrix} \begin{pmatrix} T \\ M'_T \end{pmatrix}$$

$$\sum_{\beta_{A-3}} \sum_{\beta_{A-2}} \sum_{\beta'_{A-3}} \sum_{\beta'_{A-2}} \sum_{\beta'_{A-1}}$$

$${}_a\langle \beta_{A-3}\beta_{A-2} n l j' m'_j \tfrac{1}{2} m'_t | V_{3N} | \beta'_{A-3}\beta'_{A-2}\beta'_{A-1}\rangle_a$$

$$\langle \Phi'^{(A-1)} I'_1 M'_1 T'_1 M'_{T_1} | a^\dagger_{n l j m_j \frac{1}{2} m_t} a^\dagger_{\beta_{A-2}} a^\dagger_{\beta_{A-3}} a_{\beta'_{A-3}} a_{\beta'_{A-2}} a_{\beta'_{A-1}} | \Phi^{(A-1)} I_1 M_1 T_1 M_{T_1}\rangle$$



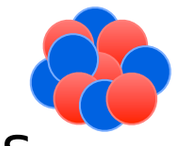
- Use $|\Phi^{(A-1)} I_1 M_1 T_1 M_{T_1}\rangle = \sum_i c_i \, |\mathrm{SD}\rangle_i$
- Very-well suited for parallel computation

NCSMC Formalism with 3N Forces

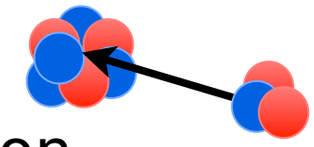
- Representing $H|\psi^{J\pi T}\rangle = E|\psi^{J\pi T}\rangle$ using the **over-complete basis**

$$|\Psi^{J\pi T}\rangle = \sum_{\lambda} c_{\lambda} |\Psi_A E_{\lambda} J^{\pi} T\rangle + \sum_{\nu} \int \mathrm{d}r\, r^2 \frac{\chi_{\nu}(r)}{r} |\xi^{J\pi T}_{\nu r}\rangle$$

Expansion in A-body (IT-)NCSM eigenstates



Identical to the NCSM/RGM expansion



leads to the NCSMC equations

$$\begin{pmatrix} H_{\mathrm{NCSM}} & h \\ h & \mathscr{H} \end{pmatrix} \begin{pmatrix} c \\ \chi(r)/r \end{pmatrix} = E \begin{pmatrix} \mathbb{1} & g \\ g & \end{pmatrix} \Big($$

Accessing targets beyond ^{4}He using uncoupled densities

3N forces contribute in

H_{NCSM}

Covered by (IT-)NCSM

h

Given by $\langle \Psi_A E_{\lambda'} J^{\pi} T | \hat{H} | \xi^{J\pi T}_{\nu r} \rangle$

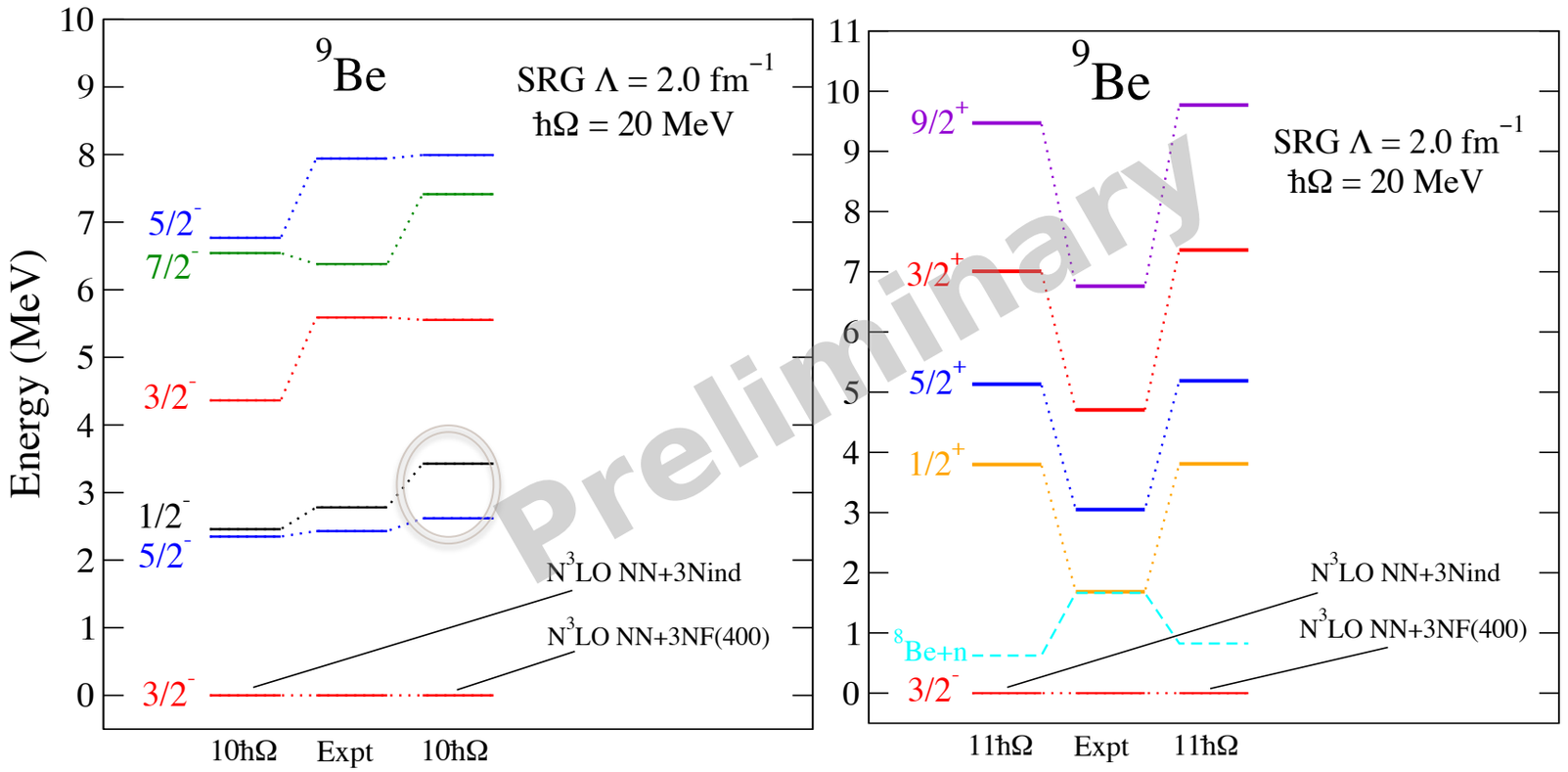
$\mathscr{H}$

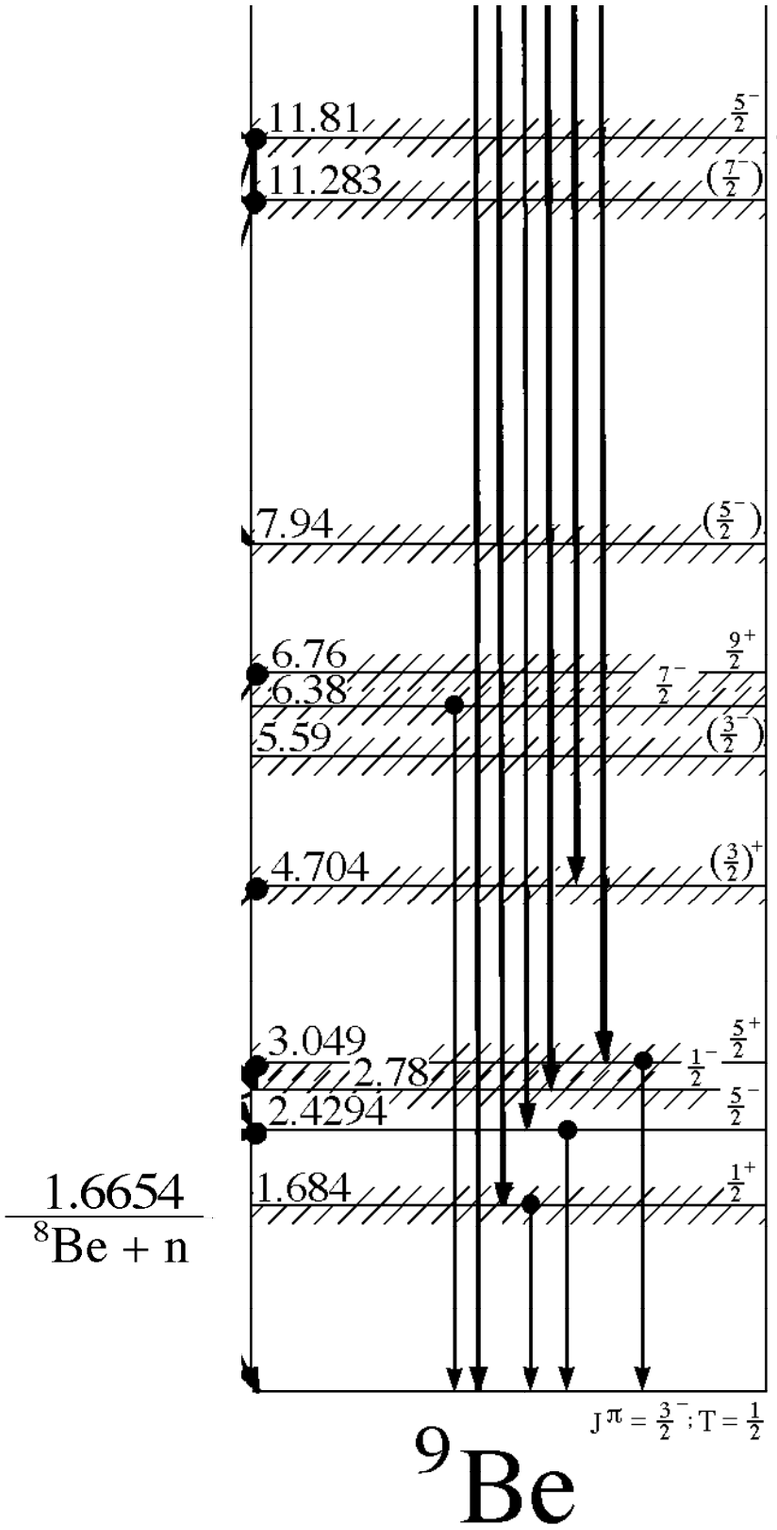
Contains the NCSM/RGM Hamiltonian kernel

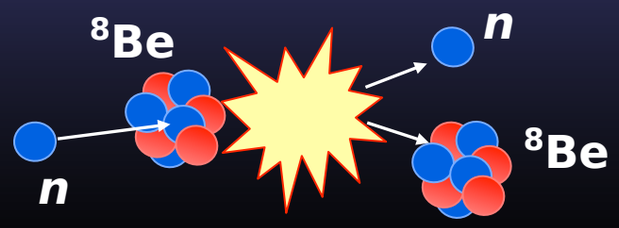
Ab-initio Description of ^{9}Be via NCSMC

Collaboration with Petr Navrátil

- All excited states are resonances
- Study the impact of the continuum by investigating neutron-^{8}Be scattering
- NCSM with 3N forces reveals large discrepancies compared to experiment

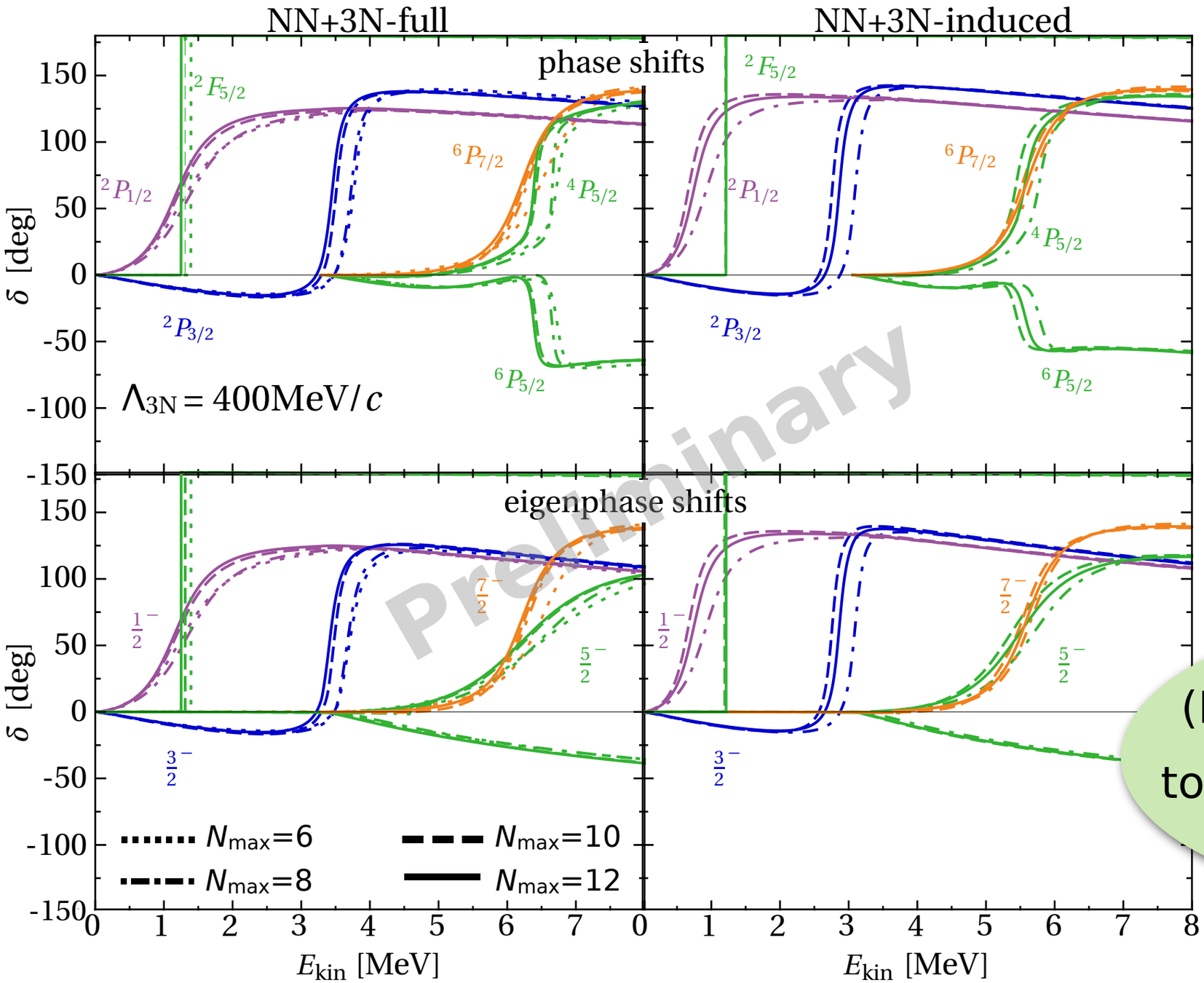






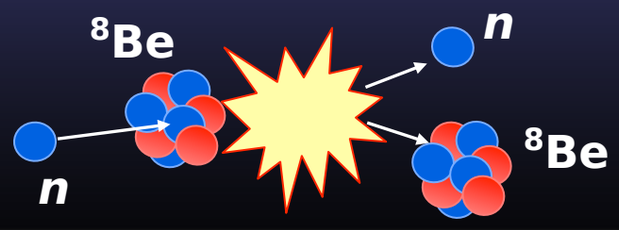
N_{max} convergence of phase shifts

Collaboration with Petr Navrátil



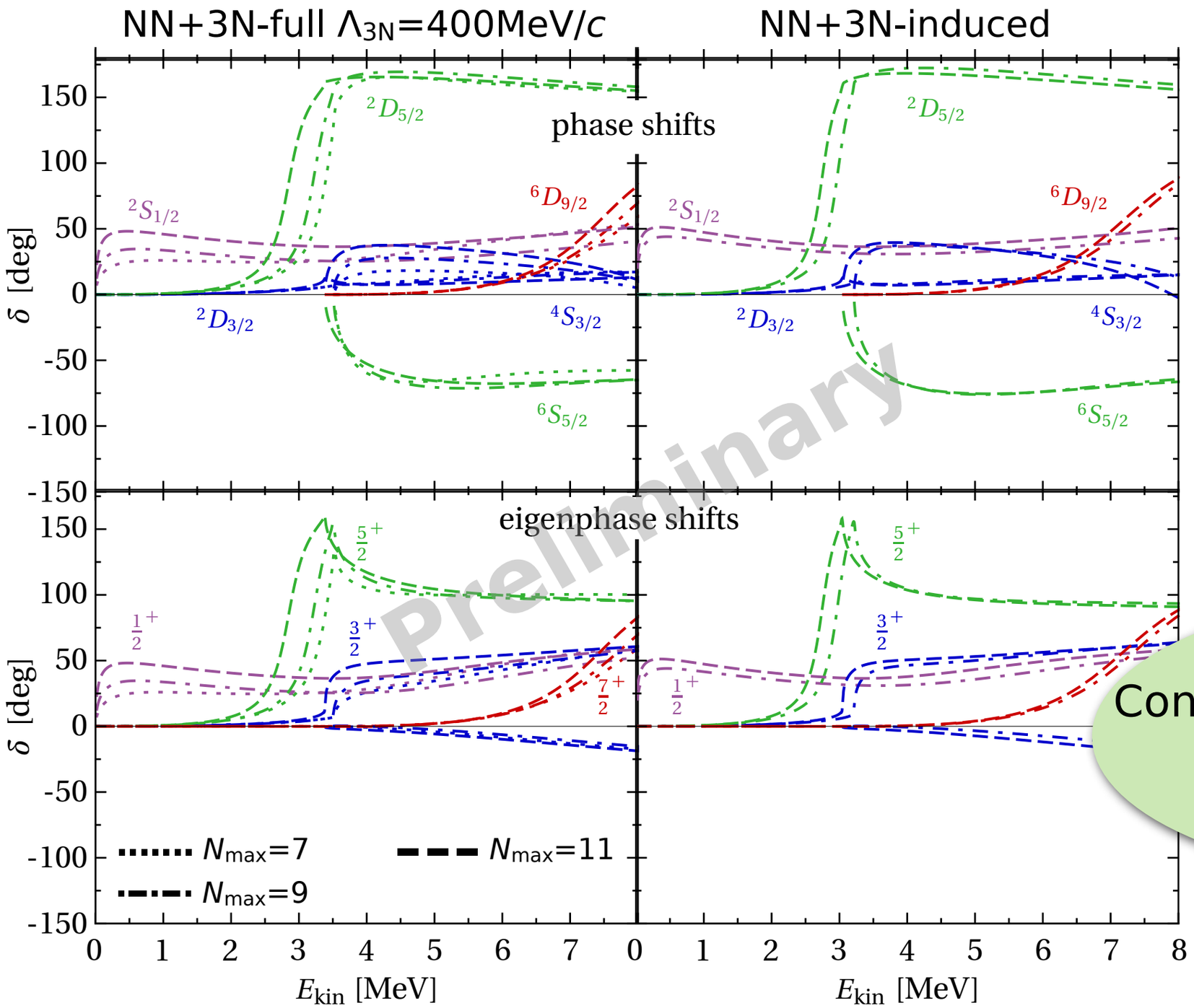
- Included states of ^{8}Be: 0^+ ground state & 2^+ excited state
- Included states of ^{9}Be: 6 negative parity states

(Eigen)phase shifts close to convergence at N_{max}=12



N_{max} convergence of phase shifts

Collaboration with Petr Navrátil

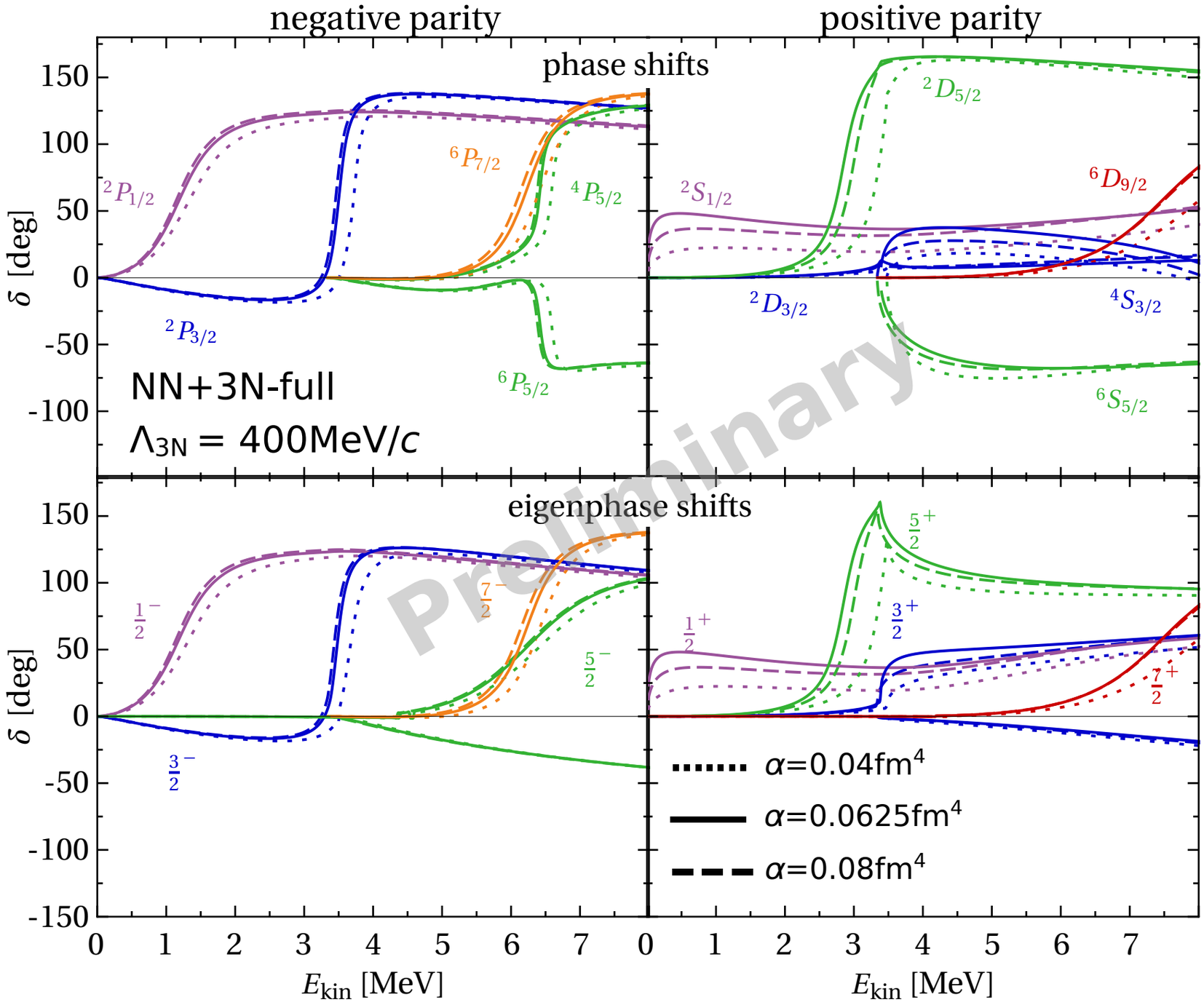


- Included states of ^{8}Be:
  0^+ ground state &
  2^+ excited state
- Included states of ^{9}Be:
  4 positive parity states

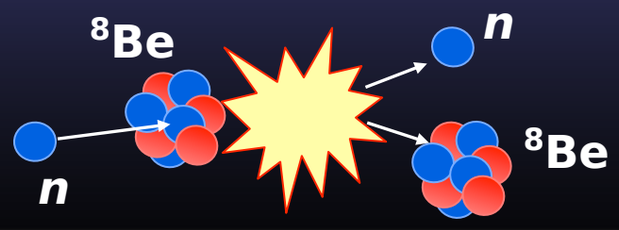
Convergence not yet reached,
N_{max}=13 underway

SRG Parameter Dependence

Collaboration with Petr Navrátil

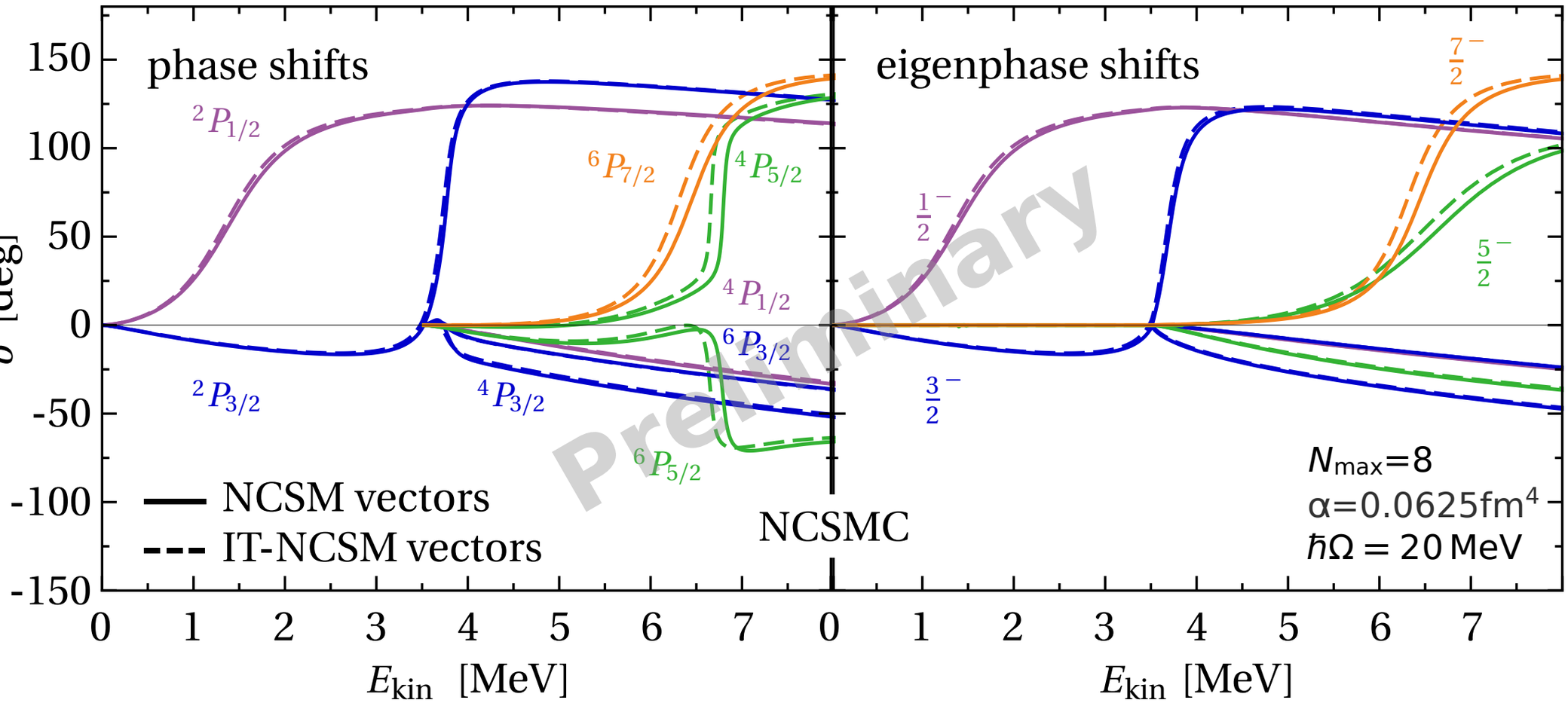


- Included states of ^{8}Be:
  0^+ ground state &
  2^+ excited state
- Included states of ^{9}Be:
  6 negative parity states
  4 positive parity states
- Negative parity:
  α=0.0625 and 0.08fm^4
  phase shifts practically
  identical
- Positive parity:
  Larger deviations due to
  lack of N_{max} convergence

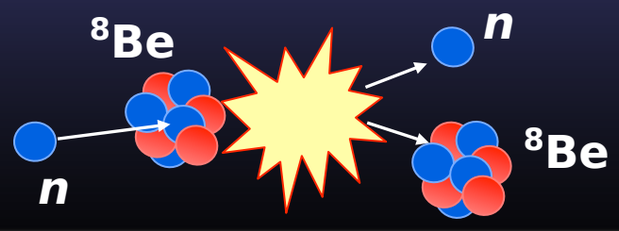


Monitoring the IT-NCSM Inputs

Collaboration with Petr Navrátil

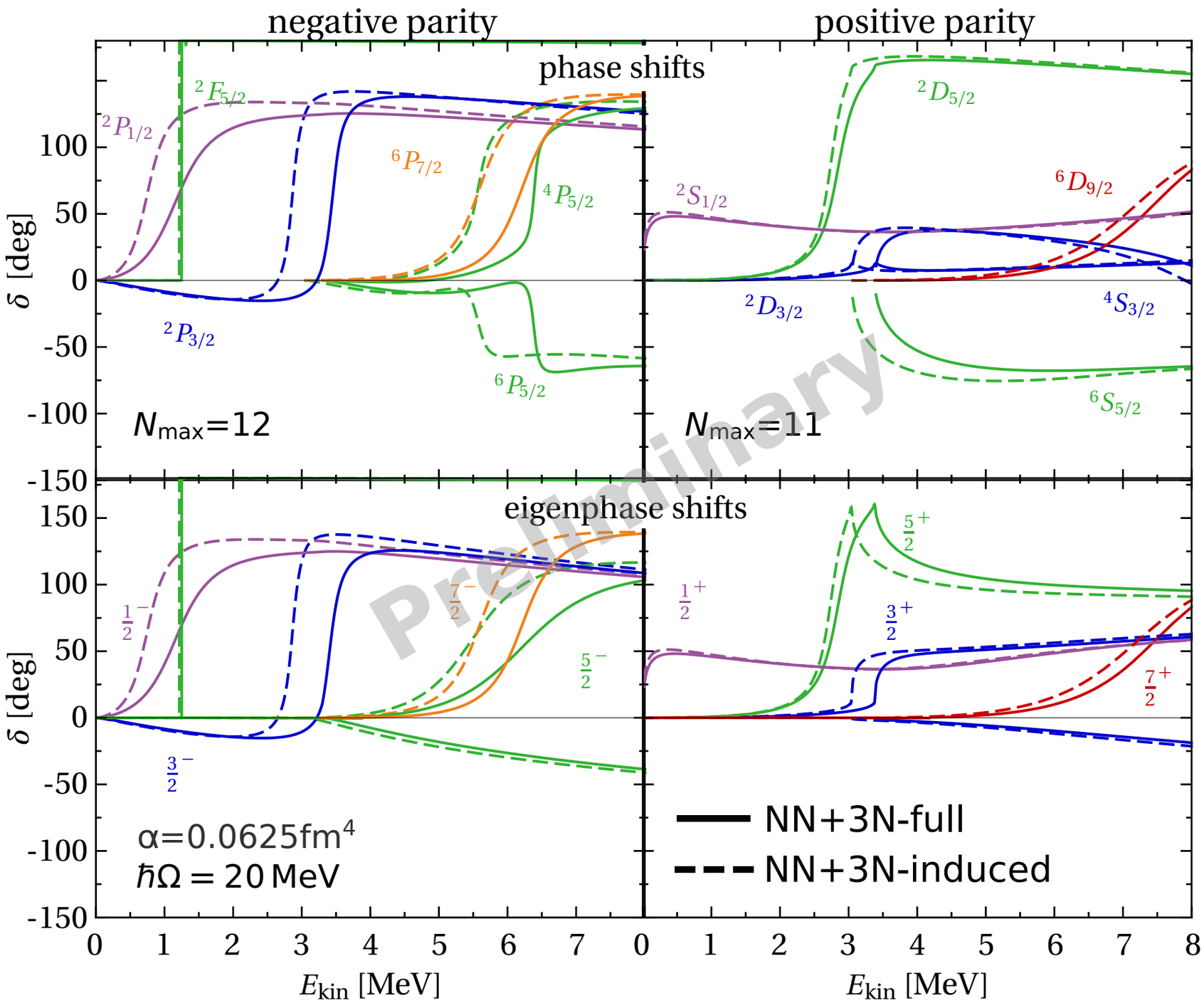


- Eigenphase shifts obtained with NCSM or IT-NCSM eigenvectors are in very good agreement
- Small deviations in the $5/2^-$ and $7/2^-$ eigenphase shifts



3N Force Effects on Phase Shifts

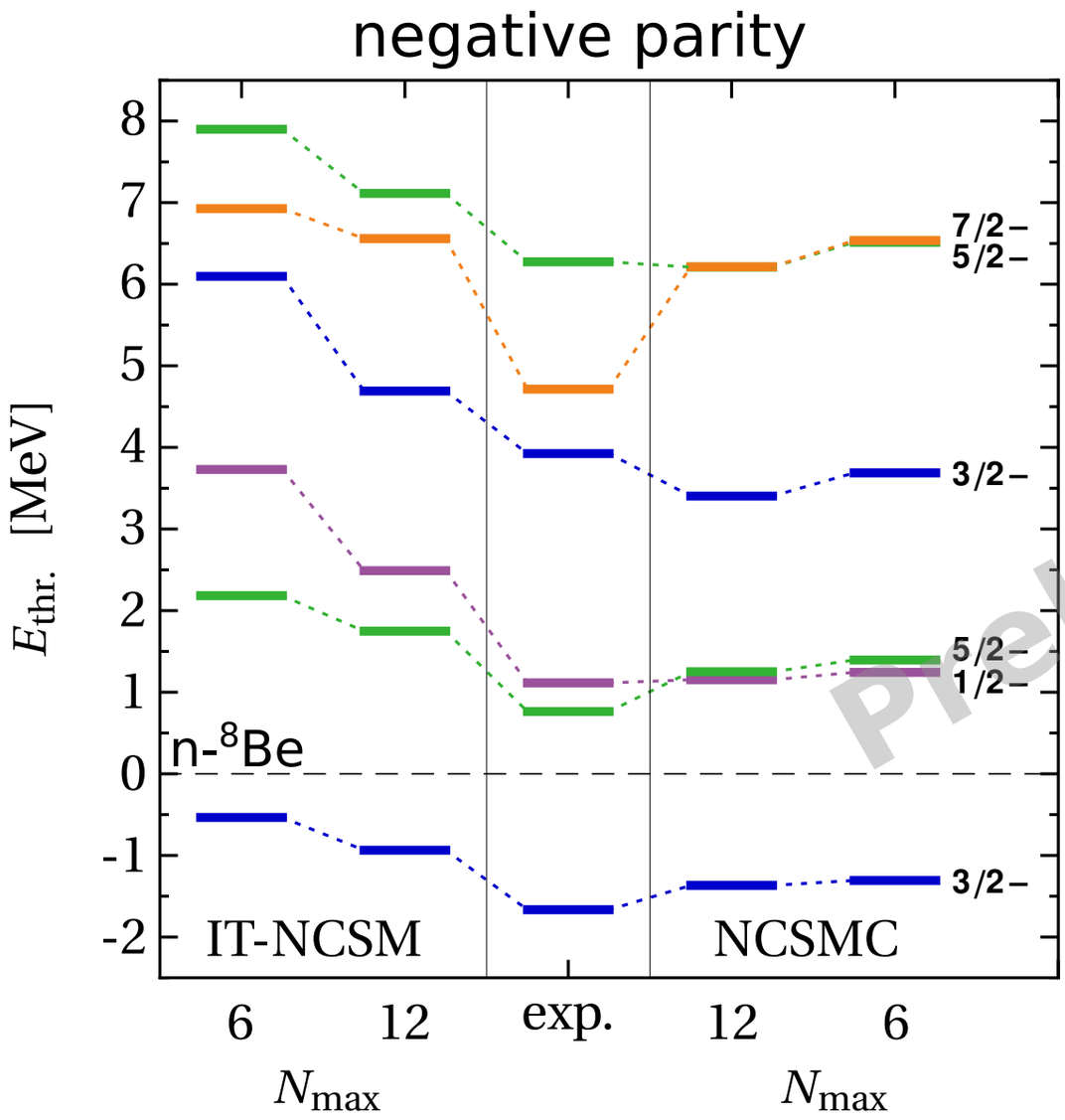
Collaboration with Petr Navrátil

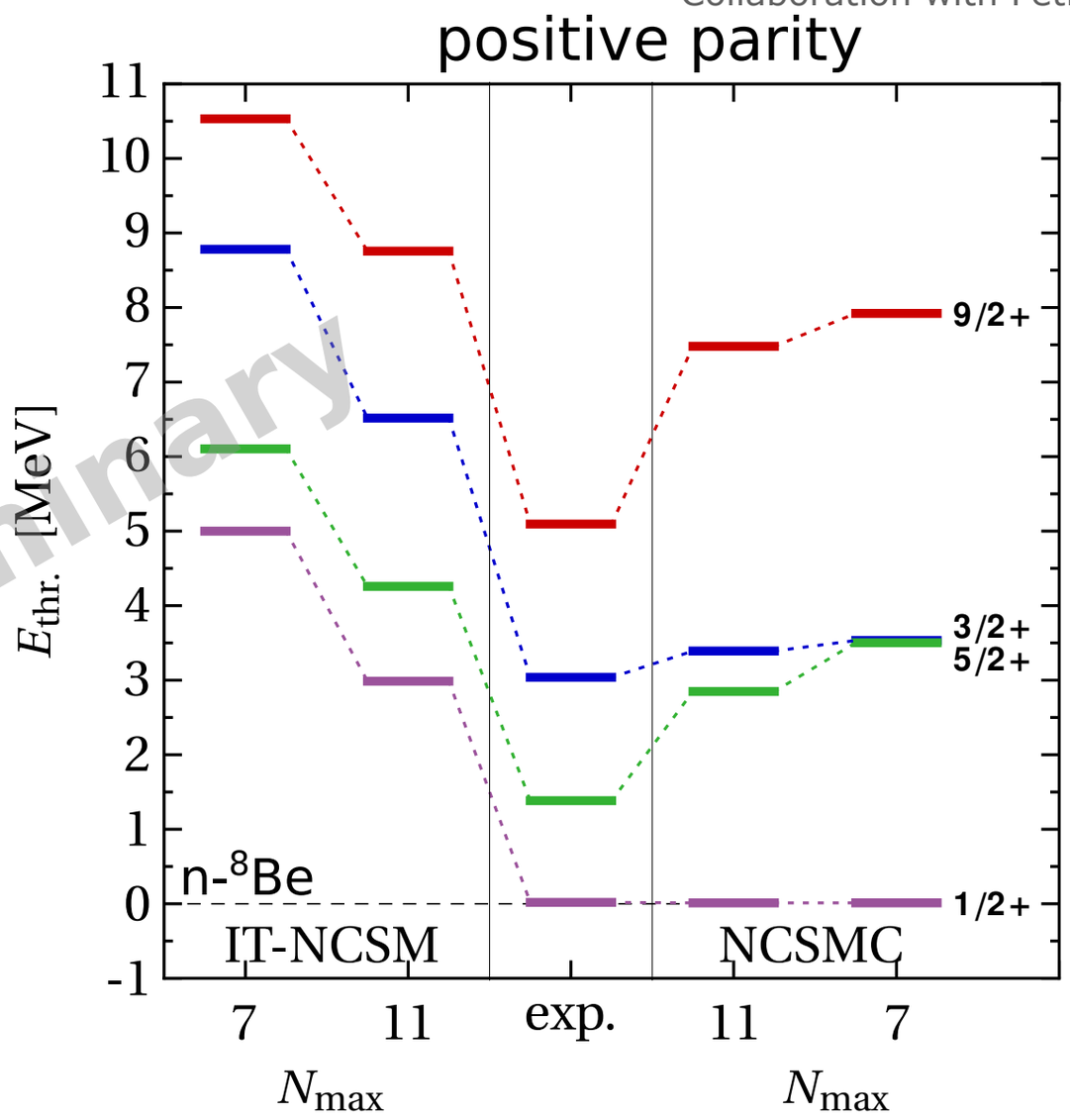


- Typically chiral 3N force shifts resonance positions to larger energies
- Narrow $^2F_{5/2}$ phase shift not affected at all
- Overall larger effects of the chiral 3N forces for negative parity
- Only a minor change in the $^2S_{1/2}$ resonance

^{9}Be Energy Levels: NCSM vs. NCSMC

Collaboration with Petr Navrátil

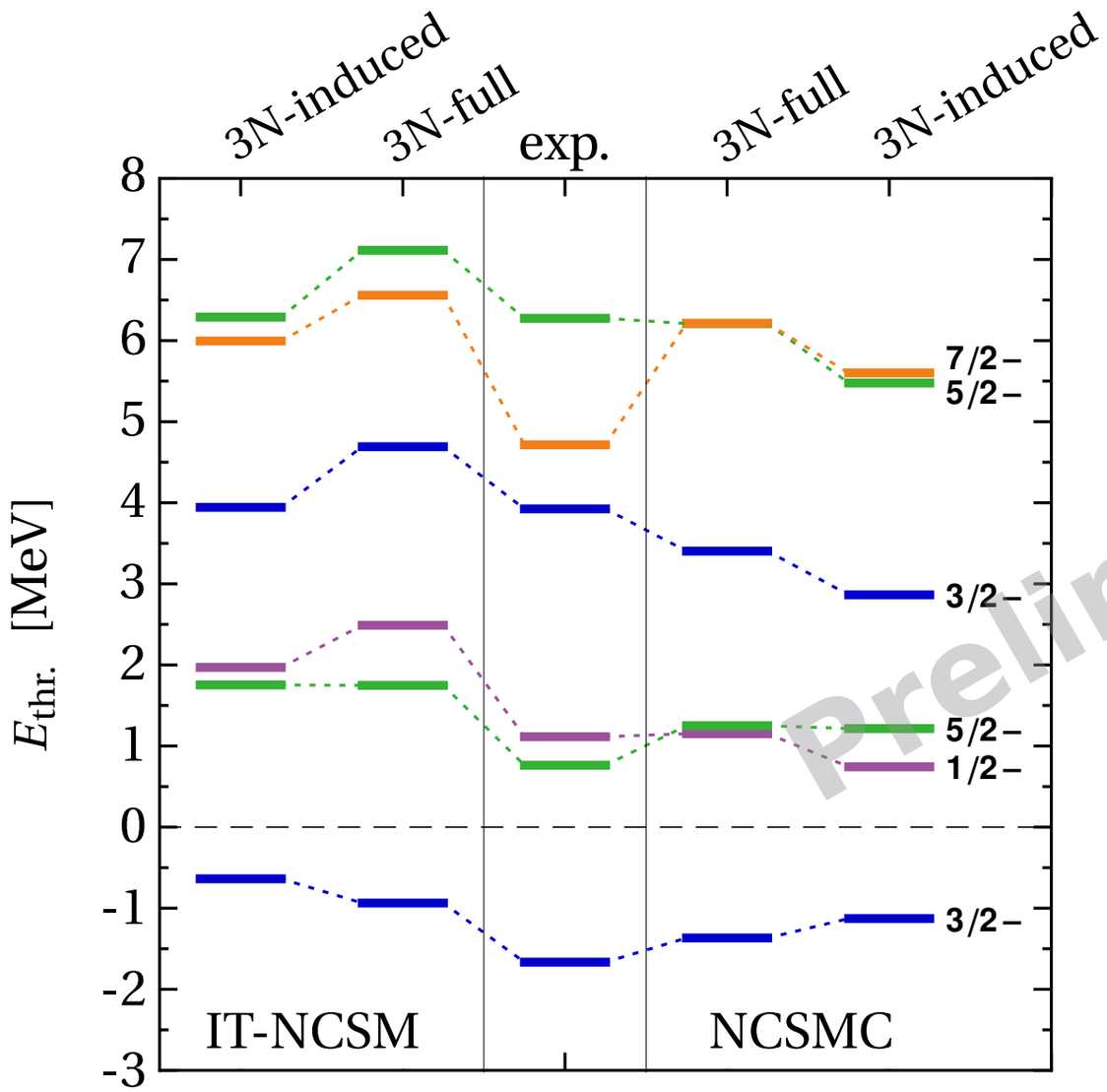


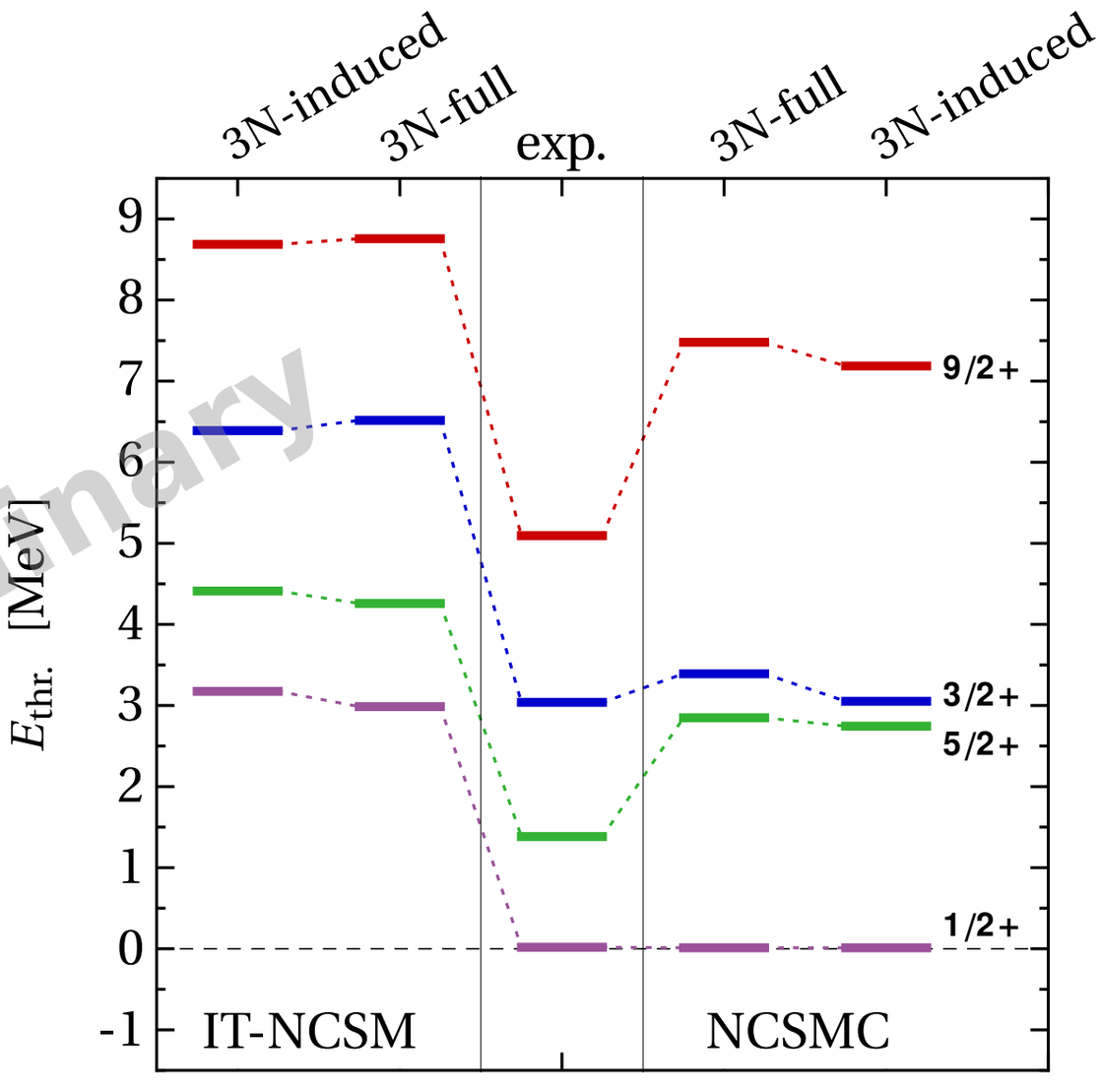


- Significant contributions from the continuum degrees of freedom
- Excellent agreement with experiment for $1/2^-$ & second $5/2^-$ as well as the $1/2^+$ and $3/2^+$ states
- NCSMC seems to be well-converged already at moderate $N_{\max}$

^{9}Be Energy Levels: NCSM vs. NCSMC

Collaboration with Petr Navrátil





- Treatment of continuum important for conclusions about 3N interactions
- First $5/2^-$ insensitive to the chiral 3N interaction
- $7/2^-$ resonance → interaction problem?

p-^{10}C scattering: Structure of ^{11}N resonances

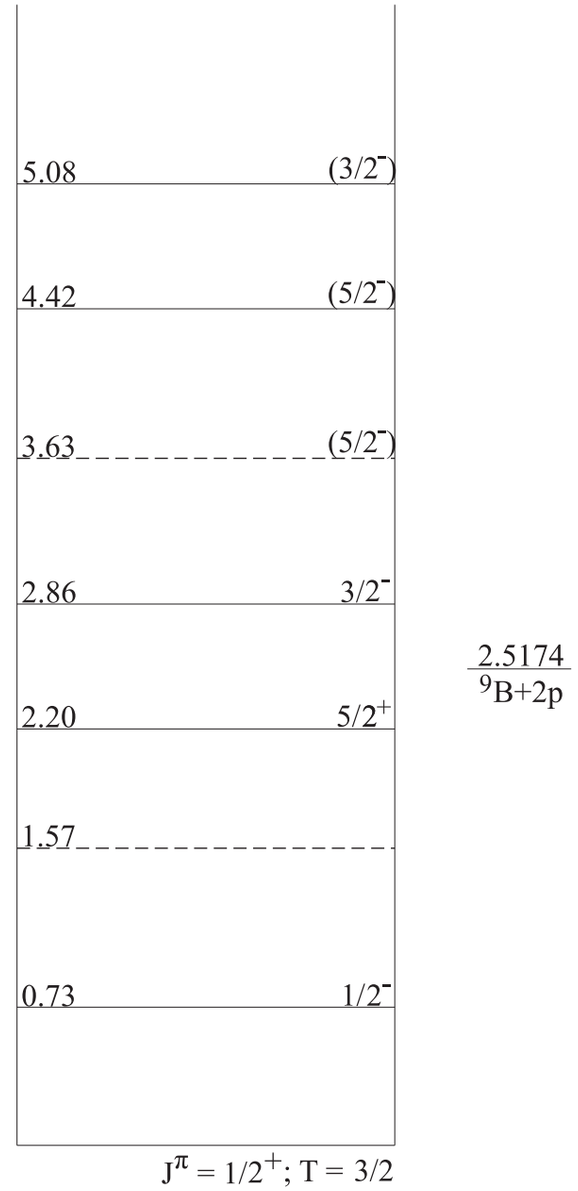
Collaboration with Petr Navrátil

- So far limited structural information about ^{11}N

- Experiment at ISAC TRIUMF
  - First ^{10}C beam at TRIUMF
  - Measurements at $E_{CM} \sim 4.1$ & 4.4 MeV

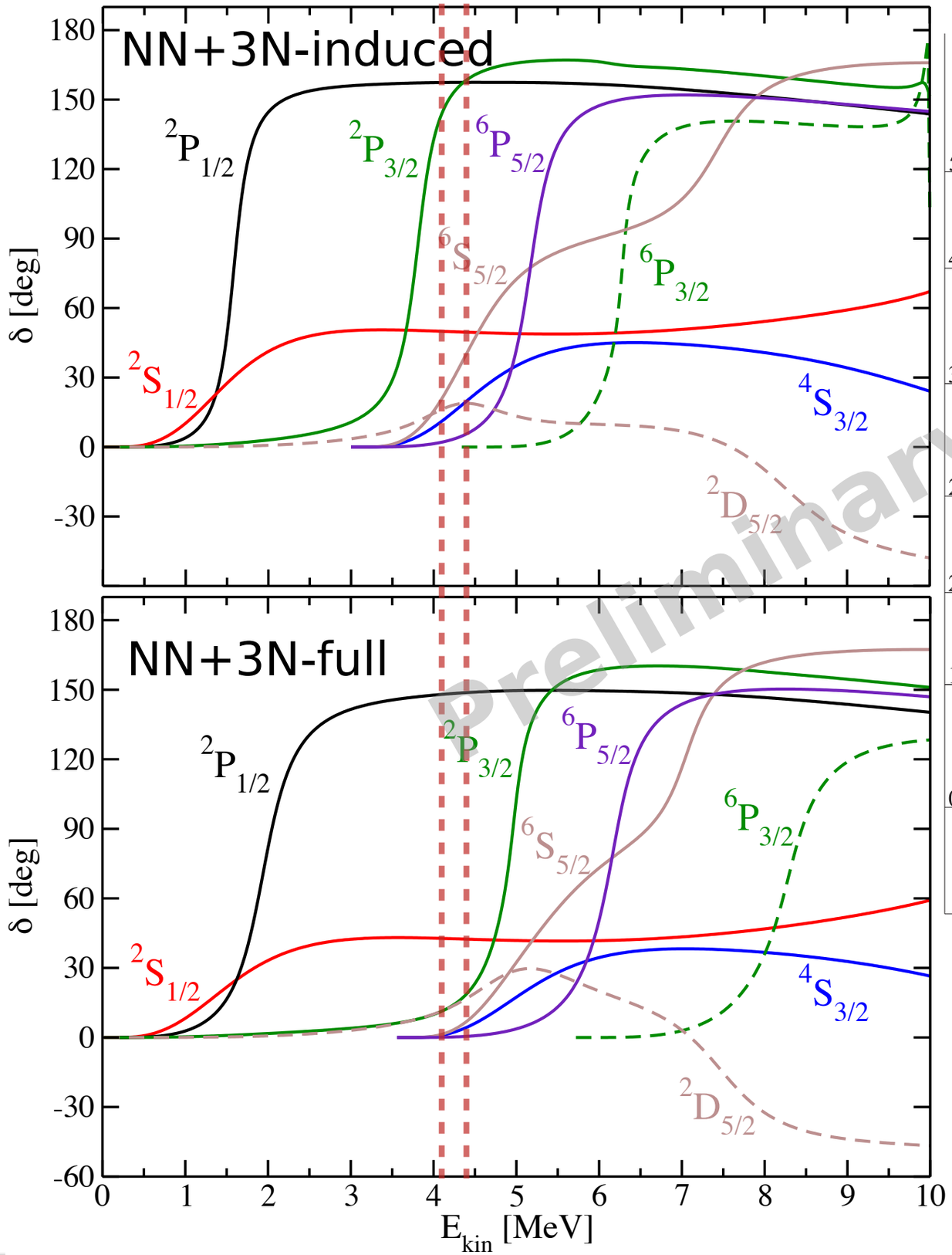
- Supported by NCSMC calculations
  - Including chiral 3N interaction
  - p-^{10}C(0^+,2^+,2^+)

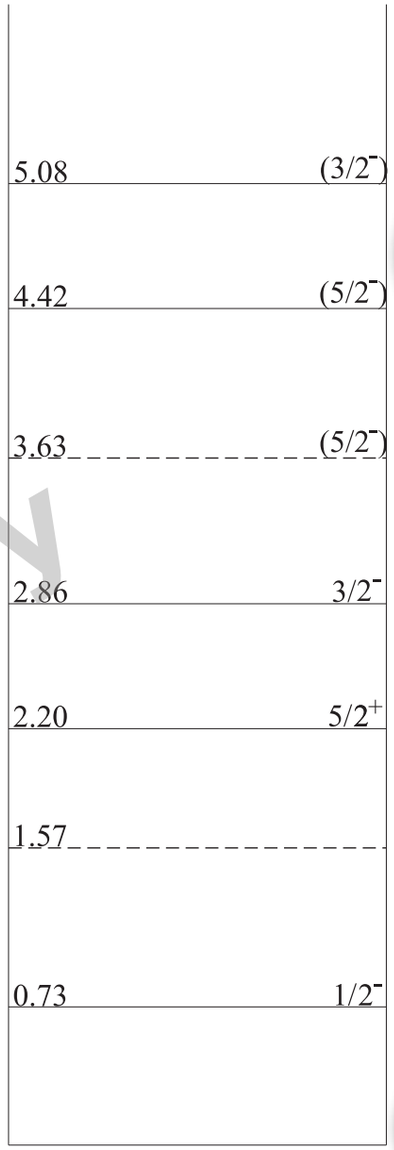
    + ^{11}N (6 ×π=-1 and 3 ×π=1 states)
  - So far N_{max}=7...



p-^{10}C scattering: Structure of ^{11}N resonances

Collaboration with Petr Navrátil





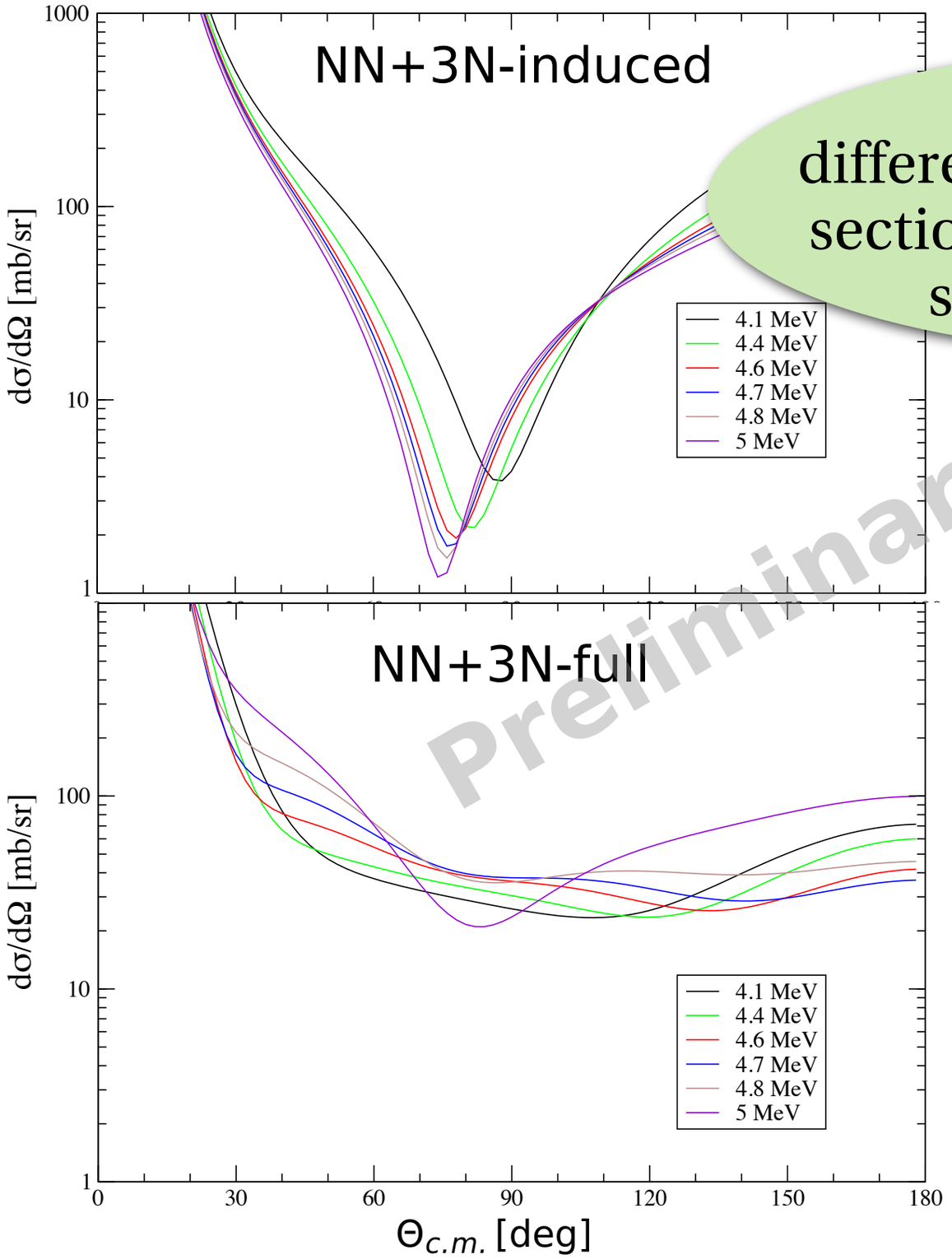
Chiral 3N shifts all resonances to larger energies

Improved agreement with experiment for ^{2}P$_{1/2}$ & ^{2}P$_{3/2}$ phase shifts

Energy shift for ^{6}P$_{5/2}$ partial-wave phase shift too strong

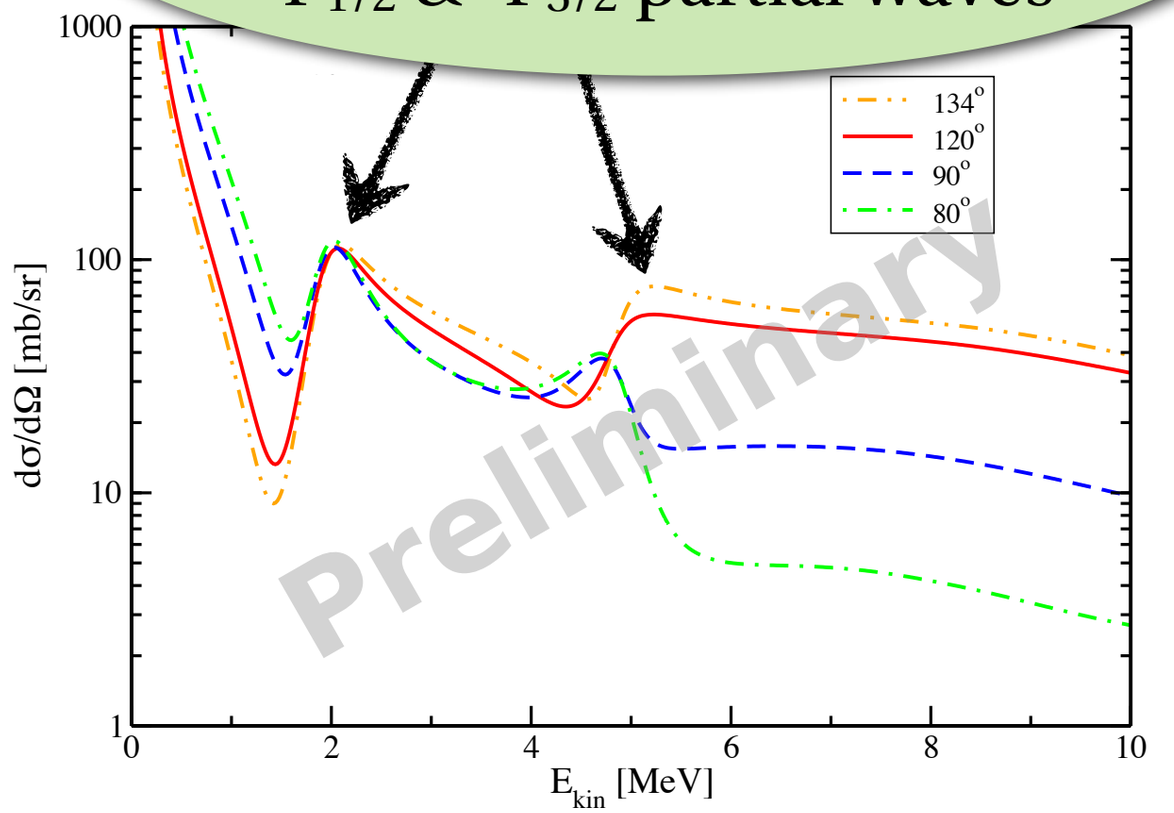
p-^{10}C scattering: Structure of ^{11}N resonances

Collaboration with Petr Navrátil



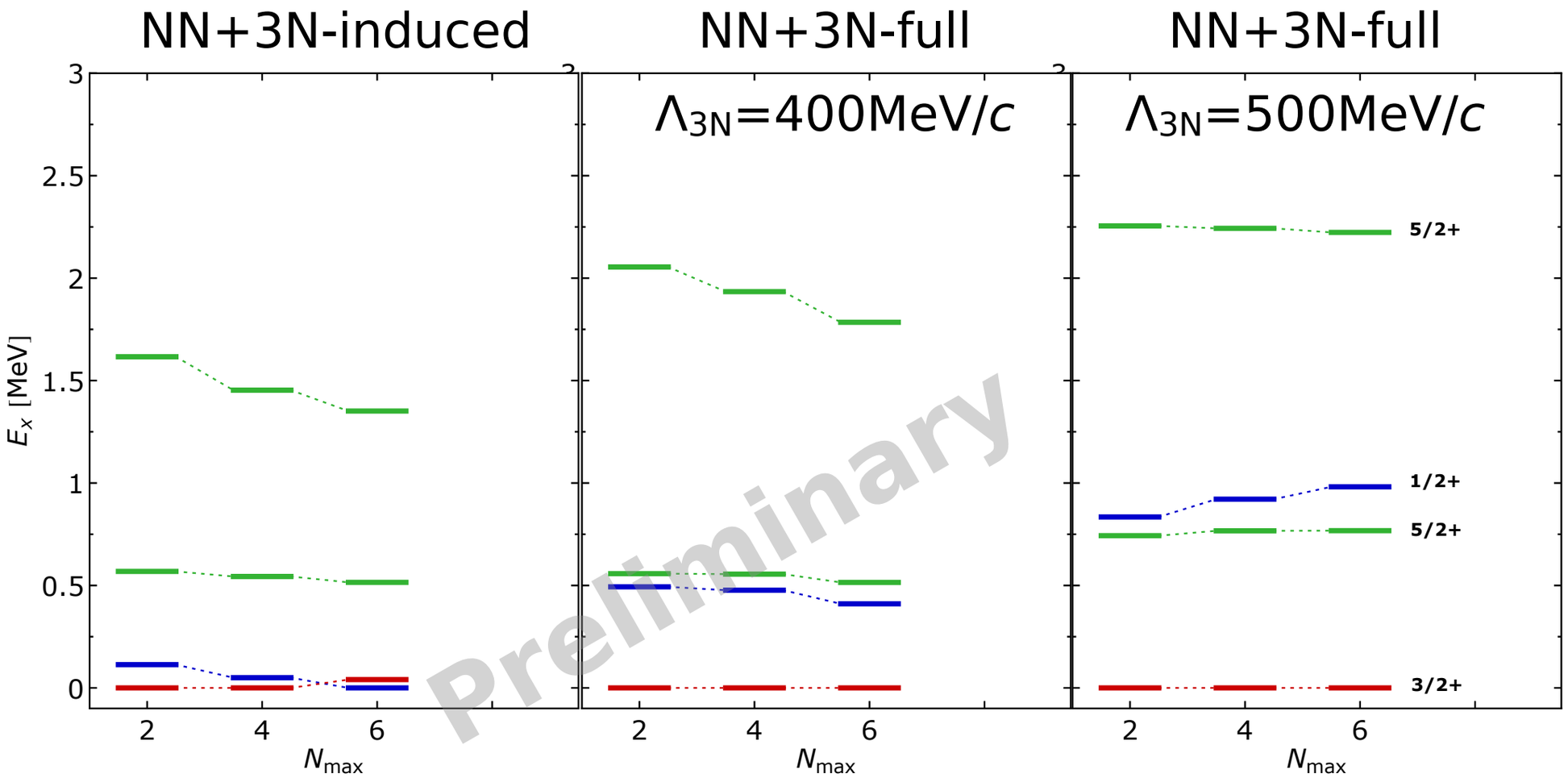
Significant differences in differential cross sections in the experimentally studied energy range

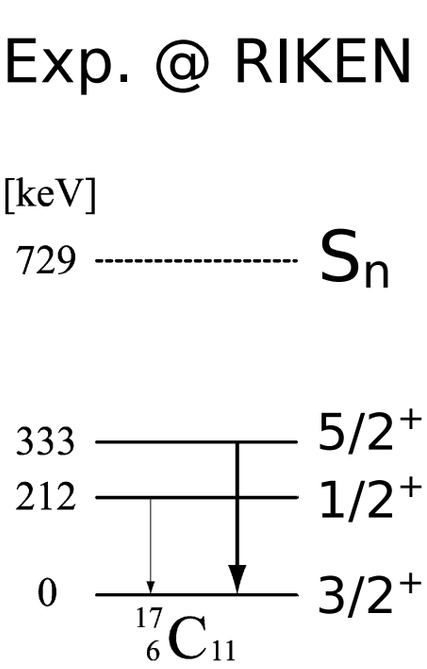
Strong variations in angular distributions connected to $^2P_{1/2}$ & $^2P_{3/2}$ partial waves



Impact of the Continuum on ^{17}C Energy Levels

Collaboration with Petr Navrátil

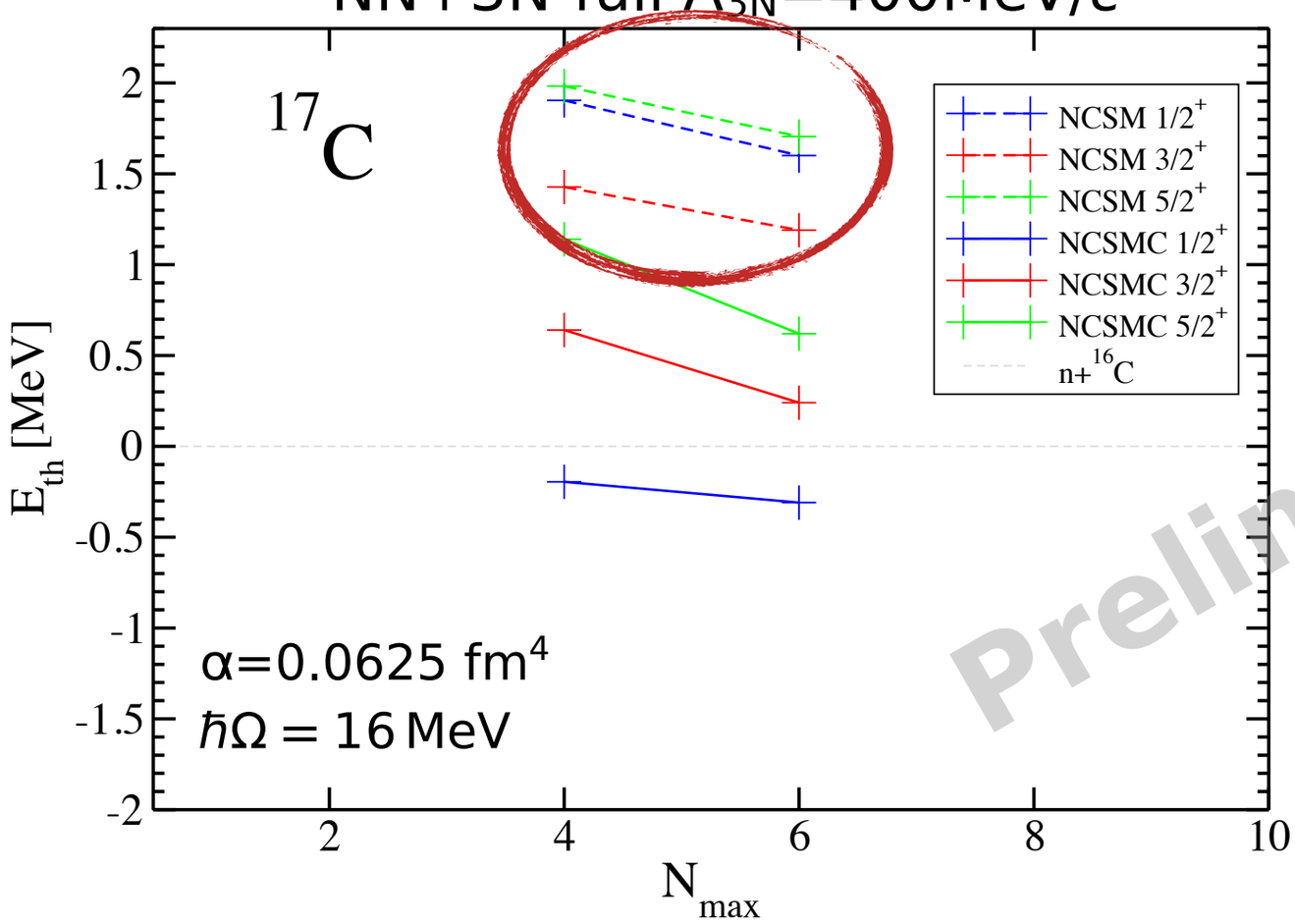


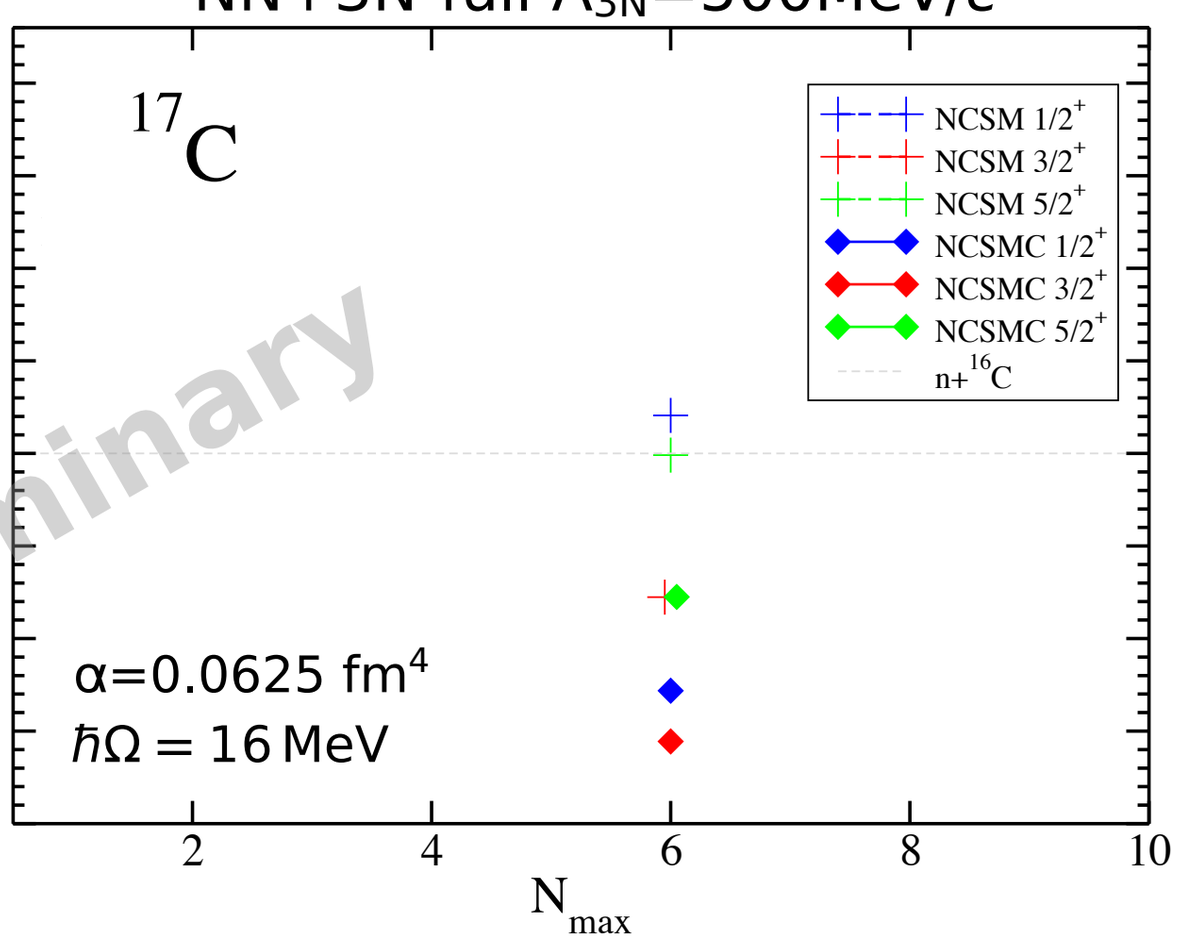


- NCSM calculations: strong sensitivity to 3N interaction
  - Even the ordering of the states is affected
- **What is the impact of the continuum?**

Impact of the Continuum on ^{17}C Energy Levels

Collaboration with Petr Navrátil





- NCSM yields the states unbound
- NCSMC binds the $1/2^+$ state and lowers the $3/2^+$ & $5/2^+$ states
- Convergence w.r.t N_{max} not yet reached

- NCSM binds $3/2^+$ & $5/2^+$ states
- NCSMC (over)binds all three states and provides their correct ordering
- N_{max} = 8 underway...

Conclusions

Conclusions

Nuclear structure and reactions accessible
with full 3N treatment via the
No-Core Shell Model with Continuum

- **Inclusion of 3N forces** challenging but **completed** for single- and two-nucleon projectiles
- New computational scheme $\Longrightarrow$ **heavier targets accessible**
- Promising results for n-^{8}Be, p-^{10}C and n-^{17}C
- Proper **treatment of continuum vital** for validation of chiral 3N interactions

Epilogue

- **thanks to my group & collaborators**
  - S. Binder, A. Calci, E. Gebrerufael, H. Krutsch, S. Reinhard, R. Roth, M. Schmidt, S. Schulz, C. Stumpf, A. Tichai, R. Trippel, R. Wirth
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    GSI Helmholtzzentrum
  - P. Papakonstantinou
    IPN
  - [illegible]

Thanks for your attention!

Computing Time

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NERSC

CSC Center for Scientific Computing Frankfurt

SFB 634 (Theory, Quantum Chaos, Nuclear Astrophys., Experiment Accelerator Development, Graduate College "Physics and Technique of Accelerators", Theory Accelerator Development, Nuclear Structure, Fundamental Experiments)

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